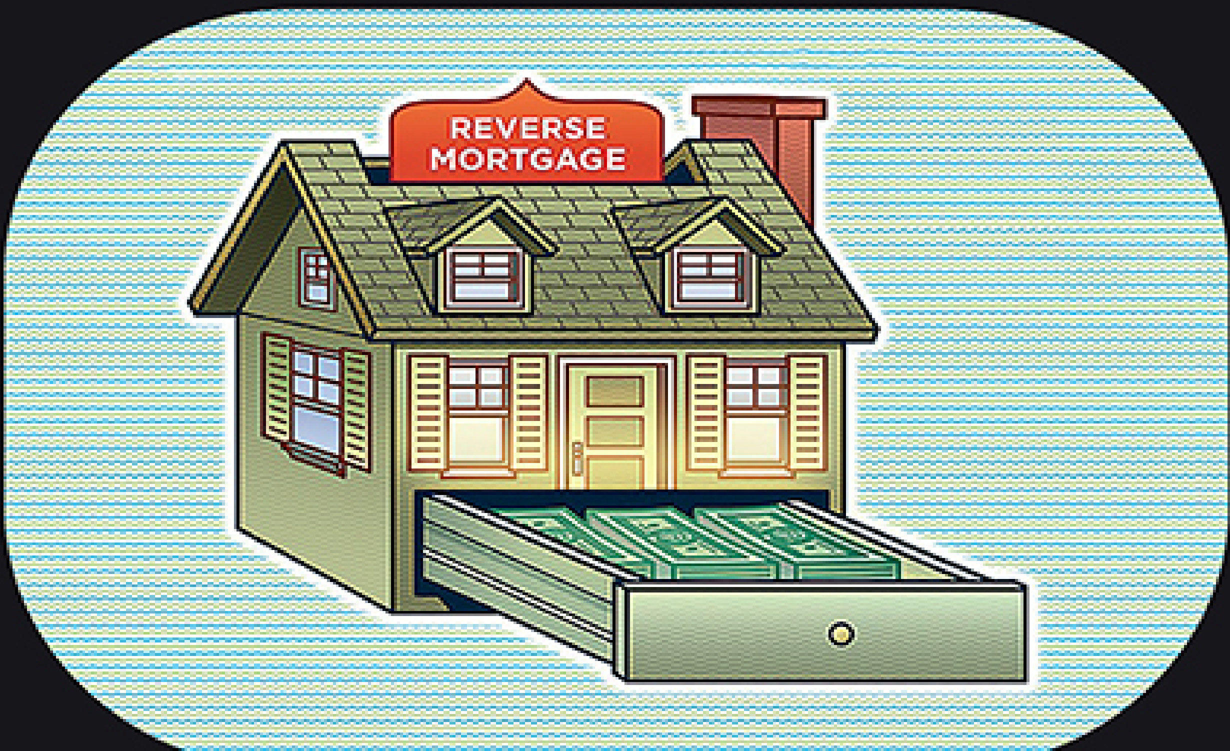
Radu S. Tunaru and Enoch B. Quaye

EQUITY RELEASE FINANCE



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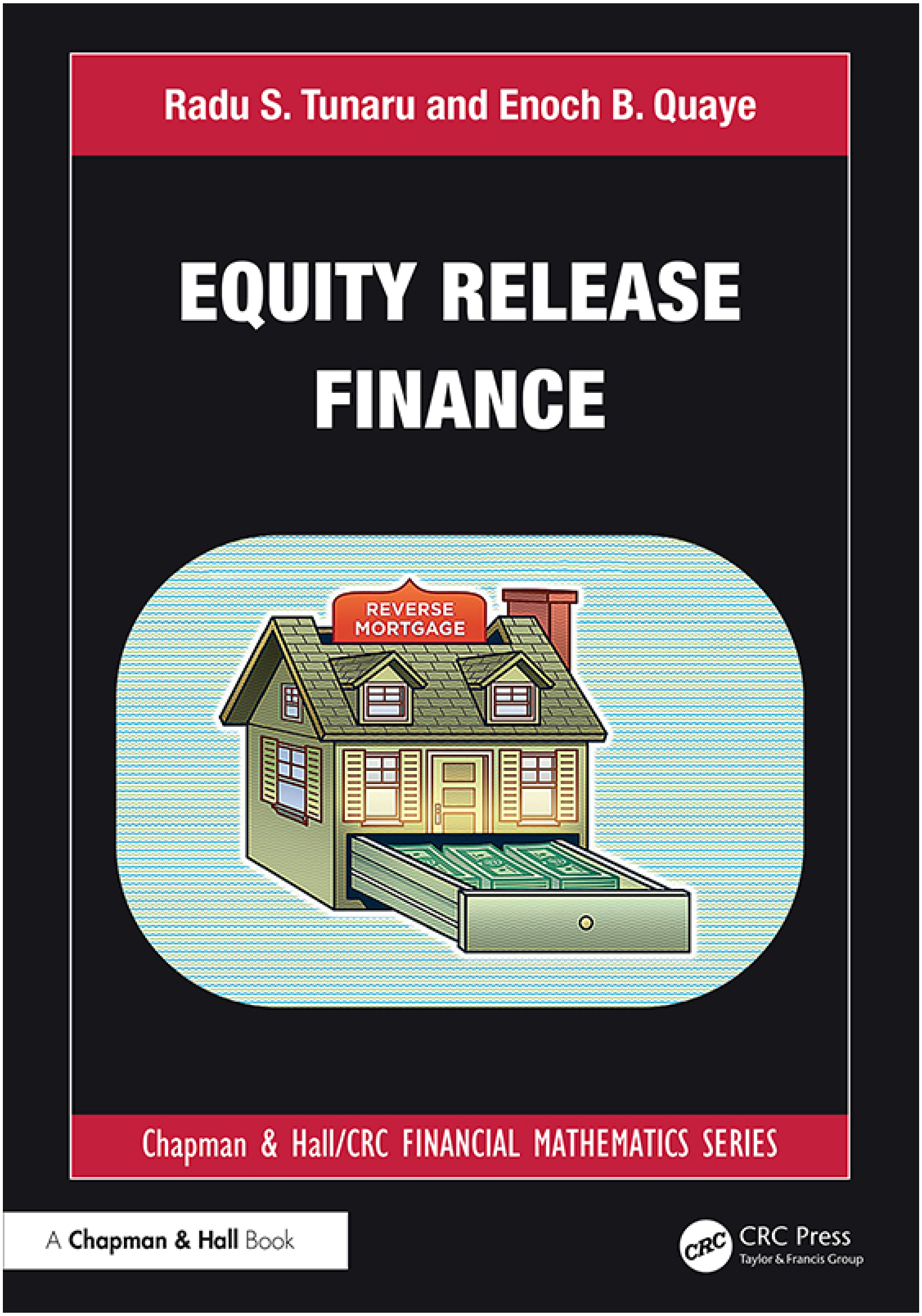
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EQUITY RELEASE FINANCE
REVERSE MORTGAGE
Chapman & Hall/CRC FINANCIAL MATHEMATICS SERIES
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Equity Release Finance

Equity Release Finance provides a self-contained introduction to the principles underpinning Equity Release Products (ERPs). The approach of the book, while academically robust, is also accessible and engaging, with a focus on practical examples and applications. It will provide an invaluable resource to a diverse audience, including Master's degree and PhD students in finance, management science, actuarial science, and risk management. It will also be of service to academics and industry professionals.

Features

- A strong practical focus makes this an effective reference for industry professionals in the field of insurance, pensions, derivatives, and risk management
- Replete with pedagogical features, the book can be used to teach Master's and/or PhD level graduate students
- The ideas presented in this book should be of interest to policy makers and regulators interested in developing a viable stable market, and also to academics, opening many avenues for further research in this area.

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Equity Release Finance

Radu S. Tunaru and Enoch B. Quaye

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To my mother, thanking her for a lifetime of efforts to look after me, from close and from distance, and in loving memory of my father, who worked for more than forty years but did not live to get a year of pension after.

To Mum and Dad, for your support, efforts, and love which have been my foundation. To my wife and children, whose love, encouragement, and belief anchor and strengthen me. To Tsentsulu, for your support and camaraderie, inspiring me always. Thank you for being my pillars of strength.

Contents

List of Figures

List of Tables

Foreword

Drawing on Professor Tunaru's extensive knowledge and experience as both a practitioner at Merrill Lynch and academic researcher in a number of UK universities, alongside Dr. Quaye's doctoral studies, the two authors have created an outstanding book on equity release finance.

Research on equity release finance is vital in tackling the financial challenges posed by aging populations, particularly the issues of insufficient retirement savings and rising long-term care expenses. By unlocking home equity for retirees, these financial products offer a potential remedy for those who are asset-rich but cash-poor. The in-depth research presented in this book helps clarify the complexities and intricacies of equity release, enabling consumers to make well-informed decisions and supporting policymakers in developing regulations that protect and consumers while encouraging market growth as it plays a vital role in boosting financial literacy and knowledge on equity release finance.

The book is organised into multiple chapters that systematically examine the complex world of equity release products (ERPs). It begins with an introduction to the concept and significance of ERPs in financial markets, emphasising on their role in improving individual cash flows and addressing long-term care and pension shortfalls. The book then explores the types of ERPs, their global demand, and regulatory landscapes across different regions, including the US, UK, EU, and Asia. Subsequent chapters cover the details of risk management, valuation techniques, and the modelling of non-negative equity guarantees (NNEG), employing sophisticated mathematical finance and econometric models. The authors also examine the rental yield computational challenges, and they address the complexities and pitfalls in ERP regulation and risk modelling, providing a thorough framework for understanding and managing these products. Furthermore, portfolio analysis and the diversification effect of a portfolio of loans is examined providing an additional insight in comparison with the calculations on a loan-by-loan basis. The authors are highly capable of utilising advanced mathematical finance and econometric models, demonstrating great pedagogical skill in explaining their application in real life problems. The book on equity release finance contribute significantly to risk management strategies and best practices in this socially crucial sector of the financial industry. The authors provide valuable insights and explain the relevant models in detail, helping academics, regulators and professionals navigate the unique risks associated with these products, such as longevity risk and house price volatility risks.

This work addresses a significant gap in the current literature on ERPs by offering a comprehensive examination of both micro and macro aspects of this emerging asset class. The authors explore the details of risk management, regulatory issues, and the practical use of non-negative equity guarantees, which are often missed or not fully covered in other books. By introducing new concepts and techniques, particularly in the valuation and portfolio management of equity release mortgages, the book makes a significant contribution, offering valuable insights for both practitioners and academics, and setting the stage for further research in this rapidly evolving field. This book can serve as a valuable resource and reference for professionals in finance, insurance, and actuarial science as well as academics and students, bridging the gap between theoretical concepts and practical applications. It can be useful to InsurTech, FinTech, RegTech entrepreneurs and journalists and nonspecialists interested in this area. The book combines practical and theoretical knowledge, making it valuable for a wide audience while also exploring more advanced techniques in modelling and risk management of ERPs.

Spyridon Vrontos
Professor of Actuarial Science and
Head of School of Mathematics, Statistics and Actuarial Science,
University of Essex

Preface

This book emerged from a long-standing interest of the first author on equity release mortgages, first encountered as a quant structurer while working for Merrill Lynch in London, as an asset class and as a part of the doctoral studies of the second author. The various problems studied under the umbrella of equity release finance financial and risk management calculus are in fact subordinated to a greater theme introduced by Robert Shiller, the Nobel laureate for Economics in 2013, which is Finance and Economics for a Greater Society, see Shiller (1993).

It is with this general aim to utilize financial products to improve life in our society that we thought that a monograph such as this would be welcome. One would hope that the vast experience accumulated collectively by researchers and practitioners in financial markets would indicate clearly what to do for this less known but expanding asset class.

A second motivation is rooted in our experience as independent researchers commissioned by the Institute and Faculty of Actuaries and the Association of British Insurers in 2018 to investigate best practices and hidden pitfalls on non-negative-equity guarantees that are a main component of the equity release markets in the UK. In particular, the interactions with practitioners and also with regulators convinced us that more research is clearly needed in this area, research not only for academic sake but also with a view to improve practices in the future, see Tunaru and Quaye (2019).

Last but not least, the first author of this book worked with equity release mortgages while he was a vice-president at Merrill Lynch in Structured Finance EMEA. There was very little written at the time on the matter and it was there seeing it in the real-world when the idea of a book that coagulates the most important ideas came about. I will be grateful to my managers for assigning me to work on this small asset class and to my colleagues at the bank who explained many of the intricate details associated with equity release mortgages. Some of the early ideas were described in Tunaru (2017).

Equity release finance are financial instruments combining many interesting facets in finance. While the risk drivers behind them are perhaps clearly understood, not many parties involved with these instruments fully acknowledge how high those risks can manifest themselves. It is well known now for example that Jeanne Louise Calment who was born on 21 February 1875, lived in France for 122 years and 164 days. Andre-Francois Raffray, had an equity release type of contract with then 90-year-old Jeanne Calment. However, madame Calment lived another 32 years to become the world's oldest woman, outliving Raffray himself who by the time of his death had paid her the equivalent of £140,000 for no benefit whatever. With advances in medical science, is it possible that several decades from now many people would live to that age? Is one simple example like this sufficient to establish a proxy of maximum living age to 120? If yes, that would add another twenty years to all calculations for equity release mortgages.

The sets of skills of both authors cover actuarial science, financial modelling, financial engineering, programming, risk management, statistics and the list is by no means closed. For this book in particular we wanted to bring into focus the model fitting exercise that is quite often neglected by practitioners but also by academics researching on finance and insurance topics.

It is difficult to cover all angles possible on this emerging interesting asset class. We tried to be as comprehensive as possible but also as explicit as possible in our discussion. When we came to the end of our writing journey we realised that this is not the end of our research in this area, and if anything, there are potential many lines of investigation that were barely scratched. Our aim was not to produce an encyclopedic work but to focus the attention on several important issues that are pertinent to equity release mortgages.

One of our best research outputs, briefly described in this book, is the paper in the Insurance: Mathematics and Economics journal, jointly with Alex Badescu, to whom we are very grateful for helping us to push the boundaries on NNEG calculation to another level. We hope that the ideas outlined in that paper may become standard practice in the not very distant future and we thank the editors, associate editors and reviewers for helping us publish our paper in the best journal on insurance.

This book would not have been possible without the support and acceptance from our families. The first author thanks his wife Diana for all the evenings and weekends taken away from family to dedicate to the cause of this book, hoping that it may help those in need of help. He also thanks his daughters for helping him overcome the darker periods that one may have to pass from time to time. The second author expresses profound gratitude to God, acknowledging His abundant and unwavering provisions that guided every step of this journey. He also thanks his wife Rhoda for her support and constant encouragement. Her love and partnership has made all

the difference. He thanks his sons for their immense patience and the joy they bring into his life each day.

Acknowledgements: First and foremost we would like to thank Chapman & Hall for accepting our proposal for a book focused on equity release finance. In particular we would like to thank Fraser Callum and Mansi Kabra from Chapman & Hall who were very patient with our delays and allowed us to construct the book the way we wanted, while also giving us some very good advices as well. We would like to thank the participants in the workshop on the NNEG Valuation organised on 28 January 2019 at Kent Business School in Canterbury, in particular Vali Asimit, Pradip Tapar, Daniel Alai, Jaideep Oberoi, for their views and comments. We would also like to thank Thomas Kenny from the Just Group, Owen Griffiths from the Legal & General, Andrew Dobinson from the Scottish Widows, for useful discussion on the relevance of parameter inputs in the market place Special thanks also to Charles Golding, Sam Gunter, Steven Findlay, Chris Hursey and Guy Thomas for their helpful suggestions during our journey for this project. We are also grateful to comments and suggestions from participants at LIFE conference in Dublin, November 2019. The results and conclusions of this research reflect only our academic views. All errors in this book are obviously ours.

Author Biographies

Radu S. Tunaru is Professor of Finance and Risk Management at the University of Reading, where he is the current Head of the ICMA Centre in the Henley Business School. Radu is on the Associate Editor Board of the Journal of Derivatives and the Journal of Portfolio Management, and he worked for Bank of Montreal and Merrill Lynch in London in their Structured Finance divisions. He published in many journals in Finance and Risk Management including Journal of Financial Economics, Journal of Economic Perspectives, Journal of Corporate Finance, Journal of Banking and Finance, Journal of Real Estate Finance and Economics, European Journal of Operational Research, Insurance: Mathematics and Economics and Review of Economics and Statistics, among others. He co-authored several papers with Robert Shiller, the Nobel laureate for Economics in 2013 and he conducted consulting for various banks, hedge-funds, start-ups and executive training houses including Lloyds and London Financial Studies. His expertise covers Derivatives Markets, Model Risk in Finance and Insurance, Credit Risk, Real-Estate Finance, Real-Options and Empirical International Finance.

Enoch B. Quaye is a lecturer in finance with the Department of Accounting and Finance, University of Bristol Business School. His Ph.D. degree specialty is in Financial Risk Management and Asset Pricing. Before then, he was lecturer in finance at the University of Kent Business School, University of Kent, and also lecturer in Statistics and Actuarial Science with the Department of Statistics and Actuarial Science, University of Ghana. He has over 10 years of industry experience as an actuary and risk management professional and has consulted extensively for corporate organizations through advice provisioning and training workshops. Enoch's research spans financial risk management, derivatives, asset pricing, and pensions.

Glossary

ARMA-EGARCH:
autoregressive moving average exponential autoregressive conditional heteroskedasticity model.

ARMA-EGARCH-rn:
refers to the ARMA-EGARCH model under the risk-neutral measure.

ARMA-EGARCH-rw:
refers to the ARMA-EGARCH model under the real-world measure.

Black76:
the Black-Scholes variant referring to pricing European options on futures.

conditional Esscher martingale measure:
it is a risk-neutral measure constructed in a specific way when we are in incomplete market.

DNS:
the dynamic Nelson-Siegel interest rate model.

Equity Release Finance:
Financial products designed to allow investors release equity locked up in a specified underlying asset.

EONIA:
the Euro Overnight Index Average.

ERC:

Equity Release Council.

ERC:

early repayment charges.

ERM:

equity release mortgages.

ERMRP:

excess ERP premium.

ERP:

equity release products.

EVT:

effective value test.

FCA:

Financial Conduct Authority in the United Kingdon.

FHA:

Federal Housing Administration in the United States.

FSA:

foreclosed sale adjustment.

***g*:**

continuously compounded rental yield.

Γ:

notation for LTV.

GBM:

geometric Brownian motion.

GBM-rn:

geometric Brownian motion under risk neutral measure.

GDP:

gross domestic product.

GMM:

generalised method of moments.

HECM:

home equity conversion mortgage.

HMD:

human mortality database.

Householder:

According the US Census Bureau 2022 housing vacancy and homeownership survey,… "The householder refers to the person (or one of the persons) in whose name the housing unit is owned or rented or, if there is no such person, any adult member, excluding roomers, boarders, or paid employees. If the house is jointly owned by a married couple, either the husband or the wife may be listed first, thereby becoming the reference person, or householder, to whom the relationship of the other household members is recorded. One person in each household is designated as the "householder."

Homeownership Rates:

According the US Census Bureau 2022 housing vacancy and homeownership survey,.. "The proportion of households that are owners is termed the homeownership rate. It is computed by dividing the number of households that are owners by the total number of occupied households."

Homeownership by Age of Householder:

According the US Census Bureau 2022 housing vacancy and homeownership survey,.. "This homeownership rate is calculated by dividing the number of owner household in a particular age group by the total number of occupied households in that age group."

H_t:

is the house price index at time t.

HUD:

U.S. Department of Housing and Urban Development.

***h*:**

is the fixed rate for constant house price growth model.

Incomplete market:

it is a market where a derivative product cannot be replicated from portfolios of primary traded assets defining the market.

IML00:

Immediate Annuities Male Lives.

IFL00:

Immediate Annuities Female Lives.

IP:

industrial production.

K_t:

is the accumulated loan balance, usually equal to $K_t = L_0 e^{Rt}$.

L_0:

is the initial loan value.

Life time income:

The income stream available after active working life. Also referred to as post-retirement income.

LIBOR:

London Interbank Offer Rate.

LTV:

loan to value ratio, defines the ratio of the face value of the loan to the value of the collateral house at the time the loan is initially traded.

LTC:

long-term care risk.

LTV:

loan value to collateral house value ratio.

MIDAS:

this is a forecasting tool that can link future low-frequency data with current and lagged high-frequency indicators, and yield different forecasting models for each forecast horizon.

MILAN:

Moody's Individual Loan Analysis.

MLE:

maximum likelihood estimation.

MM:

method of moments.

Morbidity rate:

it refers to the rate of borrowers moving into long-term care.

Multiple decrements probability:

the probability of termination of contract due to either mortality, long-term care or prepayment.

μ:

is the expected growth rate for house price returns under the geometric Brownian motion model.

NNEG:

abbreviation for the non-negative equity guarantee.

η:

is the limit total number of months to be considered.

v:

represents the percentage giving the LTV ratio.

Owner occupier:

this refers to the loan borrower.

OIS:

overnight indexed swap.

PRA:

Prudential Regulatory Authority in the United Kingdom.

$q(t)$**:**

denotes the ERM loan survival probability at time t.

R:

is the roll-up rate charged on the loan; this is the rate at which the loan balance grows.

r:

is the risk free discount rate.

REO:

real-estate-owned status.

RM:

reverse mortgage abbreviation.

σ**:**

the volatility parameter for the house price series.

SEQUAL:

Senior Australians Equity Release Association of Lenders.

SOFR:

the secured overnight funding rate.

SONIA:

the Sterling Overnight Index Average.

TALCR:

total annual loan cost rate.

TONA:

the Tokyo overnight average rate.

CHAPTER **1**

Introduction to Equity Release Finance

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1.1 INTRODUCTION

EQUITY RELEASE PRODUCTS (ERP) have been promoted repeatedly as a vehicle to alleviate long-term care costs, boost pension income, and generally open up a funding channel for a category of people, elderly home owners, who find it difficult to borrow money at an affordable rate otherwise.

A reverse mortgage, as it is usually called in the US or equity release mortgage as it is called in the UK, is a mortgage type loan that is secured by a residential property owned by the borrower or mortgagor. The cash flows operate in the opposite sense to a normal (or forward or regular) mortgage. The lender will get their money back from the proceeds from the sale of the collateral house sale or a voluntary prepayment by the borrower. The settlement payment is typically due and repayable either upon the death of the borrower or when the borrower moves out of the home, or if there is a voluntary prepayment, called maturity events. The borrowers can opt for a bullet repayment for the whole loan, or a series of repayments, or take the form of a revolving credit line. The outstanding balance accrues until the loan is paid in full. For some loans, the borrower agrees to sell a fraction (or all) of the house at a discount in return for a lump sum payment. The lender will then get a fraction (or all) of the selling price of the house proceeds when the house is sold.

As useful as these instruments are, they are not free from controversies. The UK regulator faced difficult conditions in providing rules for this market in the UK for the period when the negative equity risk was not absorbed by the loan issuers.

The ERP market was just about to take off when the subprime loan crisis hit, and the ensuing global financial crisis put a break on the advances of ERP issuance worldwide. This is quite understandable.

However, the increasing growth of pension deficits in most developed economies and the exponential increase in long-term care costs brought back the focus to ERP. In the UK, the ERP market reached approximately 4 billion pounds in 2018 with much more potential available for development. The lenders switched from specialised banks and investment banks to insurers. At this point in time, equity release mortgages (ERM) were perceived as a positive development in the finance and insurance space. This can be seen in the letter to the industry of 2 July 2018, written by the representative of the Bank of England prudential regulation authority, David Rule. In his letter, he said:

> “We continue to believe that restructured ERMs are an appropriate asset to back annuities as part of a diversified portfolio.”

For example, in Issue 1487 of Private Eye (11 to 24 January 2019) an article titled “Just about Managing” was published. The title is a play on words that could be considered anything from incisive and pertinent to derogatory and insulting. The text in the article refers to Just Group, a large insurance company in the UK that is one of the most important issuers of Equity Release Mortgages (ERM). The article suggests that negative equity in ERMs that could cause problems for the company and the entire insurance sector is not measured to the correct magnitude. The relevant text is provided here below:

> “The PRA's recognition of the problem was itself delayed by years of lobbying from firms……and by the Institute of Actuaries. While accountants have faced some political heat recently, the even more easily ignored actuaries - who measure things like likely future losses – have avoided such scrutiny.”

> “Back in the 2005, in the wake of the collapse of Equitable Life the government's Morris Review recommended major improvements in the actuarial profession. Don’t discount the possibility of another one being needed soon.”

It is great when journalists report on issues that are relevant to society. They play an important role in ringing the alarm bell and drawing attention to potential issues that could impact society. At the same time, it goes without saying that we should not take text published in newspaper journals as undisputed accounts of facts. This is the role of other categories of society. The authors of this book were involved with the discussions at the time, and we provided a substantial research exercise commissioned independently by the Institute and Faculty of Actuaries and the Association of British Insurers. Neither of us has been contacted by any journalist to offer our independent point of view.

Following the increasing noise from a distinct eclectic group of professional pessimists, see Buckner and Dowd (2018) and Dowd (2018) about the misestimation of NNEG risk in the UK, the regulator felt compelled to intervene to steer the market out of trouble. Hence, another letter on 3 April 2019 by David Rule again stated the following related to the Prudential Regulation Authority (PRA) on its expectations for conducting the (EVT):

> "In our view, it would help to demonstrate the adequacy of the calculated capital requirements. When performing this proposed validation, firms would adjust the key parameters of the EVT – such as house prices, the deferment rate and house price volatility – appropriately to reflect stressed conditions. For example, deferment rates might be expected to change following material changes in real interest rates."

In the same letter, David Rule also recognised the important role that independent research could play to advance good practice, innovation, and model validation in this exciting new area.

> "I would also like to take this opportunity to welcome the research into ERM valuation co-sponsored by the Institute and Faculty of Actuaries (IFoA) with the Association of British Insurers (ABI), and conducted by Professor Radu Tunaru of the University of Kent. We believe the research paper's advocacy of ERM valuation using risk-neutral techniques is clearly expressed and merits serious consideration by firms and their auditors. This research contributes to a growing body of

> academic thinking on how to address the valuation challenges in ERMs."

The above mentioning of our research in a letter sent by the Executive Director, Insurance Supervision at the Prudential Regulation Authority is one of the key motivations for writing this book. In doing so, we are not trying to be controversial, but to provide an independent point of view, which sometimes may be in agreement with some of the methods being followed and sometimes not.

In the appendix of that letter, which is reproduced here, essentially the main context for writing our book is outlined.

> "Below are some examples of issues in which the considers that research requires further development, framed as questions that boards might ask when considering.
>
> - *Are the mathematical techniques used based on appropriate judgements whose strengths and weaknesses are clearly explained and accessible to challenge?*
>
>   The research is based on two complex techniques, one to project property prices, and the other adapting the projected prices for risk-neutral valuation of the (NNEG). Firms could consider asking for a sensitivity analysis on the choice of these techniques to understand whether different choices might lead to materially different values. Firms could also consider asking how much confidence can be placed in the model's calibration, since the parameters of complex models are generally more difficult to estimate with high confidence and are also more difficult to interpret and validate.

- *How does the valuation approach allow for individual property risk?*

  The research does not consider individual property risk and only examines the statistical behaviour of property indices. ERMs are written on individual properties, which in general behave differently from a property index. This is not a theoretical risk: PRA is aware of a growing population of cases where ERMs have expired in negative equity as a result of individual factors, despite the significant increases in house price index levels since these ERMs were originated. The PRA expects firms to monitor and manage individual property risk, and take account of it in their valuation approaches and internal assessments of MA benefit.

- *Have appropriate data been chosen to calibrate the model parameters?*

  The research is based on data from 1991. Firms could consider how sensitive the NNEG results are to the choice of data. ERMs written today are likely to remain on balance sheets for several decades. Data covering a longer period and wider range of economic conditions are available, for example, back to the 1970s and 1950s. The PRA expects firms to satisfy themselves that the selected data are appropriate to the risk being modelled and that they are sufficiently complete so that trends and other long-term behaviour in the underlying risks can be identified.

- *Have other financially significant judgements been appropriately identified and challenged?*

  One of the most financially significant parameters is the deferment rate, which the research estimates by considering rental yields.

> Several commentators have already challenged the research's judgement to multiply the rental yield by a factor (currently 20%) representing the proportion of properties rented out – to its credit, the research highlights a challenge to this judgement made by an academic reviewer. The PRA's own view is that the challenges are well founded, the justification for the 20% factor is not persuasive and that it is necessary to consider the benefits of owner-occupation on properties that are not rented out (such as those on which ERMs are written)."

1.2 THE MAIN IDEA BEHIND THIS BOOK

The ERP can be dichotomised into reverse mortgages class and home reversion class. The former is essentially a collateralised loan that is paid in full only at the termination of the contract, which is a stochastic time. The second is a transfer of equity of the house when the ERP is issued in exchange for a lump sum and a lifetime lease contract until the termination time.

Consider an elderly couple living in the UK, both 65-year-old who would like to use an equity release mortgage to extract some cash from the equity of their house in which they live. An insurer offers them up to 30% of the current value of their house, which is taken as 300,000 pounds sterling. The lump sum agreed that is advanced to the couple is equal to roughly 100,000 pounds, so the initial LTV is $\Gamma = 0.3$. The insurer applied a loan interest rate equal to $R = 6\%$. The risk-free rate is taken as $r = 0.5\%$ p.a. The volatility of the house prices is taken as 8%, which is in line with historical standard deviation of house prices in the UK. The lender/insurer must absorb the risk of negative equity, and for that it is required by the regulator to maintain a reserve calculated from actuarial and financial markets risk management principles. The lender/insurer assumes that the mortality for the couple can be accurately estimated from the mortality tables.

How can the insurer be sure that the rate of 6% is profitable to them but also competitive? How are the main risk drivers combining to profit or losses on any given loan? What are the regulatory constraints that are influencing internal modelling for assessing risk? Is the rate of house price growth deterministic or stochastic? Can we measure everything we need to know in order to control risk? What is the academic view and what is the practitioner's view? What are the frontiers for this emerging asset class?

We shall call henceforth the above example the first standard ERP scenario, or Scenario 1. The second scenario is taken from Hosty, Groves, Murray, and Shah (2008), who provided a milestone research on ERMs in the UK. Those authors selected PNXA00 (U=2007) as the base table for mortality. The volatility of house prices is taken as 11% after desmoothing (it was only 8% prior to that). The borrowing cost was fixed as the yield on long-term government stocks at about $r = 4.75\%$ p.a. The rental yield was calculated from the IPD Residential Property Index for 2006, at 3.3% p.a. A best estimate of the growth rate of the house price index was calculated as $\mu = 4.5\%$ p.a. The product roll-up rate was taken as $R = 6.7\%$ p.a. that was comprising of 5.10% average swap rate, 0.40% the funder's margin over LIBOR, 0.25% the redemption profile insurance and risk premium, 0.07% as cost of solvency capital, 0.12% as the cost of NNEG and 0.30% in lieu of admin expenses. This would leave the lender with 0.45% profit risk margin. For a 65-year-old borrower, a 2% loading was applied to base mortality to reflect long-term care. We will call this Scenario 2.

The next scenario is from (Thomas, 2021). It has the following details. The borrower is a 65-year-old customer who has a term certain of 25 years. The current house price is $H = 1$. This is not a major simplification since we can think of 1 as 1 million, for example. The LTV is taken as $\Gamma = 0.3$. The roll-up rate is $R = 4\%$ and the risk-free rate is $r = 1.5\%$. The deferment rate (rental yield?!) is taken as $q = 1\%$ and the house price volatility, as $\sigma = 13\%$ p.a. We shall call this scenario, Scenario 3.

Some scenarios are not including all parameters that other scenarios include. In order to facilitate a like-for-like comparison, we will keep the same values of missing parameters from other scenarios. The three scenarios are presented in Table 1.1, and they will be considered for a lump sum loan of 100,000, the same mortality table and assuming that borrowers can live up to a maximum of 100 years.

TABLE 1.1 Three possible scenarios for evaluating an equity release mortgage

Parameter	Scenario 1	Scenario 2	Scenario 3
μ	0.045	0.045	0.045
σ	0.08	0.11	0.13
g	0.033	0.033	0.033
q	0.01	0.01	0.01
r	0.005	0.0475	0.015
R	0.06	0.067	0.04
LTV	0.3	0.3	0.3
Age	65	65	65
Morbidity loading	0.02	0.02	0.02

An investment banker from Goldman Sachs once said in a conference that Goldman is happy to take any risk but only after they understand it. This investment bank has survived well many crises and the ebbs and flows of uncertainty in the world. Our book is intended to help interested parties in identifying risks and learning techniques on how to deal with those risks in equity release markets worldwide.

1.3 WHO IS THIS BOOK FOR

This book could be part of a series of titles in Finance for the Greater Society. The main problem is real; solving the problem efficiently would help many people.

This book could be useful to professionals working in insurance, investment finance, risk management, regulators, and hedge funds, who are interested in improving their skills in financial and risk management calculus related to ERPs. It could also be very useful to academics and students in Actuarial Science, Finance, Economics and Policy programs at both undergraduate and postgraduate levels. Last but not least, we hope it is useful to PhD students and researchers cutting across life insurance, pensions, real-estate valuations and quantitative finance more generally.

The book is also useful for students and practitioners working in residential real-estate. In addition, we can see many ways how entrepreneurs in FinTech and RegTech could use our book as a major source for inspirations.

The book may also prove useful to journalists and nonspecialists interested in ERPs. It can provide a starting point for further research or a benchmark comparison for individual examples.

Another way to look at this book is as a fertile area where new techniques in Econometrics, Derivatives, Financial Engineering, and Actuarial Science can be applied. We think that there is a need for comparative studies for equity release instruments and these studies would inform us of more robust solutions and more pitfalls.

Many parts of our book can be read in isolation and do not require additional prerequisite background. There are also some more technical parts that may assume that the reader is familiar with basic statistical calculus, probability calculus, basic knowledge of financial instruments and markets and basic financial economics. All computations in this book have been done using standard commercially available packages such as Matlab, Excel or R.

We acknowledge that any errors in our results that may inadvertently have been introduced are ours. However, we do not accept any commercial liability for any losses that any entity may incur while using our results, in full or in part, at any point in the future and in any legal jurisdiction.

1.4 THE ORGANISATION OF THE BOOK

The book continues with Chapter 2 in which we discuss the need for ERP worldwide in the ecosystem of financial products. Although our book mainly focuses on individual and micro-level aspects of ERP in terms of modelling, from time to time, we also consider the top-down view.

In Chapter 3, we review the taxonomy of ERP around the world and how they contribute to improve conditions in society related to financing long-term care and pensions.

A difficult chapter to do, read, and embed in the entire book is Chapter 4. However, it is important to realise that the same financial/insurance instruments may be subject to different regulatory regimes and that may act as a coercive factor in the development of that particular market. Perhaps not surprisingly, the economic area with the most involved regulatory set is the EU but the market is very much in nascency there. The markets that are the most developed, the US, the UK, Korea, and Australia, also benefit from strong financial markets and more flexible regulatory conditions.

Chapter 5 focuses on risk drivers and valuation mechanisms. It gives a taste of what is needed presenting mainly the problems but not the solutions. This chapter is followed by Chapter 6, where many technical details that are relevant to the non-negative equity (NNEG) calculus are presented. This is then followed by a chapter on risk management issues. Chapter 7 is more relevant after the ERP are issued and they are on the balance sheet of some lenders.

Chapter 8 is dedicated to the analysis of various risk premia that may preoccupy the investors in ERP loans. While some old-style risk premia follow the definitions encountered in the equity space, some new risk premia definitions are also introduced. This is then naturally followed by a chapter consisting of analysis at portfolio level. Thus, Chapter 9 presents results that are generally ignored or not presented in the equity release literature. Several new concepts are also introduced here for the first time as far as our knowledge, the main idea being to follow various projections of cash flows that portfolio managers in the ERP asset class may be interested in.

Finally, Chapter 10 contains some discussions that we hope the reader will appreciate after reading our book. It also contains a few ideas for further research and some new developments in society that may impact the future evolution of ERP. The book ends with an appendix where various results that are complementing the results presented inside the main text of the book are illustrated.

1.5 SUMMARY

There is no question of the usefulness of ERP to society. Yet, as perhaps with many other things in life, in spite of clear benefits, less resources are spent on developing the ERP market. Part of the problem is the intrinsic link to the housing markets.

The fact that the real estate markets, housing, in particular, do not have proper derivatives contracts traded frequently hinders risk management. It would take a concerted effort between regulators, investment banks, and governments to start derivatives markets for property.

This book aims to keep the debate alive on main issues surrounding ERP valuation and risk management. It also wants to offer an independent view on what is rapidly becoming an area where lobbying and counter-lobbying by various groups in society influences practice.

As with all other research projects done by us, the authors, we recognise that if we had to start again now this project, we would know how to do a better job. Nevertheless, we never pretended to be perfect, and we hope that even mistakes that may have been introduced inadvertently may help improve the debate.

Below are some further readings for those who enjoy finding out more in this area. Our selection does not have an objective in mind other than to inform what other works are out there that inform the debate. We are neither supporters nor deniers of these works. We would rather you make the rather sinuous journey and read the entire book yourself and then form your own opinions.

FURTHER READING

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CHAPTER **2**

The Need for Equity Release Financial Products

DOI: 10.1201/9781003340270-2

2.1 INTRODUCTION

THE MANAGEMENT OF longevity risk poses critical challenges to lifetime income funding around the world. The imbalance between longevity, fertility, and socioeconomic advancement within a given population ultimately poses adverse social, economic, and political challenges to policy makers. Policy options on long-term care, retirement programs, and other government welfare systems designed for the aging population ultimately face funding complications and a cost explosion, thus becoming stifled in the long run. Among key stakeholders, there exists some collective uncertainty about the sustainability of social security frameworks and the adequacy of retirement income replacement cash flows in the face of the aging population. Additionally, there is an excess cost implication associated with the size of the population proportion who are inactive participants of standardised retirement schemes, including the consequence of them becoming an extra burden in social-economic planning. Moreover, pension funds are also susceptible to longevity risk mismanagement that could eventually result in the deterioration of financial capacity, risk of losing individual lifetime income, and bankruptcy. The traditional solution to these challenges typically involves policy changes such as raising retirement age, fostering national consensus on human capital development, active risk management, and regulatory oversight for pension funds, hedging financial risks, and diversifying investment into foreign markets.

Longevity risk challenges can persist even if the retirement age is raised through pension policy reforms. The immediate cause could be attributed to improved life expectancy and significant advances in medical and health care technology. An extended retirement age can give households enough room to improve their work-life financial planning horizon. Such a policy could also increase incentives to explore other viable post-retirement funding solutions. Despite the potential gains, extending retirement the age presupposes shortening of the post-retirement time line, in which case people who died before the extended retirement age would have worked until they died. The demand for lifelong income planning needs could also decrease, leading to a lower demand for alternative lifetime income planning needs. On the other hand, the cost implications of an extended retirement age also need some consideration. The health complications of old age can result in high premiums for workers' compensation benefit plans. The quality of living standards of the working and retired population may eventually be undermined when extending the retirement age is not feasible.

An alternative solution to the challenges that impact the sustainability of lifetime income involves the use of equity release finance. ERPs are financial products that allow their owners to release equity that is otherwise locked up in a specified underlying asset. Typical examples are reverse mortgage (RM) contracts as they are popularly referred to in the United States (US) and equity release mortgages (ERM) in the United Kingdom (UK). The borrower's house or property serves as the collateral underlying these loan contracts. The main benefit is immediate access to funds without the need to sell the house or having to move out. The UK Equity Release Council (ERC) stated in their Spring 2022 market report that rising house prices allowed for larger loan sizes, such that the average customer can access a sum equivalent to over seven years of the typical single pensioner's post-tax income by drawing on less than a third of their property wealth. ERMs and ERMs are typically sold to people over the age of 55 or 65 years, respectively.

In the UK, housing wealth is considered in means testing for local government support to cover residential care costs. Then additional costs are left to be covered by selling the house or by deferred payments from the estate after death or moving into long-term care. Government funding could be supplemented by a private insurance scheme that could be derived from the sale or downsizing of stocks.

The demographic and economic profile of the housing markets in developed economies provides the necessary foundation for these products when they are implemented. According to the 2021-2020 English housing survey published on 15 December 2022, about 63% of properties in the outright owner sector are owned by persons aged 65 or over while 60% of properties in the mortgage sector are owned by people aged 35 to 54 years. More important for arguments in favour of ERPs, the survey found 71% of people aged 55 to 64 years in the owner-occupier sector. About 19% of the social-rented households are in the hands of people aged 55 to 64 years, while about 11% of people aged 55 to 64 are in the private-rented sector. According to the US census bureau report on 15 March 2023, 75.1% of the owner-occupier sector are people aged 55 to 64 years, while 79.1% of the same are aged over 65 years. Figure 2.1 depicts the historical evolution of homeownership in the United States by householder age (54-64 years and 65 years and older), as well as family status (male householder, female householder, and joint couple householders).

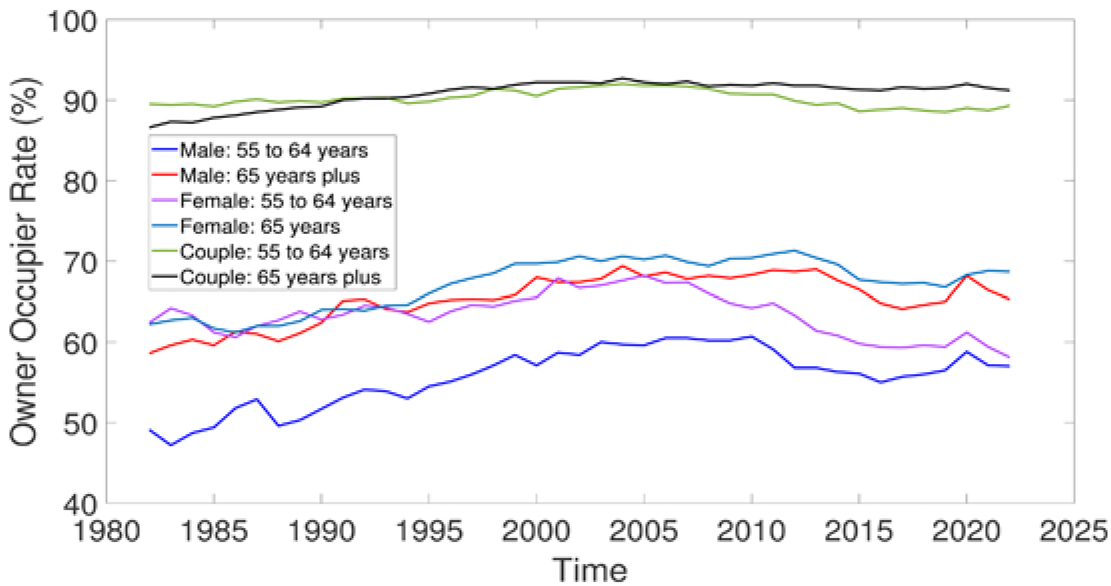

Figure 2.1 Homeownership rates for senior citizens in the United States.

Notes: This Figure plots the annual time series evolution of United States homeownership by age of householder. The time series plots are for homeowners aged 55–64 years and people aged over 65 years. The sample period is from 1982 to 2022. The plots are for the proportion of male, female, and joint (couple) owner occupied householders. The data is from the United States Census Bureau, Current Population Survey/Housing Vacancy Survey, 15 March 2023.

Nakajima and Telyukova (2013) argued that elderly homeowners face borrowing-constraints later in life that may push pensioners to finance large medical expenses by selling their houses. For this category of people an equity borrowing product may facilitate a more relaxed style of financing later in life. Essentially, ERPs help elderly homeowners to borrow against their house equity without actually leaving their house. In a very insightful study, Nakajima and Telyukova (2017) report that 1.9% of eligible homeowners had a reverse mortgage in the US in 2013. Analyzing reverse mortgages calibrated onto a life-cycle model of retirement, Nakajima and Telyukova (2017) suggest that the mean average welfare gain from reverse mortgage loans is $252 per homeowner, and $1,770 per borrower. In the aftermath of the global financial crisis there was a threefold increase in demand for reverse mortgages in the US from the lowest income and oldest households.

Figure 2.2 provides information on the relationship between age and homeownership in the United Kingdom in 2022. According to the survey results, a larger share of owner-occupier homeowners who purchased their homes outright are elderly; aged 55 and over. This constituted 85% of the housing stock included in the survey. Of this proportion, 62.8% of the original owners were over 65 years old.

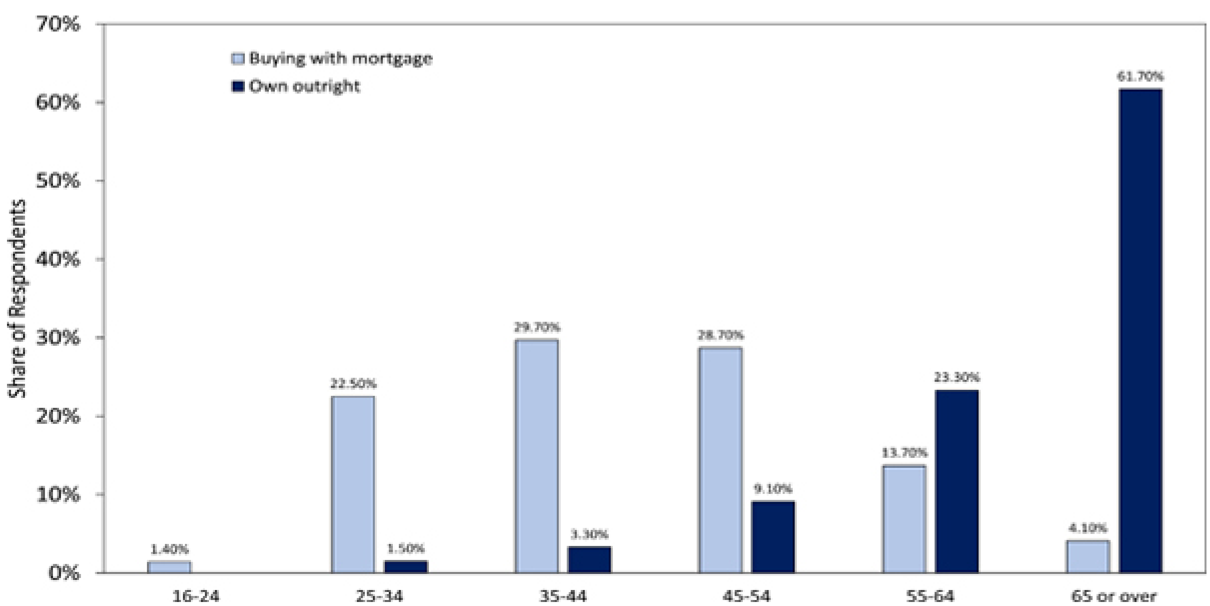

Figure 2.2 Home financing and population age distribution in England.

Notes: This Figure plots the distribution of home ownership in England in 2022, by type of home financing and age. The data is from the Department for Communities and Local Government (UK) survey conducted by NatCen over the period covering April 2021 to March 2022 for the United Kingdom. The survey included a total of 9752 face-to-face interviews and 5284 physical surveys. The data is also published in the English Housing Survey 2021-2022, AT1.3 (ID 321097) December 2022 report.

The figure naturally indicates where the ERP market lies in terms of borrower's age. The first time there are more house own outright potential borrowers than direct mortgage borrowers is in the bracket 55-64 age group. The difference between the two is substantial for the 65 and over category. This coincides also with a change in needs for the borrowers, with elderly people hitting retirement age facing increased costs of health, needing a boost for pension income to go unscathed through a higher cost of living perhaps, or simply affording to buy a holiday property or help their children and/or grandchildren with substantial costs.

One can also argue that it is this category of borrowers that indirectly help house prices to stay high, through helping younger relatives to finance the purchase of expensive properties. While keeping the house prices high benefits the ERP lenders and borrowers and the sellers of houses in the direct mortgage market, it induces an indirect pressure not liked by the buyers of houses in the direct market. Not being able to get on the property ladder will push these potential buyers into the rental market. Eventually there will be a market correction that may unfold and the relationship with interest rates is crucial in this mechanism.

2.2 HELPING THE INDIVIDUALS IMPROVE THEIR CASH-FLOWS

ERPs are suitable for seniors caught up in a low income - limited savings spiral. For older people who are cash-poor and home-equity rich, this comes as a readily available alternative to living on credit cards. Loans contracted under ERPs are often used to pay off preexisting loans of a different nature or fund long-term in home care. The latter situation is typical for a senior couple where one member is ill and needs constant care but the other one does not require it. When none of them is capable of taking care of themselves and decides or needs to move out into care homes, after which the ERM loan contract is terminated. Equity release mortgage loans are also used to purchase other properties, essentially, when the borrower(s) intends to downsize. The product could also provide protection against downsizing for the potential borrower. In the U.S. the proceeds from the loan are not taxable.

There are no income requirements for a borrower. A very important benefit for elderly people taking reverse mortgages is the possibility of increasing their income, which otherwise may be below the poverty line. This could be in the form of a single lump sum or a lifetime drawdown mortgage that allows the borrower to release the equity locked in their home as and when they prefer. By design, the product can only be used by senior people, who in general cannot access the loan markets due to their reduced income and life expectancy. Allowing the elderly to stay in their homes and receive care in their familiar surroundings will automatically decrease the need for nursing home care, which is usually subsidised by the government. Essentially, a well-designed ERM contract could save retired seniors from the burden of having to make recurring interest payments, pay penalties on delayed repayments, and potential exposure to future cost associated with standard loan contracts.

Releasing cash flows locked as equity in houses is considered a major avenue to generate funds for the higher-cost needs of retirees but also to address pension shortfalls (FCA, 2016; FCA, 2017; FCA, 2018). Importantly, in the UK, the House of Lords document titled “Ready for Aging” declares to improve the collaboration with the financial services industry so that elderly could access housing equity release without being overcharged (Lords, 2013).

An important benefit of these financial products is their inverse relationship with interest rates and declining property prices. When interest rates fall, borrowers will benefit from a reduced accrual rate. Similarly, when property prices experience a price correction or market crash that is usually associated with a recession, borrowers may continue with the mortgage and hope for a property price reversal in the future. Since they do not have to make any payments, even if for the time being they experience negative equity, they can continue to ride the housing markets that are known to be mean-reverting and buy their time to better future periods.

Some other reasons attributed to the rationale for the existence of these products are the hedge against inflation, the opportunity to invest in other asset classes, and taxes. To start with, cashing in early and paying much later on will help the borrower use inflation in her favour, that is, when inflation is positive. Of course, the argument will reverse when inflation switches to deflation. Using the equity in the house to release cash may help the smart investor put the money on the long-term in other asset classes such as equity that traditionally generate more returns than the capital appreciation in the house. For some time, the 4% rule of drawing only 4% of the initial value of the portfolio and immediately investing at least 50% into equities has been thought to allow a person's portfolio not to be depleted over a 30-year period with a confidence of 90%. Recently, Wagner

(2013) showed that taking tax-free monthly sums is a better strategy than taking out credit lines and, moreover, drawing even 6% initially would still leave money in the portfolio for a 30-year period with a confidence level between 88% and 92%. Last but not least, depending on the jurisdiction, there could be significant savings on tax.

2.3 HELPING THE GOVERNMENTS REDUCE THE COSTS OF LONG-TERM CARE

Global economies are beset with challenges posed by the rapid aging of the population and the longevity due to the improvement in life expectancy. The tendency to have a declining workforce alongside a significantly large population aged 60 years and older in the near future is a critical concern fiscal constraints on the economy. The direct result is that some within the economically active age group support a larger than expected number of seniors. This is against the backdrop of unpredictable economic growth cycles, geopolitical risks, and high unemployment rates.

Across the European Union, there is a general perspective that it is possible to combine private pensions with pension income resulting from releasing equity from owners' house properties. Eventually, in a context of aging Europe, the idea is to combine Equity Release Schemes (ERS) with pension schemes. That would require clearer regulatory specifications across ERP and pension schemes, as well as improved regulations for lenders regarding how they can generate and use capital in ERP markets and how they should manage the associated risks. A comprehensive study that examines all aspects related to ERS is in the EU Final Report of Al-Umaray et al. (2017).

Sustainable ERPs, when fully implemented, will provide significant relief to the government by reducing the financial burden associated with social intervention for retired seniors. Retired seniors own a larger proportion of the housing stock; enroling such a group in a well-structured ERP will free up funds that could be channeled to support other government expenditures. Retired seniors who own expensive homes are more likely to take pride in funding their own long-term care and other aging-related needs if the property's equity can be easily released. Furthermore, the ability to retain ownership of the collateral house involved in the ERM contract is likely to boost the product's popularity. More importantly, government efforts to ensure the safety of the borrower will help promote and maintain consumer confidence in ERPs. This could take the form of prudential oversight carried out within well-defined regulations. In the future, deliberate policy interventions aimed at promoting a culture of homeownership among the young working class will provide leverage to secure ERM contracts. Early interventions of this kind could help the government and other stakeholders avoid the direct costs of funding retired seniors’ long-term care and post-retirement needs.

2.4 HELPING THE INDIVIDUALS AND GOVERNMENTS SOLVE THE PENSION DEFICIT CRISIS

An emphasised finding in the 2016 UK national statistics report is the anticipated decline in the traditional working-age population, although it has remained stable over the last 40 years. According to the March 2019 Labour Force Survey report by the Office of National Statistics, the UK's employment rate comprises of labour force aged years from 16 to 64 years in paid work. In the European Statistical Office (Eurostat), the population projections (Muszyńska and Rau, 2012) used a working age range of 15–64 years. Working age has generally remained unchanged in many countries. Extending the working age to 70 or 75 years results in long working years and a shorter pension / retirement period, thus lowering the labour force of the country, as they enjoy fewer periods of pension benefits (Smeaton and McKay, 2003; Barnes, Parry, and Taylor, 2004).

Longevity challenges may persist even when the retirement age is extended through parametric[1] pension reforms. Increasing the retirement age suggests a longer working period and a shorter retirement period. On this basis, people with age at death below the average life expectancy would have worked to their grave without any retirement benefit. Households can improve their financial planning horizon for work and life when retirement age is extended. Increasing retirement age may also amplify incentives to explore other viable post-retirement funding solutions. The feasibility of such a policy may result in a lower demand for long-term home equity release. On the other hand, the cost implications of an extended retirement age also need some consideration. The health complications of old age can result in high premiums for workers compensation benefit plans. The quality of living standards of the working and retired population may eventually be undermined when the extension of retirement age is not feasible.

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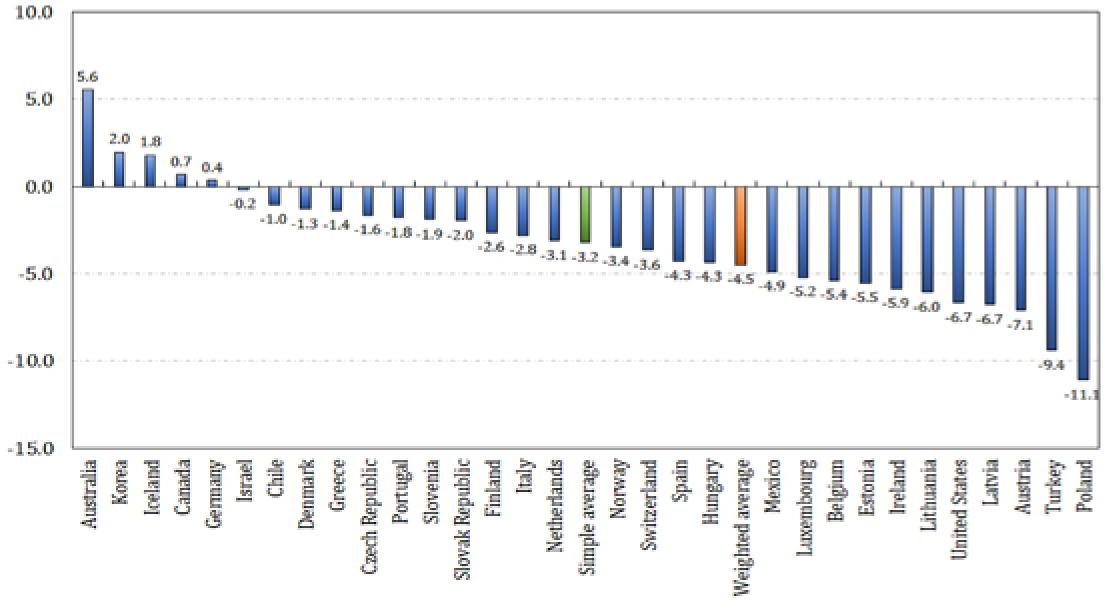
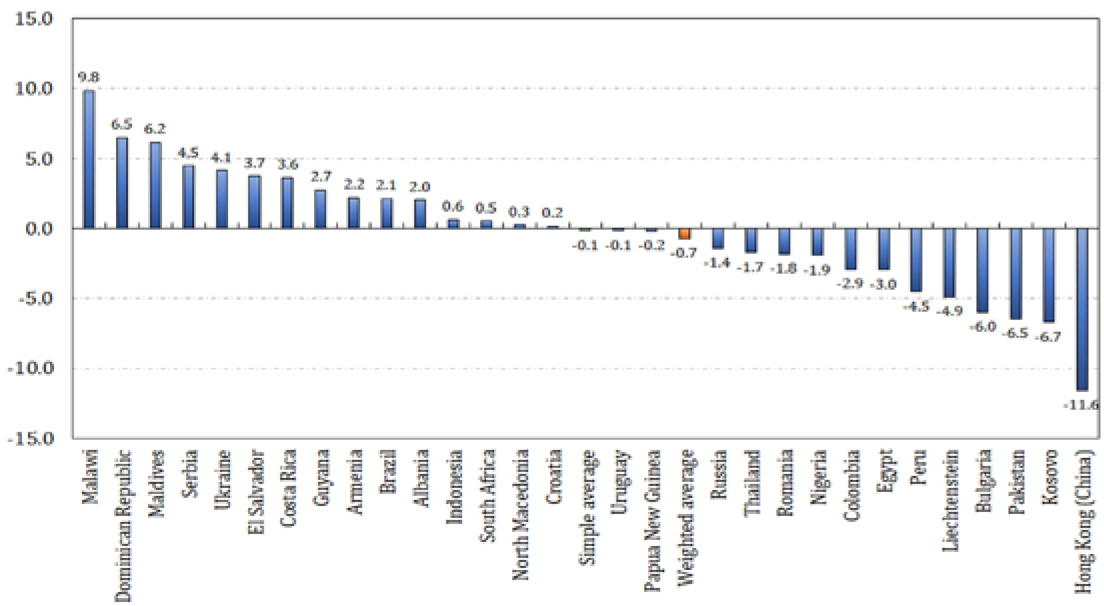
[1]Parametric pension reforms involve altering parameter values of a country's pensions programme.

Europe is on track to become the oldest region in the world by 2030 (European Commission and Directorate-General for Communication, 2017). This will have a great impact on public spending related to pensions and health care. In addition, the poverty risk rate among people over 65 years of age drops rapidly due to public pensions. There are clear benefits of reverse mortgages for governments, pensioners, and also lenders. However, ERPs are currently not widely used, and in the EU, they represent less than 1% of the entire mortgage market.

Pension funds and pension plans around the world have experienced investment losses. The (OECD, 2019), report on pension markets shows a decrease in the real investment rate of return by an average of -3.2% in OECD countries in their 2017-2018 year review. Figure 2.3 illustrates how 26 of the 31 sampled OECD countries experienced losses in real investment rates of return. The report mainly attributed these losses to bearish events in the equity market that occurred around the last quarter of 2018. (Hennecke, Murro, Neuberger, and Palmisano, 2017) outlined instances where the return on retirement savings is subdued by low interest rates on the capital markets. This threat is further enhanced as retired seniors live longer. The joint effect of impaired investment returns and improved mortality creates an adverse impact on pension funding patterns. A direct impact of longevity is captured in the 2017 World Economic Forum (WEF) report, where the retirement savings gap is expected to grow by 5% each year by 2050. (World Economic Forum, 2017) also reported that the retirement savings gap in the United States (US) was 70 trillion dollars, 1.5 times higher than the annual GDP recorded in 7 countries.[2]



(a) (OECD) selected countries.



(b) Countries from other jurisdictions.

Figure 2.3 Annual real investment rates of return (in %) of funded and private pension plans, net of investment expenses, 2018.

Notes: All the annual returns are computed over the period December 2017–December 2018 with an exception to Australia (June 2017–June 2018). Source: (OECD) Global Pension Statistics.

[2]The countries studied in the 2017 World Economic Forum report are Australia, Canada, China, India, Japan, Netherlands, United Kingdom, and United States.

So far, these observed statistics characterise the state of global retirement systems prior to the covid-19 pandemic in 2020. The financial environment has experienced significant changes from the norm. The impact of the covid-19 pandemic has been severe in global economies in 2020 with a recession despite extensive fiscal measures to contain its negative impact on the global economy (Mogaji, 2020). The outbreak can potentially lead to a prolonged incidence of long-term unemployment, an increase in government debt levels, lower growth rates, and a decrease in capital market values (OECD, 2020). The uncertainty surrounding time-to-recovery from the negative impact of covid-19 is further worsened by nonexistent vaccines and lack of reliable treatment. Jorda, Singh, and Taylor (2022) critically analyses reasons why the long-term effects of the pandemic are expected to last approximately 40 years with economic environments characterised by substantially low real rates of return. The resulting long-term challenges faced by governments include, among others, rising aging-related spending on pensions, health care cost, and long-term care. It is not clear how governments around the world will fund stimulus efforts to avoid the recession. However, tackling population morbidity remains an active part of the government strategy to combat covid-19-related mortality, Muszyńska and Rau (2012) demonstrated how health improvements and progressive prevention of disability will not compensate for the aging of the workforce.

The short-term impact of the covid-19 pandemic on pension systems can be associated with the crash of the US equity market on 24 February 2020, which preceded a loss of $20 trillion in the global equity market (Mitchell, 2020). In the substantially low real rate of return environment, Jorda et al. (2022) and Mitchell (2020) predict increases in real wage rates in the labour markets. Meanwhile, the space of the pension market is not fully protected from the negative impact of the pandemic, and complications before covid-19 still persist with a tendency to worsen over time. Mitchell (2020) discusses how the funding rate for the Dutch retirement plan has decreased from 105% (before the pandemic) to below 70%. Discussions in Mitchell (2020) also suggest that defined benefit (DB) pension schemes will face substantial under-funding complications. Early or voluntary retirement can also increase if unemployment persists for a long time. The direct impact of the pandemic on the country-specific funding rate is not immediately observed, as various countries rely on regulatory-based valuation methods of pension liabilities. Defined contribution (DC) pension plans are expected to be affected by the pandemic as the level of joblessness increases in the near term.

Consequently, it has become necessary to work towards ensuring retirement protection for senior population groups. The key to this step is the effort made to preserve and improve the economic well-being of pensioners while ensuring uninterrupted funding frameworks for social security (Choua, Chow, and Chi, 2004).

Among other critical aging needs, seniors struggle with insufficient incomes and retirement savings and larger debt[3] (La Grange and Lock, 2002; Chou, Chowa, and Chib, 2006; Twomey, 2015; Boyer, De Donder, Fluet, Leroux, and Michaud, 2019). This results from the use of leverage to finance daily living, versus the need to preserve capital, generate additional income, and live long. Reliable funding for post-retirement income liquidity is a growing concern for both senior population and public policy makers. Proposed solutions to retirement liquidity constraints traditionally explore the use of state pensions, retirement plan annuities, personal savings/liquid assets, and other alternative discretionary wealth, e.g. sale of homes, renting or downsizing homes, etc. Recent events show that the elderly population is at risk of losing significant value in their pension plans, home equity, and asset value depreciation (Bhuyan, 2010). According to Hennecke et al. (2017), alternative forms of liquidation of housing equity impose much greater financial and psychological burden on the elderly; some of these alternatives include the sale of one's home or renting out, and downsizing (moving to smaller homes). Currently, equity release mortgages (ERM) constitute private savings that boost retirement income security, provide a viable medium to smooth lifetime income, and support alleviation of challenges presented by the aging population on public budgets (Fornero, Rossi, and Brancati, 2016).

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[3]Per the 2014 income distribution study by the UK Department of Work and Pensions, the oldest pensioner age group in the UK is most likely to be in relatively low income (i.e. before housing cost) group. These individuals may also be exposed to rising healthcare costs and difficulties in maintaining financial independence, typically relying on credit cards to cover basic living expenses. Employers' inclination to shift from defined benefit (guarantee retirement benefits) to defined contribution plans (match employee contribution) primarily transfers the retirement savings responsibility and associated risks solely to the employee, thus inflicting retiring households with massive inadequate savings (Chatterjee, 2016).

ERMs as they are known in the US or reverse mortgages (RM) in the UK, are collateralised loans that allow senior borrowers to convert equity that is locked in their houses into lifetime income while aging-in-place; thereby retaining the "possession value" of the house. ERMs fall under a special class of lifetime mortgages with common identifiable features which include; (i) sale to senior population members typically aged 62 and older (Equity Release Council, 2017) although Boehm and Ehrhardt (1994b) sets average borrower age at 75 years, (ii) embedded with non-negative equity guarantees and non-recourse clauses (in the UK), (iii) contract duration is not fixed-term, (iv) issued loans demand no regular repayments from borrowers. As a lifetime loan contract on the borrower's house, repayment of the accumulated loan becomes due when the borrower dies, prepays early, or moves into permanent long-term care.

Ho et al. (2022) employ state-of-the-art economic and actuarial modelling to recover the views of Australian retirees to use housing wealth with different marital status, wealth portfolios, and preferences. They find that the Pension Loans Scheme is perceived as most beneficial if households mainly look to improve the pension income by some amount. On the other hand, private-label reverse mortgages are more appealing if borrowers like a large lump sum at retirement age. This means that households with lower house price growth expectations are better off buying home reversion schemes, while when households have strong bequest motives, they would be more optimal not downsizing or using home equity release.

2.5 RECENT VIEWS ON ERP

Sharma, French, and McKillop (2022b) discuss the difficulties impeding the equity release market because of several demand and supply side constraints. They identify first the following supply side constraints: the no-negative-equity risk, the high insurance costs, the absence of risk pooling mechanisms and the regulatory disincentives, see also Chatterjee (2016); Nakajima and Telyukova (2017) The demand constraints are: the transaction costs, the bequest motives, the implications for state benefits and product complexity, see Davidoff, Gerhard, and Post (2017).

It is fair to recognise that in Europe the most developed ERP market is the UK one. Recent figures reported by Sharma et al. (2022b) state that in 2019, 45,598 households aged 55 and over got ERP with a notional over £3.4 billion in home equity. In Europe the focus has been mainly on introducing regulatory frameworks for this new asset class and there are some small ERP markets emerging from Czech Republic, Finland, Germany, Italy, Netherlands, Poland, Slovakia, Sweden and Spain. All countries with a large enough ERP market require that an NNEG is embedded in the loans to customers, except Germany. In the UK the regulator still considers the ERP market as “highly concentrated,” see FCA (2018). This lack of development leads to issues related to transaction costs, interest charges and LTV ratios. Insurance companies are the main lenders for ERP but Solvency II increased regulations stiffens the growth of this market.

The Solvency II supervisory framework for insurance and reinsurance companies, was introduced by the European Insurance and Occupational Pensions Authority (EIOPA), in January 2016. Solvency II allows matching of assets and liabilities that can lead to decrease in Solvency Capital Requirement (Rae et al., 2018). Hence, a maturity matching adjustment can generate a regulatory reduction for insurance companies for holding long-term assets that match their long-term liabilities. Importantly, an increase in the share of lifetime mortgages for an insurer could lead to a reduction in capital requirements, but only if there is a perfect maturity match in the long-term cash flows. The multiple risks embedded in an ERP makes this matching highly unlikely to be achieved.

The NNEG is an important tool to manage risk for ERP. It is also a main contributor to establish consumer confidence since the negative equity risk is absorbed by the lender and not the also borrower. However, this can be only illusory since lenders address the NNEG risk using lower LTVs. In the US, lenders charge an NNEG insurance premium, see Pu, Fan, and Deng (2014). Interestingly, for the US, Nakajima and Telyukova (2017) estimated that by eliminating the NNEG protection there would be an increase of reverse mortgage issuance by 73%. This figure looks very high at a first glance but one should also take into account that ERP loans are very small compared to the potential size of this market.

A different approach has been taken by Chen (2007) who proposes to solve the risk diversification of ERP loans by implementing a reinsurance strategy. While the idea of using proportional reinsurance contracts is noted, the econometrics of the jump-diffusion processes for capturing the house price dynamics is somewhat unproven and the application of Black-Scholes formula for pricing the negative equity put option is also not very robust.

House price dynamics can be influenced by regional effects. Huang, Yang, and Chang (2020) use different models for London, Manchester and Coventry, as well as for the entire UK. They rely mainly on the ARMA-GARCH jump model that should in theory capture jump persistence, autocorrelation and volatility clustering. As in Tunaru and Quaye (2019) they value the NNEGs using the conditional Esscher transform technique developed by Buhlmann, Delbaen, Embrechts, and Shiryaev (1996). They clearly point out to the presence of house price model risk that may greatly impact the NNEG values and these effects could be very different for different locations.

One other important point regarding ERP is to what extent the collateral house is a consumption asset, after retirement in particular. Jang, Owadally, Clare, and Kashif (2022) demonstrate that it is possible to have both non-housing and housing consumption increasing post retirement when deferred annuities and home reversion contracts are available than when they are not.

2.6 SUMMARY

Reverse mortgages in the US and equity release mortgages in the UK are financial products that were introduced to help with two main problems faced by modern societies, increasing cost of long-term care and pension deficits leading to decreasing pension income. With a projected increase in the aging population in most countries around the world, these two problems are more present than ever.

Unfortunately, the size of the ERP market locally and globally is still in nascency, judging by comparison with the number of potential applicants, after decades of much-broadly discussed growth. There are no financial instruments that resonate more with Finance for Greater Society than ERPs.

There could be many reasons why ERP markets do not develop naturally. There are problems related to product design, problems coming from the regulatory side, cultural and educational problems, and risk management problems faced by issuers. In this book, we try to provide valuable insights in all these directions, but we mainly focus on the latter because we believe that more efficient modelling and capturing of risks embedded in ERPs can only unlock the potential growth in these markets that is clear for all finance and insurance specialists.

We argue that in the next decade, ERPs will become relatively standard instruments, and governments will push the agenda for further expansion of volumes in these markets. This could be interpreted as part of neoclassical liberal economics if we think that individuals who are asset rich cash poor should take more care of themselves, but we could also interpret this drive as a more left-socialist view if we think that in this way more money becomes available to those with lesser means because of lack of assets.

There are many risks that come together when issuing these types of loan. The recent covid-19 pandemic has taught us that there could be more risks to think about and that common assumptions may not hold under all circumstances. Many disciplines come together when helping the ERP market to thrive, from Economics and Finance to Statistics and Computing Science. Regulations and technology have not matured yet, but social needs will push this agenda forward in the next decade.

The further reading list below provides just a glimpse of recent work in this area, but it does show the importance of equity release finance worldwide. We highly recommend Nakajima (2012) for an introduction to reverse mortgages in the US and Hosty, Groves, Murray, and Shah (2008) for an introduction to the equity release mortgages in the UK. Other important ideas are contained in the following additional readings that we recommend.

FURTHER READING

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CHAPTER **3**

Equity Release Products

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3.1 INTRODUCTION

EQUITY RELEASE PRODUCTS (ERP) combine insurance features with structured finance and risk management features. For this reason, they are difficult to price and risk-manage. Furthermore, there are other difficulties posed by the different regulatory requirements in various countries. In this chapter, we would like to improve our understanding of the features embedded in ERP and to gauge the differences in design, if any, across various international setups. Furthermore, we would like to determine the demand for these products worldwide and gain insight into the drivers of this demand.

We hope that more new ERP designs will be introduced over time in many parts of the world. The recent covid-19 pandemic has brought into focus the high cost of long-term care and the fact that such a pandemic could produce immense clusters of deaths in the old age category, well outside of the curves implied by mortality models. Almost immediately after that, interest rates surged to levels never seen for many decades, with inflation reaching double digits in many countries. The current war in Ukraine presents another risk that was not considered before. The risk that the collateral house may be destroyed and again the possibility of large clusters of unpredicted deaths are challenges to deal with. The current designs of ERP do not take into account these risks that we have seen materialised, so more work on this front is needed.

ERPs are used not only to offset the costs associated with long-term care long-term care. They can be used and are used to boost pension income. When evaluating the adequacy of retirement savings, economists and financial planners must decide whether the equity of the house can be used to calculate future income during retirement; see Shan (2011a). The collateral house is viewed by many economists as both a consumption good and an investment good. It is still highly debated how to optimally treat housing equity when it comes to pension savings. Sinai and Souleles (2008) argue that the proportion of what they call "consumable housing equity" is between 60% and 99% for elderly homeowners, and is very sensitive to their age.

Under new regulations, Robert Merton has endorsed the ERP as a viable source of funding for the elderly. Merton and Lai (2016) argue that RMs could be an efficient vehicle to transfer intergenerational wealth, in a hassle-free way for elderly homeowners. Furthermore, Merton and Lai (2016) discuss a structural design of RM that aims to improve risk sharing between the borrower and the lender, while also highlighting the important role of the regulator. Cocco and Lopes (2014) outline improvements to ERP design that may help expand this market for those in need. People would benefit from simple rules to make long-term decisions related to retirement

(Binswanger and Carman, 2012). At the same time, education will remain an important factor that links demand with knowledge or understanding of the characteristics of the financial product to the end user (Davidoff et al., 2017).

The nomenclature can sometimes be confusing, and different terminology is used for similar products. In general, there are subtle differences in the way various products are applied and operated in different countries.

In Section 3.2, we discuss the differences between reverse mortgages, as they are called in the US, and equity release mortgages, as they are called in the UK, highlighting commonalities and differences. Then, in Section 3.3, we continue with a review of the zoology of ERP. In Section 3.4, we present an analysis of the demand for ERP worldwide. The last section presents a summary and further interesting readings on the same topics.

3.2 REVERSE MORTGAGES OR EQUITY RELEASE MORTGAGES

In the US, loans designed similar to mortgages that allow a percentage of the equity of the collateral house to be recovered in cash are called *reverse mortgages*. The name suggests that the cash flows are, in a sense, in the opposite direction to a standard mortgage, where a borrower has to pay a stream of payments to own the house. With reverse mortgages, the borrower already owns a house or property, and they receive rather than pay a stream of cash flows until a prespecified termination event.

A similar financial instrument in the UK is called an *equity release mortgage* or ERM. It operates in a similar manner. The two instruments operate under different regulatory regimes. The difference between the two is in the covenant regarding what happens if the value of the collateral house at the termination event is less than the balance accrued on the loan. In the UK, this risk should be absorbed by the lender. In the US, there is an insurance mechanism and a premium involved to cover this possible loss.

The two approaches are not necessarily equivalent. A lender in the UK could use a higher roll-up rate (loan rate R) that may include an insurance premium recalculated as a running fee (δ). Since the balance to which the insurance would apply is, in essence, stochastic, varying with the house prices, it is not a trivial exercise to a) calculate this insurance premium and b) transform it into a running percentage interest rate that can be embedded into the roll-up rate.

Another important aspect is that in the UK, any risk management calculations for the ERMs must be done on a loan-by-loan basis. Therefore, a lender cannot take advantage of the diversification benefits that a portfolio might bring.

3.3 A TYPOLOGY OF EQUITY RELEASE PRODUCTS

3.3.1 General Classifications

Equity release products are experiencing some kind of quiet renaissance around the world. Governments and financial/insurance markets recognise that these products can help the elderly and retired people boost their pension, which has been eroding over the last decades, or to pay for long-term care, which is becoming more and more expensive. Here are some products that have been used in the real world.

Lifetime mortgages are contracts that allow the borrower to take a loan (lump sum) collateralised with their own home while still owning it. Products are also called *equity release mortgages* or *roll-up mortgages*.

A *retirement interest only mortgage* is a loan (lump sum) secured against the home of the borrower(s). The loan requires the borrower to pay the lender interest monthly, but the principal of the loan is not paid back until the borrower dies or moves to long-term care. The collateral house can be taken away if interest payments are not made.

A *home reversion loan* is a contract in which the borrower(s) sells the entire or a fraction of their property for less than the market value and continues to live in that house as a tenant.

An ERP classification can be made according to the type of cash that the borrower can receive on one of these loans. The most utilised loans are

- a one-off *lump sum*
- a periodic (monthly, quarterly, annually) amount of cash, also called a *tenure* payment method, for a given period of time or for as long as the borrower(s) live in the house
- a *line-of-credit* or creditline account allowing the borrower to decide when and how much cash can be withdrawn
- combinations of the above
- a *deferred loan* requiring the borrower to use it for payment on house repair and renovation.

The lump sum option is the most expensive since interest is charged from the first day the contract is issued.

There are few hybrid contracts within the equity release mortgage class. A *Line-of-Credit loan* pays an initial lump sum and allows the borrower(s) to take out further drawdowns subject to maintaining a suitable LTV relative to their age. If the market value of the collateral increases, the borrower can draw more funds. A *Shared Appreciation Mortgage* is similar to a standard reverse mortgage, with the difference that the monthly interest charged is below the standard rate normally charged. For this type of contract, the lender takes an equity in the property and agrees to receive a share of any increase in the value of the property.

A *Reversion Product* is a loan in which the borrower sells the property to the company that takes legal title and gives the homeowner a life lease. The loan principal is generally higher (the LTV of a 65-year-old may be up to 45%), with no interest being charged. The lender will receive the entire sum from the sale of the property upon the death or entry into long-term care of the homeowner. For some variants of this product, borrowers may sell only a share of their property, and therefore retain a share of the equity.

A *HECM for purchase* is a loan in which the borrower can purchase a new home using the bank loan and the remaining deposit in cash. Borrowers should switch to the new property as their principal residence. A *HECM saver* is a variant of the standard HECM that carries a lower upfront mortgage insurance premium to compensate for possibly larger notional loans.

Last but not least, an *Indexed Reverse Mortgage* is a reverse mortgage in which future payments will be indexed with inflation. In light of recent global problems with inflation created by geopolitical uncertainty, this product deserves perhaps further research.

Another classification of home equity options has been discussed by Addae-Dapaah and Leong (1996) (see also (Bridge, Adams, Phibbs, Mathews, and Kendig, 2010) and it can be described as follows:

1. Home Reversion schemes

   (a) reversion plan
   (b) sale leaseback

2. Reverse Mortgages

(a) fixed term
(b) line of credit

and these can be

(a) tenure-lender insured
(b) special purposes loan
(c) federal government

3. Deferred Loans

(a) interest only
(b) roll-up

3.3.2 Other Equity Release Products Characteristics

Reverse mortgages should be the primary debt against the house that is used as collateral. For a reserve mortgage, the sum of money that can be borrowed, sometimes called the principal limit, is determined as a proportion of the value of the house. There are no additional credit requirements on the borrowers, other than their usual various taxes and maintenance costs of the property. The principal limit is the maximum gross amount advanced to the borrower(s). In the past and to date, the principal limit was between 60% and 70% of the appraised value of the house at the time the loan was first issued.

The termination of the contract and repayment of the loan occurs when the last borrower dies, sells the house, they do not live in that house anymore for more than a year, or simply prepays the loan, as in the case when the borrower(s) win the lottery and their utility has changed. The lender, usually an insurer, may also trigger repayment in some jurisdictions (in the US in particular) if the borrower is defaulting on paying the property tax or house insurance or fails to repair the property to maintain it to a minimum required standard. Keeping property in good condition not to lose value over time due to dilapidation is a contractual obligation of the borrower(s).

Home equity conversion mortgages (HECMs) are nonrecourse loans; that is, the lender is not entitled to get more than the balance or the value of the property, whichever is less. For repayment, the lender will get the house collateral and there is no further debt. If the value of the house that is sold is larger than the accrued balance, the lender will only keep the amount owed, and the excess is returned to the borrower's estate.

The reverse mortgage can be seen as a portfolio of income security and a crossover put option that allows the borrower to put the house as collateral in the loan back to the lender if the accumulated outstanding balance is larger. The lender protects itself against the crossover put (called negative equity in the UK) by buying insurance. The insurance premium is passed over to the reverse mortgage buyer either as an upfront cost or a running fee applied on top of the rollover interest rate or a combination of both.

In the market, two categories of reverse mortgage can be found: those that are publicly guaranteed by government entities and those that are privately guaranteed by private labels. In the US, the Department of Housing and Urban Development (HUD) guarantees reverse mortgages that come from the HECM programme. The HUD provided two ways to protect the lender against put option risk. First, the HUD offers to acquire the loan from the lender whenever the outstanding balance accrues to a pre-specified notional limit. Second, the lender may design the contract such that it takes a shared appreciation position on the collateral property, and the HUD will simultaneously guarantee that position.

The rolling interest rates stipulated in ERPs can be fixed, and many borrowers seem to prefer this design. However, there are also ERPs with a structured rolling rate that can increase gradually, reaching possible double-digit figures such as 12% or 15% per year. There are also ERPs that have annually adjusted rates that are directly related to a reference interest rate, such as the one-year constant maturity treasury, the one-month and one-year LIBOR (when LIBOR rates still existed), the ten-year Treasury rate in the US, and the certificate-of-deposit (CD) rate in Korea. Since these rates can sometimes be quite volatile and can create risk management problems, the respective ERPs have an additional covenant requiring ERP payments to not vary by more than a few percentage points within any given year.

The legal jurisdiction where an ERP may be issued may have an impact on the overall costings. For example, upfront costs to pay lawyers and agents for arranging the deal, a monthly charge for securing the funding of the loan, monthly servicing fees in case the ERP requires funds collection and management on a recurrent basis from the same borrower(s).

In the UK, for reinsurers, the PRA's valuation rules are described in Valuation 2.1 of the PRA Rulebook. For a fair valuation, the requirement is to value the “assets at the amount for which they could be exchanged between knowledgeable willing parties in an arm's-length transaction”. Accounting rules depend on the type of reporting. Companies reporting under UK GAAP, FRS 102 employ the fair value as “the amount for which an asset could be exchanged, a liability settled, or an equity instrument granted could be exchanged, between knowledgeable and willing parties in an arm's length transaction”. However, IFRS 13 defines fair value as “the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date”.

Some lenders may include some covenants in the loan contract that allow them to subtract funds from the monthly payment that can be used to pay off all these carrying costs. There could also be an extended list of default conditions that cover the borrower's declaration of bankruptcy, abandonment of the house, and misrepresentation of persona. There could also be *acceleration* conditions that can trigger the final payment, such as the borrower deciding to rent parts of the property, changes in the deeds of the property, and the collection of additional loans where the house can be used as collateral.

For a reverse mortgage, the default option can enter the money domain even if the borrower does not take any action. The negative equity condition may be on and off during the life of the loan, being determined by the house prices and the level of accumulated balance.

When premiums were originally set for HECM loans, there were no actual exit data, so the assumption made was that loan exits would occur 1.3 times the rate of mortality; see (Rodda, Lam, and Youn, 2004). In the UK, equity release mortgages also include, as a possible exit event, the possibility of moving to long-term care, also called morbidity. Again, due to the lack of availability of morbidity rates, the premia were calculated using a table of adjustment factors based on known mortality rates.

Changes in design make it very difficult for borrowers to compare different types of reverse mortgages. Total annual loan cost (TALC) summarises all costs incurred on a reverse mortgage in one annual average rate. This rate can be used for comparison purposes between lenders and products.

3.4 DEMAND FOR ERPS AROUND THE WORLD

The size of the RM market has not seen uniform growth worldwide in developed countries. Nakajima and Telyukova (2017) highlighted that only approximately 2% of eligible homeowners had an RM in 2011. This proportion has increased only slightly in 2017, with 55,000 senior borrowers taking RM out of the total of 2 million adults over 65 years of age. This is surprising since RM, together with LTC insurance, provides consumption transition benefits to older adults Davidoff (2009). RM in the UK has increased substantially between 2012 and 2019, reaching about four billion pounds of notional outstanding loans (Equity Release Council, 2018). In Australia, the market size grew from $0.9 billion in 2005 to $3.32 billion in 2011, with the total number of outstanding loans increasing from 14,584 in 2005 to 42,410 in 2011, Deloitte Australia (2018).

Equity release mortgages remain popular in the United States (US), Canada, the United Kingdom (UK), Germany, Ireland, Italy, Netherlands, Poland, Spain, Sweden, Australia and New Zealand. ERM markets vary slightly between countries by scale of implementation and securitisation. In the US, securitised mortgages are backed by the government. Australia, Ireland, and Sweden also have securitised mortgages. The scale of implementation could be assessed based on the number of key stakeholders, that is, funding sources, regulator involvement, and the volume of contracts associated with the borrowers. With respect to product design: there are lifetime mortgages, home reversion plans, products with or without NNEGs, fixed rate or floating interest rate loan, etc. The mortgage loan payment (type of borrowing) may be lump sum, drawdown, or annuity (level) payment. The youngest global eligibility age is 55 years. Despite the fact that most contracts do not involve medical underwriting, some UK ERM designs require medical underwriting, although not common.

The stifled growth in ERP take-up rates among elderly citizens could be design specific when looking at the supply side. For example, in the US, we can identify the issue of high premia charged to consumer, while the UK market has the NNEG clause stipulating that any excess of the accrued loan amount above the sale value of the property after the exit event will be written off by the lender, subject to certain conditions. Other possible demand-side explanations relate to poor product knowledge of ERM risks (Davidoff 2015; Davidoff et al., 2017) moral hazard perhaps highlighted through default in payment of property taxes (Shiller and Weiss, 1999) and homeowners insurance (Moulton, Haurin, and Shi, 2015). Using data on home equity conversion mortgage loan levels prior to the financial crisis of 2008, K. Chen and Yang (2020) documented that increased housing prices increased demand for reverse mortgages among senior citizens, who rationally hold negative expectations about future appreciation of the housing price and therefore try to capitalize on their home equity gains through reverse mortgages. Similar conclusions were advocated earlier by (Shan, 2011b).

3.4.1 The HECM programme in the U.S.

In the US, ERPs were offered in two categories. One is the Reverse Equity Conversion Mortgages (RECM) and the other one is the more successful Home Equity Conversion Mortgages' (HECM). The former has been used sparsely since 1988. The home equity conversion mortgage (HECM) is a loan that is typical to the US, and it is the only reverse mortgage that is covered by the federal government, being issued by the Federal Housing Administration, more exactly by the Department of Housing and Urban Development (HUD), on the primary market, and also being supported by the Government National Mortgage Association (GNMA) on the secondary market.

The first reverse mortgage was first traded in the US as early as 1961. As a financial product, it was clearly defined in 1987, when Congress approved the Home Equity Conversion Mortgages (HECM). The aim was to make real estate assets more liquid for the elderly. The HECM loans that are insured by the Federal Housing Administration are securitised in HECM mortgage-backed securities, or HMBS. They are guaranteed by Ginnie Mae.

Other privately issued reverse mortgages may have smaller arrangement fees, but they will have higher interest rates. HECMs are available to borrowers who satisfy a series of conditions and are used for a specific declared purpose. The youngest homeowner's age must be 62 years old, and the collateral house backing the loan contract must be the single-family primary residence or an owner occupied two- to four-unit building. The collateral house must be debt-free, requiring potential borrowers to seek independent financial advice. The HECM programme was introduced in 1989 with a predicted annual volume of about 25 billion dollars in the next decade.

In the first quarter of 2023, the HECM lending limit has been increased to 1,089,300 dollars, with the youngest and oldest borrowing groups, respectively, receiving 38.2% and 57.0% in the loan-to-value ratio (LTV). The nationwide lending limit and LTV have also been subject to several revisions in the recent past. The limit for was $625,500 from 2009 to 2017, $636,150 in 2017, $679,650 in 2018, $726,525 in 2019, $765,600 in 2020, $822,375 in 2021, and $970,800 in 2022. Post 2009 borrowers could only get up to 56% of the home value. Between January 2011 and September 2012, around 110,000 loans were financed for a principal of approximately

$17.4 billion. In June 2013, there were 624,318 outstanding HECM loans in the United States. The HECM covers 90% of the reverse mortgage market.

There are some differences between HECM loans and traditional home equity loans or home equity lines of credit (HELOCs), traded in the United States. A HECM loan does not have a fixed maturity date, being due and payable only after the borrower dies, moves out permanently (for long-term care purposes or otherwise), or the house is sold. HECM loans do not have strict underwriting requirements compared to home equity loans and HELOCs that require borrowers to demonstrate that they have sufficient cash flow income and also sufficiently high credit scores.

The amount of cash income that the borrower can draw from an HECM loan is calculated as follows. First, the Maximum Claim Amount (MCA) is calculated as the lesser of the appraised value of the house or the county-specific Federal Housing Administration (FHA) mortgage limit for a one-family residence under Section 203 (b) of the National Housing Act. Then, the initial principal limit (IPL) is computed by multiplying the MCA by a factor between zero and one, which depends on the age of the borrower and the funding interest rate. This funding rate is a proxy for the future interest rate and is generally equated to the sum of the 10-year Treasury rate and the lender's margin, typically between 100 and 200 basis points. The principal limit factor increases with the age of the borrower and decreases with the interest rate. Shan (2011a) gave an example where the factor was equal to 0.281 for a 65-year-old with an expected interest rate of 10% and equal to 0.819 for an 85-year-old with an expected interest rate of 5%. Afterward, the net principal limit (NPL) is determined as the amount the borrower can draw as a lump sum at closing, subtracting from the IPL the upfront costs associated with HECM loans and a set aside for a monthly servicing fee. The upfront costs include the initial mortgage insurance premium (MIP), the origination fee, and other closing costs. The initial MIP was around 2% of the MCA on the market for a long time, and the origination fee was capped at the maximum between $2,000 and 2% of the MCA. The service fee set aside is computed as the present value of the monthly service fee applied by the issuer. These upfront costs and the servicing fee are financed rather than paid by the borrower out of pocket.

For a given NPL, HECM borrowers can draw their loan cash flows in several ways. Under *Tenure plan*, the borrower receives equal monthly payments for as long as they live and occupy the house as their principal residence. Under *Term plan*, the borrower receives equal monthly payments for a fixed period of months as specified and under *Line of Credit plan*, the borrower is receiving mortgage proceeds in unscheduled payments or in installments, at times and in amounts specified, until the line of credit is emptied. There is also the possibility of *Modified Tenure and Modified Term plans* whereby the borrower can combine a line of credit with a tenure plan and another one with a term plan, so that the borrowers can change their payment plan at any time at a small cost.

The origination fees and servicing fees are also usually ignored when modelling the risks posed by HECMs. They have been in decline in the US but still represent several thousand dollars in the final payback of the loan. Moulton et al. (2015) discovered that in 2014, about 12% of reverse mortgage borrowers in the HECM programme defaulted on their property taxes or homeowners insurance. A possible explanation is that, unlike the standard mortgage market, there were no underwriting guidelines for HECMs up to 2014. The high default rate triggered several policy measures, including underwriting guidelines. Key drivers of default may include the initial loan amount, the property tax burden, and the history of prior default by the borrower. More specific to the latter, the current HECM design setup does not involve borrower creditworthiness checks. Essentially, the success story of HECM can be tied to policy changes around key drivers of delinquency and its impact on participation rate.

All HECMs use periodic payments based on either a fixed or variable interest rate and a fixed or variable notional for the loan; if the borrower is using a credit line, then she may draw cash every month. The credit line of a HECM can grow with time until all the credit is used. The rate of growth is usually the same as the rate on a savings account to avoid arbitrage.

In 1989 Fannie Mae generated a secondary market by purchasing HECM loans from lenders, loans that satisfied some standards like lump-sum draws against a credit line, fixed interest rates to advance loans from the lender to the borrowers, and flexible interest rates (one-year Treasury plus a spread) for balance accumulation. Fannie Mae developed the Home Keeper programme aimed at helping elderly homeowners use equity to maintain or improve their homes.

As the market for reverse mortgages developed, as was the case with forward mortgages, private labels entered the market to take advantage of a growing market and easier access to funding through the financial markets. These private lenders introduced *jumbo reverse mortgage* loans, but the emergence of the financial crisis stopped their growth for many years after the subprime crisis.

A HECM loan is a *non-recourse* loan. This means that if the borrower (or estate) does not reimburse the loan balance in full when due, the property will be foreclosed and the borrower (or estate) will not be personally liable for any deficiency resulting from foreclosure (Shan 2011a). The loan also becomes due and payable when the borrower refinances into a new HECM loan, but refinancing a HECM loan is perceived as costly, and the proportion of HECM loans that are refinanced is quite small.

An important feature of HECM loans is that they are insured by the FHA insurance programme so that the HUD insures the borrower against the risk that the lender cannot pay anymore. There is also insurance for the lender against the risk that the loan balance exceeds the value of the property. This dual insurance programme is financed by HUD charging an initial issuing MIP, set at 2% of the MCA and then an on-the-run monthly MIP of 0. 5% per annum applied to the outstanding balance of an HECM loan.

The current regulations for HECM, see Twomey (2015), stipulate that “the owner's obligation to satisfy the loan obligation is deferred until the homeowner's death[1], the sale of the home or the occurrence of other events specified in the regulations by the Secretary”. When the homeowner dies, the house that is the collateral in the reverse mortgage is given to the borrower's heirs by law. The loan becomes due for payment, and the heirs decide whether to keep the house, sell the house, or give up the house to the lender. In the US the regulations state that the lenders should start proceedings for closing the loan within six months of the mortgagor's death if the amount due is not fully paid. Twomey (2015) provide an interesting discussion of reverse mortgages used in conjunction with bankruptcy.

In the US, the balance of HMBS guaranteed by Ginnie Mae was 54 billion USD as of June 2016. (Nakajima and Telyukova, 2017) pointed out that only approximately 2% of eligible homeowners had a RM in 2011 in the United States (US). This proportion has increased only slightly in 2017, with 55,000 senior borrowers taking ERM out of the total of 2 million population of adults over 65 years of age. This is surprising since RM, together with LTC insurance, provides consumption transition benefits to older adults (Davidoff, 2009).

3.4.2 Equity Release Loan Programme in Korea

The Far East countries seem to be very interested in this type of financial product, perhaps not surprisingly given their increasing longevity rates. Korea is such an example. For ERP in Korea, the minimum age of a borrower should be 60 under the Korean government-insured reverse mortgage programme; for more details on this particular programme, see Ma and Deng (2013).

The reverse mortgage market in South Korea began to grow when the government amended the Korea Housing Finance Corporation (KHFC) law in 2007 to allow KHFC to guarantee reverse mortgage products provided by private financial institutions. The KHFC appeared in 2004, being set up by the government and central bank of South Korea. The reverse mortgage loan started to trade on the Korean market after 2007, but started to increase in 2014, after more intense advertising. Choi, Lim, and Park (2020) reported that the ratio of Korean citizens older than 65 years of age in the total Korean population has increased from 7.2% in 2000 to 14.9% in 2019, and it is expected to increase further to 46.5% by 2067. The global ratio of over 65 years was 9.1% in 2019 and is expected to increase to 18.6% by 2067, according to the United Nations.

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[1]If only one spouse is party in the contract as a borrower, upon the death or move into care of that spouse, the reverse mortgage became due. New rules implemented in 2015 allow the spouse who is not listed in the contract to stay in the house as long as it is her/his primary residence.

Borrowers must be a minimum of 60 years old, and the collateral house price must be less than 900 million South Korean won-the equivalent of 800,000 USD. The applied roll-up rate was calculated as the 3-month certificate of deposit + 1.1%. KHFC-sponsored reverse mortgages have an incentive to reduce property tax by 25%.

There were slightly more than 5500 new reverse mortgages sold in 2015 with a total loan amount of KRW 369.3 billion, roughly 2.5 times what it was in 2011. The loan amounts of all reverse mortgage products that were sold until December 2015 sum to KRW 31.56 trillion, see Kim and Li (2017). As of June 2015, homeowners in the program have an average age of 72 years, the average monthly payment per person is KRW 980,000, and the average price of the homes in the program is KRW 279 million. There were more than 25,000 borrowers who benefitted from this program. This is only 0.8% of the population older than 60 years. To be eligible for the reverse mortgage program, the homeowner must be at least 60 years old. Since many Koreans retire (forced or voluntary) at around age 50, private banks saw an opportunity to market a bridging reverse mortgage, which converts to the standard one once the homeowner reaches 60, but with limited success.

Ma and Deng (2013) presented an actuarial-based model for pricing the Korean reverse mortgage with constant monthly payments and also with graduate monthly payments indexed to the growth rate of consumer prices. They found that the TALC rates for the graduate payments scheme are more advantageous to the borrower, and any shock to house prices may impact the younger borrowers more severely.

Korean reverse mortgages have a different design, being equivalent from a modelling perspective to a portfolio of long position in both the European call and put options, unlike reverse mortgage products in Hong Kong, see Han, Wang, Xu, and Choi (2017). Korean reverse mortgages have a straddle feature which allows building in a hedge strategy for the borrowers. Therefore, this different ERP design is very appealing to elderly borrowers because they can also cash in a regular cash flow to cover living expenses without being impacted by the volatility of the house price. It should be noted that the loans in Korea are non-recourse loans and if the collateral house price is above the total outstanding balance, then the heirs will get the residual payment from settling the loan.

3.4.3 Equity Release Loan Programme in Japan

Japan is recognised as the country with higher longevity, and this gives Japan a great motivation to expand a reverse mortgage market to generate extra cash to help elderly citizens. The economy experienced a persistent decline in property prices after the asset bubble at the end of the 1980s and the beginning of the 1990s. These low property prices essentially did not allow the growth of the reverse mortgage market. Insurers and other financial institutions were unwilling to underwrite the credit risk associated with these loans in Japan. The state of reverse mortgages in Japan is described in Kobayashi, Konishi, and Takeishi (2017).

Consequently, it was felt that the local government would accept the challenge and contribute to the development of a reverse mortgage market in Japan. The first reverse mortgage loan in Japan was issued by a local government unit, Musashino City, in the western part of the Tokyo Metropolitan Government in 1981. Later, however, some consultants hired by Musashino City suggested closing the reverse mortgage program in 2014. The local council did that on 31 March 2015. However, the idea was liked by the government, and the Ministry of Health, Labour, and Welfare started a national government reverse mortgage program in 2002. This program went well and in 2007 was expanded to provide social security assistance for households in need of social security assistance. The idea worked and was franchised by several local government councils.

Japan had a particular characteristic of reverse mortgage loans issued by private companies. The Tokyo Star Bank introduced a “deposit collateralised reverse mortgage” in 2005, aimed at “tangible asset rich, financial asset rich, but cashflow poor” elderly homeowners. Hence, their target market was also *financial asset rich* people, something that was not of particular interest before. The main destination for the use of the cash received from these loans issued by the Tokyo Star Bank was medical expenses. The interest rate applied to this type of loan was zero until the amount drawn exceeded the amount of deposits of the borrower. In November 2015, Tokyo Star Bank had almost half of the outstanding reverse mortgages; see Kobayashi et al. (2017).

Chuo Mitsui marketed ERPs with fixed annual installments and also allowing disbursements, with a capped limit, on an as-needed basis. A survey conducted in February 2016 indicated that 20.8% Japanese financial institutions were thinking about marketing reverse mortgage products.

3.4.4 Equity Release Mortgages in Australia and New Zealand

Several ERPs are available throughout Australia, with the borrower receiving payment as a lump sum, a line of credit, regular income, or a combination of these. Australia is another country with an aging population. The population over 60 years changed from 13% in 1971 to 18% percent in 2005. As in Japan, the market for ERP in Australia is made up of over 65s who have property assets and financial assets. The market size grew from $0.9 billion in 2005 to $3.32 billion in 2011, with the total number of outstanding loans increasing from 14,584 in 2005 to 42,410 in 2011, (Deloitte Australia, 2018).

Based on studies by the Senior Australian Equity Release Association of Lenders (SEQUAL) and Deloitte (2013), the number of ERP loans in Australia was more than 40,000 in December 2013. In 2010 Australia had

10 different reverse mortgage products, 6 were from SEQUAL accredited lenders, 3 were from non-SEQUAL accredited lenders, and 1 was from the Australian government through Centrelink. The largest reverse mortgage issuer with 70% of the market was Royal Bank of Scotland, after it acquired ABN AMRO Bank.

LTV was between 15% and 20% for 60-year-old borrowers, and increased by 1% as the borrower ages. Hence, the maximum LTV is between 25 and 30% for 70-year-old borrowers.

In New Zealand with an ERP loan, the borrower can receive either a lump sum (called reverse mortgage) or periodic payments (called reverse annuity mortgage). The loan is collateralised with the house and is not payable until the borrower dies or moves into another house.

3.4.5 Equity Release Mortgages in UK

There are two main types of ERPs contracted in the UK market. Lifetime mortgages are the equivalent of reverse mortgage in the United States. The other is defined by home reversion schemes. The first equity release loans were traded in 1965. However, in the 1980s, these types of financial products were affected by the scandal associated with the misselling of investment bond plans and roll-up plans. That was mainly a problem with design and the fact that the NNEG was not covered by the issuer.

An industry group called the Safe Home Income Plans (SHIP) initiated a self-imposed regulation in 1991. After that, more than 350,000 people have taken out an equity release plan from the members of SHIP, accounting for nearly 17 billion pounds sterling in collateral housing wealth. SHIP then morphed into the Equity Release Council in 2012.

The drying of funding caused by the global financial crisis in 2007-2008 led to the withdrawal of one of the top three equity release providers

(Sharma, French, and McKillop, 2022a). The Financial Conduct Authority (FCA) has criticised lenders for lack of competitiveness and also lack of product innovation (FCA, 2016). Recently, with the implementation of Solvency II under the EU directives, Prudential Regulatory Authority (PRA) had to increase the debate on risk management of equity release schemes, particularly with regard to NNEG and its valuation; see Kenny et al. (2018).

In January 2019, the maximum loan-to-value (LTV) offered to a 55-year-old borrower was 18.5%, 29.6% for a 65-year-old customer, and 47.1% if the customer was 90 years old (ERC, 2019). The costs involved in setting up an equity release plan include administration fees, solicitor fees, surveyor fees, and adviser fees, and are generally between £2000 and £3000. Borrowers are also charged a fixed or variable (but capped) interest rate. Historically, the average interest rate has been around 6.5%, but decreased to 5.2% in the second half of 2018 (ERC 2019).

The ERP market in the UK is one of the most active markets in the world. ERPs are offered to homeowners with a minimum age of 55 years, following a regulated process of financial advice and independent legal advice.

The ERP market in the UK is to some extent small compared to the reverse mortgage market in the US, although the UK has better rates of elderly homeownership and a clear demand driven by long-term care costs (FCA, 2018). The UK market is spread mainly across a few regions with London, South East and South West constituting up to 59% of overall lending, while in the north regions that are poorer by income, there is only 14% of the ERP market (Sharma et al., 2022a). The latter authors also demonstrate that the concentration of this market in the southern areas is not due to the lack of demand in the northern areas, but rather due to the risks faced by suppliers. They also show that these regional variations in NNEG risk are not captured in national pricing models. Last but not least, they also argue that EU Solvency II capital requirements are essentially adding more frictions to supply ERMs in these areas.

One important characteristic of the UK market is the non-negative equity guarantee that essentially requires the lender to absorb the risk that at termination of the loan the redemption value could be impaired by the possibility that the house price may be lower than the accrued balance.

The total lending for 2022 reached £6.2 billion, representing an almost 30% increase from £4.8 billion in 2021. This implies that the equity release market in the UK almost doubled in size since 2017 when the lending was about £3.06bn.

In Figure 3.1, the figures collected by the Equity Release Council show a sustained increase in this market between 2013 and 2018, followed by a flat issuance rate period until 2020 and then a revamped period until the end of 2022.

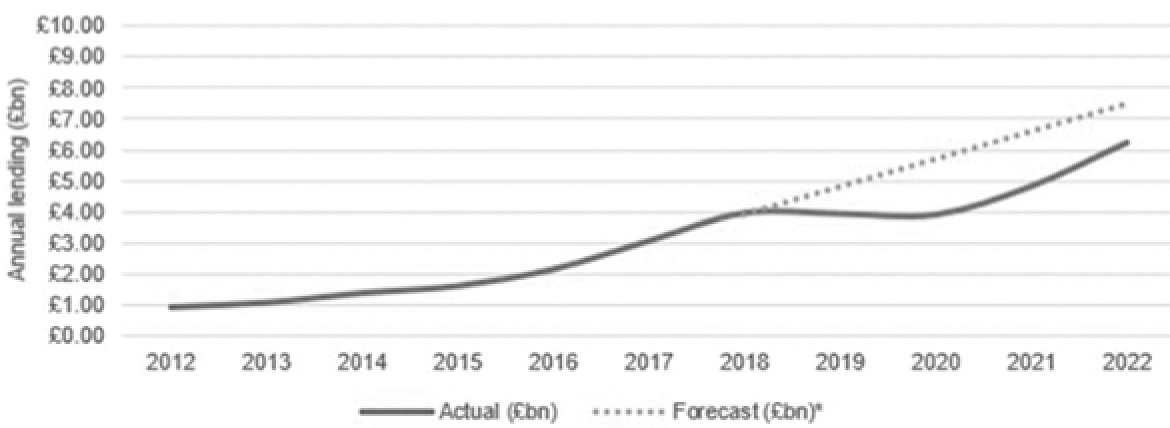

Figure 3.1 Annual equity release lending activity, 2012–22. Source: Equity Release Council 2023.

In contrast to the US reverse mortgage market, the UK equity release market is dominated by insurance companies and other financial boutiques focused on retirement products. Most firms and advisers are members of the Equity Release Council (ERC), a trade body for the UK market. It is compulsory for ERC members to cover the NNEG risk on their own balance sheet, and that has become an industry standard. The risk management of the NNEG risk is undertaken by offering prudent loan-to-value ratios and incorporating a premium in the interest rate charged to the borrower. Although UK premiums are not made public, estimates for the cost of the NNEG as a percentage of the initial lump sum range from 0.1% to 6.5% for a 65-year-old female borrower depending on assumptions (Tunaru and Quaye, 2019).

ERMs in the United Kingdom (UK) have increased substantially between 2012 and 2019, reaching about four billion pounds sterling in notional outstanding loans (Equity Release Council, 2018).

3.4.6 Equity Release Mortgages in the EU

The earliest form of ERP has been available for more than two centuries in France; (see Ward, 2004). Based on the French viager financial laws, a house owner could sell the property below the current market value to receive a life-annuity from the buyer and be allowed to remain in their house for their remaining life. The lenders liked this contract because they were able to avoid financial intermediation costs and dealing with banks.

Reverse mortgages were first considered in Germany in 2006 when the German Savings Banks Association (DSGV) and the Association of German Public Banks (VB) considered the idea but ultimately opted against it. Then, in 2009, Deutsche Kredit Bank (DKB) in cooperation with Immokasse introduced a reverse mortgage product at the national level to borrowers who are at least 65 years of age, with a maximum LTV of 35%, rollover rate fixed at 6.9%, applied to a one-off lump sum loan. DKB protected borrowers from overindebtedness with a no-recourse clause that limited the maximum amount to be repaid to the current market value of the property. Immokasse was declared insolvent in 2013 and unfortunately DKB exited the market after selling only about 100 reverse mortgage contracts.

The market was revived in 2010 by Investitionsbank Schleswig-Holstein (ISH), a public bank, which started issuing reverse mortgages, but with some important changes. Their loans did not have a no-recourse clause for the borrowers. This meant that borrowers were liable for repayment not just with their property but also some of their other assets in their patrimony. The loan was issued as a lump sum or a monthly payment up to their statistical life expectancy, or a combination of the two forms. Five years after exceeding the statistical life expectancy, borrowers were expected to repay the loan. However, borrowers could refinance with another loan that would not have to be repaid before death or reaching the age of 110. The story kind of repeated itself; ISH sold only 44 loans and exited the market in 2013. In 2011, the insurance company R+V also started to sell a reverse mortgage, designed as a split into multiple streams of payments, see Bartsch, Buhlmann, Kirschenmann, and Schmidt (2021). One part was paid directly to the borrower while the rest was divided to pay two pension savings accounts and bad debt insurance. When a lending threshold was reached, the payments from the loan were to cease and the first pension account would resume the payments. After some time, the second pension account was meant to cover the accruing interest payments to avoid an ever-increasing loan amount that might surpass the property value at some point. In general, borrowers were guaranteed a lifetime of payments. Potential borrowers had to be between 65 and 80 years of age and own debt-free property worth at least €250,000 at current market prices. This ERP was available to people living in metropolitan regions. R+V also did not have success.

The reason why reverse mortgages did not work well in Germany could be explained by the success of home reversion plans in the same country. However, in reality, the overall number of home reversion plans sold is between 500 and 1,000. In 2004, Stiftung Liebenau, a non-profit foundation, started a home reversion plan by which pensioners sell their home in exchange for a lifelong right of residence and a lifelong or limited monthly payment. The collateral house must have a minimum market value of €200,000 in an attractive region and the borrowers must be at least 65 years old. In 2013, Deutsche Leibrenten Grundbesitz AG also started a home reversion plan, selling about 400 loans in 2020. Deutsche Leibrenten does not impose a minimum market value, and it works with a higher minimum age of at least 70. In Germany, there were only 200 reverse mortgages in 2015 Bartsch et al. (2021) and there was no major reverse mortgage provider at that time.

3.5 SUMMARY

According to World Bank information, as of 2016, the population 65 years or older will increase from 34 million (or 26. 86% of the total population) to 39.1 million (or 36. 31%) in 2050; see also Merton and Lai (2016). In Singapore, there were 0.691 million (12.29%) in 2016 and 2.2 million (33.92%) predicted for 2050. The estimates for Korea are 6.9 million (13.57%) in 2016 and 17.7 million (35.35%) projected for the 2050. The worst situation is for China, with 137.8 million people 65 years or older in 2016, representing 10%, and is predicted to be 368.2 million, or 27.50% in

2050. For all these densely populated countries, the pension schemes provided by their governments will not be sustainable.

There is a “puzzle” as to why not many retirees have taken ERP loans, even in countries like the United States, the United Kingdom, and Korea, where government backing or strong consumer protective legislation have been available for a considerable time. Some of the reasons that hinder the development of ERP markets are high costs, high regulatory barriers, lack of education moral hazard, and adverse selection.

ERPs are spreading around the world, and more countries have introduced such financial products or are preparing to introduce these products and related legislation. The US and the UK are the economies with the most performant ERP sectors. Other countries have arguably even greater needs for the ERP sector to function well, but local cultural differences prove to be a more difficult barrier than previously thought. Germany is a negative surprise, but efforts at the European Union level to drive forward the advancement of the ERPs market may lead in the near future to more significant developments in this economic region as well.

Merton and Lai (2016) propose a structural design for an ERP contract that may be applicable across geopolitical borders. They emphasize key design criteria for the contract itself and an innovative approach for funding ERP loans.

The following further reading list provides a short but important collection of articles for those willing to start doing research in the equity release finance area. These references focus more on the financial economic intuition behind the ERPs and do not open much horizon regarding the modelling, which is discussed in later chapters.

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CHAPTER **4**

Regulation of Equity Release Instruments

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4.1 INTRODUCTION

The reduced growth in take-up rates of ERPs among elderly citizens could be design-specific when looking at the supply-side. For example, in the US, we can identify the issue of high premia charged to consumer; while the UK market has the non-negative equity guarantee (NNEG) clause stipulating that any excess of the accrued loan amount above the sale value of the property after the exit event will be written off by the lender, subject to certain conditions.

Other possible demand-side explanations relate to low product knowledge on the ERP risks (Davidoff, 2015; Davidoff et al., 2017), moral hazard perhaps highlighted through default in payment of property taxes

(Shiller and Weiss, 1999) and homeowners insurance (Moulton et al., 2015).

Regulation can have a dual role. From the evolution point of view, regulation can help establish confidence in a market in nascency and therefore it can help ERP market development. Once the market matures, regulation is primordial in assuring the stability of this relatively new asset class. Moreover, regulation should ensure that exogenous shocks such as pandemics, climate change events, political events and more broadly societal wide events, do not destabilize the ERP market directly and the financial system indirectly.

Regulation must be understood also in the context of local cultural development and other international network legal subordination. There is also some differentiation between common law countries and Roman law countries. In addition, as with any other set of regulatory measures, the question of "who regulates the regulator?", while remaining answered, it should stay in the mind of any researcher.

This chapter explores the design structure of equity release product market space in the United States (US), United Kingdom (UK), Euro Area (EU), Asia, and the far Eastern countries. We engage a critical discussion on unique and common features shared by the various market jurisdictions in these selected countries. The final section of the chapter discusses common problems and pitfalls related to regulation of ERPs.

4.2 REGULATION OF EQUITY RELEASE PRODUCTS IN THE US

The main ERP in the US is insured by the Federal Housing Administration (FHA) and it falls into the Home Equity Conversion Mortgages (HECMs) class of loans. Other ERP alternatives include: (1) home equity lines of credit (HELOCs), home (2) equity loans, and (3) cash-out refinancing.

The FHA is the institution empowered to update and regulate reverse mortgages to protect borrowers. The most important rules regarding the borrower of reverse mortgages in the US are:

1. borrowers must be 62 years of age or older.
2. borrowers ought to own their home.
3. borrowers ought to own their home outright, or have a substantial amount of equity.
4. borrowers ought to live in the home as their primary residence.
5. borrowers ought to complete a financial assessment

A home can only be one classified as single-family, and if the property is multi-family then one unit must be occupied by the senior homeowner. Moreover, vacation homes and second homes do not qualify as collateral for reverse mortgages, whereas manufactured homes and condominiums may be eligible for a reverse mortgage loan.

After the loan is issued the borrowers are required to:

1. Immediately using reverse mortgage loan funds to pay off any other mortgage you may have. Continuing payments on your home insurance, property taxes, and basic home maintenance. Complying with all the loan terms, such as continuing to live in the home as your primary residence.

The Home Equity Conversion Mortgage (HECM) loans are non-recourse. Therefore, if the loan accrued balance is not repaid after maturity, no assets other than the collateral house can be used to pay off the reverse mortgage loan. Moreover, if the loan accrued balance is greater than the value of the collateral house, the lender cannot recover more than the amount the selling price of the collateral house. In addition, the FHA has set additional rules to protect borrowers and develop a sustainable reverse mortgage loan market.

Prior to the loan approval, borrowers should complete a counseling session with an FHA-approved counselor. In the first year of a reverse mortgage loan, the borrowers may only access 60% of the approved loan amount, or the amount required to pay off your current mortgage plus 10%, whichever is greater. Subsequently, the borrower may access the remaining loan amount.

Importantly, the regulator insists that lenders are not allowed to require borrowers to add extra loans or financial products, as a pre-condition of granting the ERP loan. Furthermore, lenders should also complete a financial assessment of potential borrowers and perform a thorough analysis of borrowers' income against their expenses. If this ratio indicates that there could be some problems in paying various other liabilities (taxes, insurance, etc.), the borrowers are permitted to set aside a fraction of the loan funds in order to pay their financial liabilities.

Two new rules were implemented in 2014 and 2015 in the US regarding non-borrowing spouses. Following the death of a borrower, the non-borrowing spouse may continue living in the house that was used as collateral, but the non-borrowing spouse will inherit the responsibility for the reverse mortgage loan as well as the house's ownership. This rule may impact the final terms of the loan, the age of the non-borrowing spouse may change for example the amount available to borrow.

The rule on the financial assessment requires lenders to examine all reverse mortgage loan applicants, and analyse their income, taxes, assets, payment history, and other outstanding debt.

Upon granting the loan, the borrowers must satisfy some conditions, on an ongoing basis. These areas follows. The borrowers must occupy the home as their primary residence and if they decide to move out or even leave the house for over a year, the loan becomes due and payable automatically. Furthermore, the loan must also be repaid if the borrower move to a long-term care home.

2. Tax and insurance

Under the reverse mortgage terms, the borrower must pay annual property tax and maintain a homeowner's insurance policy. These requirements accompany almost all home loans, so anyone who has held a forward mortgage will be accustomed to these ongoing property charges. Failure to pay property tax or maintain homeowner's insurance will make the loan due and payable.

3. Maintaining the home

The final requirement of an FHA-insured reverse mortgage is maintaining the home's condition. The home must remain in good repair throughout the loan, as determined by the loan servicer. Upkeep the house, paying property tax and insurance, and remaining in the home will ensure the borrower is in good standing on the reverse mortgage and can age in place if they choose to.

4.3 REGULATION OF EQUITY RELEASE PRODUCTS IN THE UK

The UK equity release market dates back to the 1970s, mainly supported by building societies. Until 1991 there was no legislation regulating equity release, but in 1992, a ruling under the Equitable Life Assurance Society v Hyman case ruled that the company's directors could not claim unlimited power over pension investments and then the Retirement Income Act of 2000 and Equity Release Council Regulations 2003, introduced a cap on the liability side for equity release products. Its ups and downs could partly be linked to industry-specific events such as mis-sellings that led to significant market impairment.

In the UK, the ERPs are governed by the Financial Services Act or the Consumer Credit Act. There is additional self-regulation under the Mortgage Code or Safe Home Income Plan. Advisors and sellers of ERP are regulated by the Financial Conduct Authority (FCA), the UK's financial services regulator and watchdog. The Equity Release Council is an organisation related to the UK's equity release industry that considers for consumer protection five product standards: fixed or capped interest rates (for lifetime mortgages), the right to remain in the property, the right to move to another property, the “no negative equity guarantee” and the right to make penalty free payments.

In the UK there are two main types of ERP: a lifetime mortgage and a home reversion plan. A lifetime mortgage is essentially a collateralised loan with the collateral house being the borrower's house where the borrower has their primary residence. Roll-up interest accrues on a compound interest basis unless the borrower pays the interest in full each month. For a home reversion plan the borrower relinquishes ownership of the entire or a proportion of their house in exchange of an upfront sum of money. They are allowed to stay in the collateral house rent-free. For both types of ERP, repayment to the lender comes out of the sale of the house when the customer passes away or moves into long-term residential care.

The UK Prudential Regulation Authority (PRA) expressed concerns in 2018 that firms investing in ERP should hold reserves in lieu of the cost of the no-negative-equity guarantee. Its consultation paper CP 13/18, published 2 July 2018, provided a benchmark for valuing the guarantee. The CP 13/18 paper recommended modelling the NNEG as a value of the portfolio of put options expiring at each period when cash flows are accounted for, each option being weighted by the probability of mortality, morbidity and pre-payment. The paper also recommends that the option valuation is done using a “version” of the Black–Scholes pricing formula.

It seems that the UK PRA recommended that the price of the option should reflect the cost of deferred possession of the property somehow independent of any proper assumptions about future property growth.

Since the NNEG clause is predominantly associated with the RM market in UK, we will situate our conceptual framework on current requirements in Product Standards (PS) within the Statement of Principles of the Equity Release Council (ERC) and Bank of England's Prudential Regulatory Authority (PRA). An issuer of a RM has to consider many factors that contribute to the price dynamics of the RM and subsequently, other cashflow valuations. The main factors are age of borrower(s), initial house price, loan-to-value (LTV), house price growth, risk-free rate, roll-up rate to be applied on the disbursed loan, mortality rate of borrower(s), long-term care (LTC) incidence, early prepayment rates, current yield curve, forward yield curve, funding issues if necessary, idiosyncratic risk due to postcode house price differences, ratings requirements if any, regulatory requirements (Solvency II) and most likely the list is not exhaustive.

For UK reinsurers, the PRA's rules on valuation are described in Valuation 2.1 of the PRA Rulebook. For fair valuation the requirement is to value assets at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction. Companies reporting under UK GAAP, FRS 102 employ the fair value as the amount for which an asset could be exchanged, a liability settled, or an equity instrument granted could be exchanged, between knowledgeable, willing parties in an arm's length transaction. On the other hand, IFRS 13 defines fair value as the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date.

In his letter to Mark Carney, the Governor of Bank of England, Philip Hammond from HM Treasury, stated:

> In discharging its general functions, the PRA must also have regard to the regulatory principles set out in Section 3B of the Act, which are:[…] the principle that a burden or restriction which is imposed on a person, or on the carrying on of an activity, should be proportionate to the benefits, considered in general terms, which are expected to result from the imposition of that burden or restriction.

The key word here is "proportionate" and this is why it is imperative to allow insurers to conduct internal calculations on the risks associated with ERMs. In their document Prudential Regulation Authority (2017), the PRA made it clear that they will gauge the allowance made for the NNEG risk against its view of the underlying risks retained by the issuer. Their assessment is spanned by the following four principles,

1. Securitisation where firms hold all tranches do not result in a reduction of risk to the firm.
2. The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG of the present value of deferred possession of the property providing collateral.
3. The present value of deferred possession of property should be less than the value of immediate possession.
4. The compensation for the risks retained by a firm as a result of the NNEG must comprise more than the best estimate cost of the NNEG.

The first two points are clear. The second point reflects the concept introduced in the PRA's discussion paper DP1/16. According to the PRA, the best estimate cost of the NNEG is “the mean of a stochastic distribution of possible future guarantee costs, where random variables used in the stochastic projection have been calibrated based on a best estimate of their true distributions.”

In Prudential Regulation Authority (2018) there is a substantial section on feedback to responses received on various risk-calculation issues on ERMs. On point 2.29 the PRA considers that the Black-Scholes formula is still appropriate for NNEG put option valuation, but in CP13/18 they also made it clear that other option pricing frameworks may be used as long as it can be demonstrated that valuations meet the four principles enumerated above. Black-Scholes formula has been reiterated in Prudential Regulation Authority (2019), that describes the final methodology on managing illiquid unrated assets and equity release mortgages. The formula is described with two fixed values for the main two parameters that are difficult to estimate, the volatility of the house price $\sigma = 13\%$ and the **minimum** deferment[1] rate $q = 1\%$.

[1]The PRA's 1% deferment rate became effective from 31 December 2021 where it completely phases out the current nil deferment rate regime.

It is quite extraordinary to see not only the model to be used for valuation being specified by the regulator but also the exact values of its parameters. While we understand that the regulator should think about potential system wide spread problems that may impact the financial stability of the financial system and the economy more generally, the questions emerging are how the regulator decided that the Black-Scholes is appropriated and how they decided on the actual pointwise estimates of the parameters to be used in the Black-Scholes model. The risk-averse attitude of the regulator, which is understandable given their role, when coupled with the wrong parameter estimates, could lead to a negative impact on the ERP market. Given that the Black-Scholes relies on the geometric Brownian motion for the dynamics of the underlying asset, house prices in this case, the volatility increases *proportionally* with time. Therefore, an inflated value of the volatility parameter will inflate the values of the NNEG valuations. The longer the time to maturity for the options the larger the excess value inflated. As it is very clear from standard market practice, this cost due to capital reserving for NNEG, is passed on to the borrower in the form of a higher roll-up rate. This in turn will make the ERP not highly feasible to be used by those in need, who in turn will ask local councils and government to help.

Furthermore, there is another compounding effect of the current regulation. The lenders cannot take advantage of the portfolio effects and they have to calculate the NNEG and sum up all values for their loans in the portfolio. As one could notice, the regulation was issued before the covid-19 pandemic. This extraordinary period demonstrated that it is possible for ERP issuers to face clusters of termination events combined with possible stagnation and even deflation of house prices. For these periods it makes sense to add the NNEG capital reserves. However, for those periods, the weighting of each individual option by the conditional probability of dying or moving into long-term care will be all wrong since they will be calibrated on national mortality tables prior to the pandemic. Secondly and importantly, we cannot assume that the NNEG capital reserving should be based on a permanent pandemic situation.

The equity release loan can be seen as a portfolio of an income security and a crossover put option that is automatically applied at termination, effectively posting the house as collateral in the loan back to the lender even if the accumulated outstanding balance is larger. The termination time is obviously determined by one of the following four types of risk: mortality, long-term care, prepayment and refinancing Szymanoski (1994). Depending on the market conditions, it may work out in both parties interest to have a longer termination time. A priori, there is no reason to believe that the longer the contract is alive the worse it is for the lender. The advantage or disadvantage position in the contract is contingent on the relative *cumulative* growth of house prices versus loan balance.

Moral hazard may increase the NNEG risk if the borrowers forfeit on their obligations to maintain the state of the property, as mentioned by (Shiller and Weiss, 1999). To minimise this risk most ERMs products require borrowers to maintain the insurance for the property, pay the running fees on the house, maintain the property, be the sole residents in the property, do not leave the property unoccupied for longer than six months.

4.4 REGULATION OF EQUITY RELEASE PRODUCTS IN THE EU

ERP design generally remains similar, although not entirely harmonised within the European Union (EU). Product regulation frameworks are mainly country-specific, despite the existence of some shared regulations when it comes to general financial and consumer protection. This introduces some degree of subjectivity into ERP regulations. The evaluation of the regulatory framework may also be done on different fronts, either social or macroeconomic. Diverse regulatory advances have been made across the EU. ERP regulation in Italy, for example, has transitioned through notable rules/decrees. These, respectively, include a) law decree No.203 -30 September 2005, b) law No.248 -2 December 2005, c) law[2] No.44 -2 April 2015 and d) decree No.226 - 22 December 2015. These transitions are largely modifications of their predecessors. Decrees c) and d) jointly form a recent foundation towards a meaningful identification of regulatory operating rules introduced by Italy's Ministry of Economic Development. Policy-related issues put forward by (Baldini and Causi, 2016) tends to suggest that law No.226 of 22 December 2015 is essentially tilted towards fostering marketing of ERPs rather than pushing forward a set of macro-economic or social objectives for sustainable ERPs.

The comparison of regulatory frameworks across the EU is based on the support it offers to sustain the social purpose of ERPs. It is worth looking for provisions made for consumer protection. Provisions such as financial counseling for potential borrowers, product education to improve awareness of market actors, including several others. The UK provides these key support through relevant rules issued by the Financial Conduct Authority (FCA). ERP loan originators in the UK are also monitored by the Prudential Regulatory Authority (PRA) of the Bank of England.

The slow rising borrower demand may be met by pre-existing and new lenders as the market gradually expands. New entrants into the ERP loan originator landscape across the EU are from the banking and non-banking sector.

House ownership has been encouraged in developed countries of EU so that elderly are less dependent upon government subsidies for this category of population. The main idea behind this policy is the theory that there is an inverse relationship between levels of house ownership and welfare spending, (Kemeny, 1981). This theory has received empirical support from a study showing that the level of European households wealth is largely driven by the owners' pension expectations (Muller, 2018).

ERPs are useful but they need to be regulated at the European level (Settanni, 2017). The EU is trying to connect housing wealth to the pension inadequacy. Despite its conceptual success the ERP are not widely spread in EU. There are only a few countries that developed a proper legal framework. From the EU, Ireland and Spain are more pro-active, and Italy has recently changed their legal system to promote ERP. The UK is quite developed by comparison.

The reverse mortgage or "hipoteca inversa" as it is called in Spain was regulated first in 2007 as a loan or credit guaranteed by a mortgage on the applicant's primary residence. In Italy, the reverse mortgage or "prestito vitalizio ipotecario" was regulated only since 2005, and even then the law did not have a significant impact to improve the market expansion. Thus, the law was amended again in 2015. Ireland was mainly focused on the lifetime mortgage contract and that was regulated only from 2008. Other European countries do not have such legal framework.

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[2]https://www.gazzettaufficiale.it/eli/gu/2016/02/16/38/sg/pdf.

In Germany, legal and regulatory uncertainty are seen as major obstacles to a working market for ERP. Bartsch et al. (2021) argue that there is no single comprehensive piece of legislation that clearly regulates the terms of

ERP (reverse mortgages as they are called in Germany) contracts or the responsibilities of the loan providers. The problem seems to be that reverse mortgages are governed by the same laws regulating loans, property, and debt in general. The implementation of the European Mortgage Credit Directive into German law did not help either, requiring that lender can only provide a loan if the credit check analysis shows that the borrowers will fully comply to pay back the loan contract. But the credit check is dependent on the current complete financial situation of the loan applicant, including any future streams of income. In essence then the role of the value of house collateral is greatly diminished and credit check analysis leads to non-necessary constraints when taking the decision to grant the loan. It is thus unclear from a legal point of view whether a reverse mortgage borrower is complying with their contractual obligations.

4.5 REGULATION OF EQUITY RELEASE PRODUCTS IN ASIAN AND FAR EAST COUNTRIES

In Australia, for the majority of ERPs, the government guarantees that even if the loan balance exceeds the property value, the borrower will still receive the regular payments. However, there are some reverse mortgages that are not federally insured and there is some variation in the type and degree of state regulation across the different states. In addition, some private lenders have additional internal policies impacting their ERPs.

The Senior Australians Equity Release Association of Lenders (SEQUAL) is a self-regulatory body of ERP lenders and accredited consultants. They comply with the Privacy Act, Trade Practices Act any other relevant Code or Regulation at law, see Bridge et al. (2010). SEQUAL introduced accreditation standards and a code of conduct that specifies minimum standards. Although there is a “no negative equity” policy for

SEQUAL members, this is not directly enforceable by law. The Uniform Consumer Credit Code is law biding for lenders. One Commonwealth law allows taking action against lenders who mislead borrowers during the advertising or sale of ERPs.

In New Zealand, the government passed a code of practice for the ERP sector, insisting on disclosure of terms and conditions, risk management practices and recoveries. Notably, in New Zealand some local councils allow low-income older people defer payment rates but then interest accrues on the outstanding amount and the debt becomes payable when the house is sold.

While most ERPs do not require borrowers to make any payments until they die or move out, in Japan there were some products designed to allow periodic interest-only payments. These loans were designed for financing home renovations. It is also interesting that the Japanese government guarantees the cash-flow of regular rental payments if an older couple agrees to let the collateral house to a young family that is in need of accommodation.

4.6 COMMON PROBLEMS AND PITFALLS RELATED TO REGULATION OF ERP

Some problems are common to all market spaces irrespective of product uniqueness or prudential regulatory requirements. Issues concerning product knowledge, reason(s) for unlocking and consuming home equity, and understanding of ERP contract terms mainly pertain to borrowers and form the central part of demand side problems. Problems in this category directly affect the borrowers ability to make an optimal[3] financial decisions and are common to the global market space. Problems and pitfalls related to regulation of ERPs does not only originate from a *lender-borrower* relationship. The *regulator-lender* relationship is another source of problem to consider.

On this front, regulatory requirements concerning pricing, capital reserving, funding, and cross-over risk (event that the outstanding loan balance exceeds the market value of the collateral property upon termination) are common supply side problems. Other problems include treatment of adverse selection, interest cost, and moral hazards. The following paragraphs will discuss each of these pitfalls.

Some conditions are worth considering for the survival of ERPs in the EU. Among several others, the wealth situation and ease of meeting basic needs among the senior population is critical to ERP market growth. Sustainable government policies that address these two problems will most certainly result in declining market growth and demand for ERPs within the EU.

Lenders can incur losses if the borrower(s) remain in the home for many years and if the rate of house price appreciation is relatively low. ERPs are by majority no-recourse loans and the borrowers' induced date of termination of the contract may lead to adverse selection and moral hazard in the reverse mortgage market. Adverse selection occurs then borrowers live much longer than expected. There is a famous case of well-known French citizen Jeanne Calment who lived until the age of 122. Her lender Andre-Francois Raffray bought her apartment forward when Calment was in her eighties.

Shiller and Weiss (2000a) highlighted the moral hazard aspect of home equity conversion. A borrower of an ERP may have no incentive to maintain and repair the collateral property. However, in most countries it is a legal requirement that the home owner must maintain the property to a minimum required standard. Another second moral hazard that is difficult to control through regulation is that by providing additional payments to an elderly, the borrower may stay for longer in the same property.

Model risk has only recently been considered from a regulatory standpoint of financial markets, mainly in the aftermath of the global financial crisis of 2007-2009. The Federal Reserve states that “model risk should be managed like other types of risk” and that “banks should identify the sources of model risk and assess the magnitude”. The European Banking Authority also says “Institutions should include the impact of valuation model risk when assessing the prudent value of its balance sheet. [..] Where possible an institution should quantify model risk by comparing the valuations produced from the full spectrum of modelling and calibration approaches”. The cost of using wrong models in financial markets can be quite high as discussed in Farkas, Fringuellotti, and Tunaru (2020).

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[3]Suboptimal financial decision is only observed when the loan is unable to cover the major vulnerabilities the borrower is facing.

4.7 SUMMARY

Regulations of ERP around the world have common features but they evolved also with some differences. This asset class is aimed to help elderly citizens and it combines characteristics of a consumer loan with an insurance product.

Several market events that contributed to borrowers’ dissatisfaction pushed regulators to think seriously about the negative equity part associated with this type of financial product. The legal solution differs in the most developed ERP markets, the US and the UK. While in the former, the negative equity is covered through insurance of that particular risk so that the cost is externalised and passed on tot the consumer as a running fee, in the latter jurisdiction it is required to be absorbed by the lender and it is internalised first and passed to the borrower as part of the roll-up rate applied to the loan.

Regulation is used to start-up an ERP market by providing some confidence to elderly borrowers that they are not overcharged. Even when up to date regulation is passed into law, local cultural considerations may still act as a friction to the development of a viable ERP market, such as in the case of Italy.

However, the world-wide spread of ERP will also trigger regulations to be perfected in the future. In the UK the major actors in this market also contribute substantially in the consultative process on regulation. This ground up movement may be the reason why in the UK the ERP market is the fastest growing.

The following references proposed for further reading contain more details that were not captured in this book. In particular the latest consultation papers published by the Bank of England PRA on important issues such as securitisation, resecuritisation, matching adjustments are very useful for real ERP market activities.

The SS1/23 and PS6/23 in particular, we find them extremely interesting since they focus on model risk, a topic that is usually neglected by risk managers of banks, insurers and other financial institution. Given that the ERP market does not benefit from liquid asset trading to convey market information transparently and directly, model risk gathers an increased dimension for risk managing assets and liabilities in the ERP market.

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CHAPTER **5**

Equity Release Instruments: Risk Drivers and Valuation

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5.1 INTRODUCTION

THERE ARE SEVERAL risk factors contributing to the valuation of an ERP. In this chapter we will discuss those risks and how they may impact the valuation from a lender perspective who will issues the bonds and then has to carry them in their portfolio until maturity. Furthermore, we will consider some preliminary valuation issues and emphasize a general procedure for the valuation of ERP loans.

In addition, we will look at discrete time modelling versus continuous time modelling approaches. The former seems more realistic when thinking of mortality risk and house price risk, while the latter seems more pertinent when thinking of interest rates and even default risk.

We will demonstrate that the unconditional volatilities in housing markets is low and in single digits as percentage, contrary to some perception by other researchers in the area who are advocating for double digits volatilities. We will delve more into this issue in the next chapter where we will be describing how to apply conditional volatility models for house price time series. While estimation risk is always present when using econometric models, ignoring the current empirical evidence may be detrimental to the development of a fully functional ERP market.

House price volatility is not the only parameter that needs careful estimation. Risk-free discount rates depend on the risk-free interest rate. Which interest rate should be used as the “risk-free” rate has been debated in the industry before and after the LIBOR debacle. Another equally important quantity is the rental yield produced by housing in a given area. Similar to the volatility of house prices, prior misconception coupled with mixed international evidence may place the value of this parameter in a much wider range than thought.

We will also discuss some important concepts in the ERP literature such as the cross-over point, negative equity, total annual loan cost rate, that are relevant to all ERP international markets and some other concepts like set-up upfront premium and insurance monthly premium, that are more relevant to the American market.

5.2 RISK DRIVERS OF ERPS

There are many risk factors that can influence the value of an ERP at any one time as well as the final payoff. Some of them are less recognisable and some of them although clearly identifiable they are perhaps more difficult to quantify. There are factors that depend on the borrower's characteristics and factors that are more about the economic environment where the borrower lives.

In this section first we will enlist the risk factors and then we discuss each of them in turn.

5.2.1 Key Risks for Reverse Mortgages

Here is an inventory of risks involved in trading reverse mortgages or ERP.

- *Uncertain Maturity*. There is no fixed maturity of this product. The timing is driven by

  1. the death ("mortality") or
  2. entry into long-term care ("morbidity") of the borrower,
  3. or their decision to repay voluntarily (prepayment).
  4. other possible conditions under the terms of the contract

  The uncertain maturity makes it very difficult to value the no-negative-equity guarantee or the insurance no-negative cover premium.

- *Mortality Risk*. The life of such as mortgage can be lengthy (a 65-year-old will, on average, live for a further 15 to 20 years). This risk is usually calibrated using past historical data, or national mortality tables. Other academic papers also suggest using stochastic mortality models.

- *Morbidity Risk*. The loan must be repaid following the death or entry into long-term care of the last surviving borrower. This is a risk for which it is very difficult to get historical data to do any calibration.

- *Adverse Selection Risk*. This refers to the fact that a borrower who feels that her/his longevity risk is genuinely less than the longevity risk for the average representative of a group category will have an incentive to enter into a reverse mortgage. This also works in reverse for borrowers that feel they have a larger than average longevity risk.
- *House Price Appraisal Risk*. The maximum allowable LTV varies based on the age of the borrower(s) and the appraisal value of the house. The appraisal is a subjective process and neighbourhood characteristics may impact the appraised value, in an upward or downward manner.
- *House Price Risk.* Another risk factor is the residential property market. For a pool of loans that is diversified geographically, the real-estate price risk is partially diversifiable but there is still exposure to economic recessions at a national level. The individual collateral house price risk may deviate from the area or regional house price risk as reflected in a real-estate housing index. This *idiosyncratic* house price risk while real, it is usually dealt with by using a hair cut applied at the termination of the loan and settlement of the contract.
- *No Negative Equity: Guarantee NNEG*. This is also called crossover risk and it appears when the value of the house is lower than the value of the loan balance. It is written in the contract that the borrower or his/her heirs are not liable for losses due to property market falls. Crossover risk can be either passed onto an insurer if an assignment option exists in the covenants of the contract or it can be securitised with a portfolio of zero-coupon bonds of various maturities.
- *Transfer Risk.* The loans may be transferable, subject to maintaining a suitable LTV for the age of the borrower at the time of the transfer. The

transfer may be in a different geographical location.

- *Interest Rate Risk and Leverage*. The loan interest is added to the capital loan balance so the leverage increases with time. The lender needs to match his assets with liabilities in terms of duration and convexity. There is one rate that is used for loan balance accrual and another rate that is used for discounting. The former is much larger than the latter but it should also be noted that during the life of the contract, R cannot change while r it may.
- *Legal Risk.* The lender gets an equity stake in the property while the borrower retains legal ownership until the loan is paid off.
- *Prepayment Risk.* The borrower may voluntarily prepay the loan, subject to penalty charges.
- *Interest rates and Business Cycles.* Interest rate risk, extends over a much longer and more uncertain period than forward mortgages rates in some countries, such as the UK in particular.
- *Cash-flow risk.* The cash flow generated by a portfolio of these assets is very low early on in the transaction and thus there could be liquidity problems.
- *Operational maintenance risk.* The weighted average life (WAL) of these loans gives rise to operational risk in that the condition of the properties that form the security could deteriorate after origination, particularly if the borrower is elderly and/or infirm.
- *Consumer Law Risk.* The market is defined by elderly people, a vulnerable group, and the lender may be subject to various strict consumer protection measures.

- *Default Risk.* This is more related to bankruptcy law and situations when the loan is a cross-over of reverse mortgages with bankruptcy.
- *Quick sale discount* In the end the responsibility for the sale of the property upon exit may be passed, under some conditions, to the lender who may prefer to reduce the sale price on a vacant property, trying to maximize the price for the owner within a specified timescale. In this situation the value of the NNEG will increase farther and this risk may be correlated to the general direction of the market in house prices.

5.2.2 Mortality Risk Modelling for ERPs

The sellers of reverse mortgages have considered for a long time that longevity risk is diversifiable. Hence, by pooling a large numbers of loans we could use mortality tables to determine the terminations of loans.

We let τ_x denote the time until death of the homeowner aged x at inception of the ERP contract, where $x \geq 60$. Time until death is a random variable, since the death of the borrower will occur at some age greater than the known age at inception. The time-until death can also be termed as the future life-time of the borrower. Dickson, Hardy, and Waters (2019) provide an excellent discussion on this concept. Essentially, we can conceptualize this notion in a simple example. Suppose that the borrower is aged x at inception of the contract, and dies τ_x years after. This means that $x + \tau_x$ will be the borrowers age at termination of the ERP contract. Actuaries model this future lifetime random variable in various ways be it continuous or discrete. Since τ_x is random, its distribution function in the continuous setting can be written as $F_{\tau_x}(t) = P(\tau_x \leq t)$. Furthermore, we can define the *survival probability function* that the homeowner lives longer than a time point t as

$$S_x(t) = 1 - F_x(t) = P(\tau_x > t)$$

(5.1)

The distributional modelling for the survival probability function depends on some necessary assumptions. According to Dickson et al. (2019), the following three assumptions are necessary and valid for all distributions.

- The survival probability function must be differential for all t.
- $\lim_{t \to \infty} tS_x(t) = 0$ and
- $\lim_{t^2 \to \infty} tS_x(t) = 0$

There is a limiting age when dealing with the future lifetime of the borrower. This limiting age can be linked to the life table of the country. For example writing $S_{60}(45) = 0$ suggests that the borrower who is aged 60-years at inception of contract cannot survive beyond 105 years. It is possible to relate the future life-time random variable τ_x of a borrower aged x to the standard life table used in the country of reference. To do this, we write τ_0 to denote the borrower's future life-time at birth. In the case of ERP loan borrowers, we know $\tau_0 > x$, suggesting that the borrower has survived to the required inception age at which they purchase the contract. The time till termination of the ERP loan contract will therefore coincide with the borrower age at death, written as $\tau_x + x$. For example, if the ERP loan borrower dies within the next t years from contract inception, then $\tau_x \leq t$ and $\tau_0 \leq x + t$. We are able to use life table values for our mortality calculations if the two latter events i.e. $\tau_x \leq t$ and $\tau_0 \leq x + t$ are equivalent, given that the loan borrower survives to the contract inception age *x*. Following Dickson et al. (2019), we can write, for all $t > 0$ in the case of all borrowers:

$$\begin{aligned} F_x(t) = P(\tau_x \leq t) &= P(\tau_0 \leq x + t | \tau_0 > x) \\ F_x(t) &= \frac{P(x < \tau_0 \leq x + t)}{P(\tau_x > x)} \\ F_x(t) &= \frac{F_0(x+t) - F_0(x)}{S_0(x)} \\ S_x(t) &= \frac{S_0(x+t)}{S_0(x)} \end{aligned} \tag{5.2}$$

Actuaries use a concept called the instantaneous death rate defined by the force of mortality at age *x*, and written as

$$\lambda_x = \lim_{\Delta t \to 0^+} \frac{P(\tau_0 \leq x + \Delta t | \tau_0 > x)}{dx} = -\frac{S_0'(x)}{S_0(x)} \tag{5.3}$$

The instantaneous death rate concept accounts for circumstances where the borrower aged *x* dies in the next moment. This explains why Equation (5.3) takes the limit as Δt approaches zero.

If the instantaneous rate of mortality is assumed to be constant then $F_{\tau_x}(t) = 1 - e^{-\lambda t}$ and the corresponding probability density function is $f(t) = \lambda e^{-\lambda t}$. The life expectancy is then calculated as

$$E(\tau_x) = \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}$$

(5.4)

Thus a constant rate of mortality implies a life expectancy that is independent of the current homeowner's age.

For our exemplifications, we use the mortality data derived from the Continuous Mortality Investigation Research (“CMIR”) 00 mortality tables, to which Norwich Union is a contributor. The tables are referred to as Immediate Annuities Male Lives (“IML00”) and the Immediate Annuities Female Lives (“IFL00”), adjusted for cohort effects (i.e. where rates of improvement in mortality have been different for people born in different periods historically). The tables show the probability of death during any year for an individual of a particular age who is alive at the start of that year. Actuarial experience suggests that females live longer than males.

The Table 5.1 showing the evolution of mortality rates over time we can see that longevity increased for both males and females, and more for the latter.

TABLE 5.1 Longevity expectations based on Immediate Annuities Male and Female Lives.

Expectation	of life	at birth	Expectation of life	at age 65
	Male	Female	Male	Female
1841	40	42	11	12
1900	49	52	11	12
2000	76	80	16	19
2020	79	83	18	21

Notes: Derived from Continuous Mortality Investigation Research 00 tables.

In many instances the loan is given to a living couple. The loan will survive as long as one of the couple survives. Hence, there is correlation built-in as couples can take care of each other and survive longer. There is also a selection bias, people taking up reverse mortgages have more money to look after themselves and therefore live longer than their peers.

Another common assumption made about mortality Brockett (1991) is that the death of the homeowner is uniformly distributed in the interval [0,*d]*. Then, if *x* is the current age of the borrower, conditional that $\tau > x$ the cdf of τ is $F_\tau(t) = \frac{t}{W-x}$ with the density is $f(t) = \frac{1}{W-x}$. The life expectancy in this case is equal to $W/(W-x)$. The above shows that the assumption made for mortality rates can have a big impact on the expected lifetime of borrowers.

5.2.3 Deferred Termination Probability of ERPs

The ERPs are calculated as the present value of weighted cash flows received by the borrower where the weights are the deferred termination probabilities. The concept of deferred termination probability arises from the fact that the ERP loan will remain active for a period after inception until its termination. This active period is what we term as deferred termination period. It is essentially the same as saying the termination of the loan contract occurs at some future date, given that it remains active for some time after inception.

The deferred termination probability is denoted by ${}_{t|j}q_x$, where. *x* is the age of the borrower at contract inception. The calculation of ${}_{t|j}q_x$ is the probability that a borrower aged *x* terminates a contract between time $x+t$ and $x+t+j$. Dickson et al. (2019) presents a nice derivation of the deferred mortality concept. In a discretised mortality framework, the deferred termination probability can be calculated with values in the standard life tables.

5.2.4 Multiple Decrement Concept

It is possible to setup a multiple decrement model to evaluate the probability of termination. In Figure 5.1, ${}_tq_x^{(1)}$ denotes the probability that an active borrower aged *x* dies between age *x* and $x+t$; ${}_tq_x^{(2)}$ is the probability that an active borrower aged *x* voluntarily prepays between age *x* and $x+t$, and ${}_tq_x^{(3)}$ denotes the probability that an active borrower aged *x* moves into long-term care between age *x* and $x+t$. An active ERP loan contract is considered to be in state (0) i.e. the *Active state*. For any decrement $d=1,2,3$ we let ${}_tq_x^{(d)}$ denote the probability that a borrower aged *x* at inception of the contract fails within *t* years due to decrement (d).

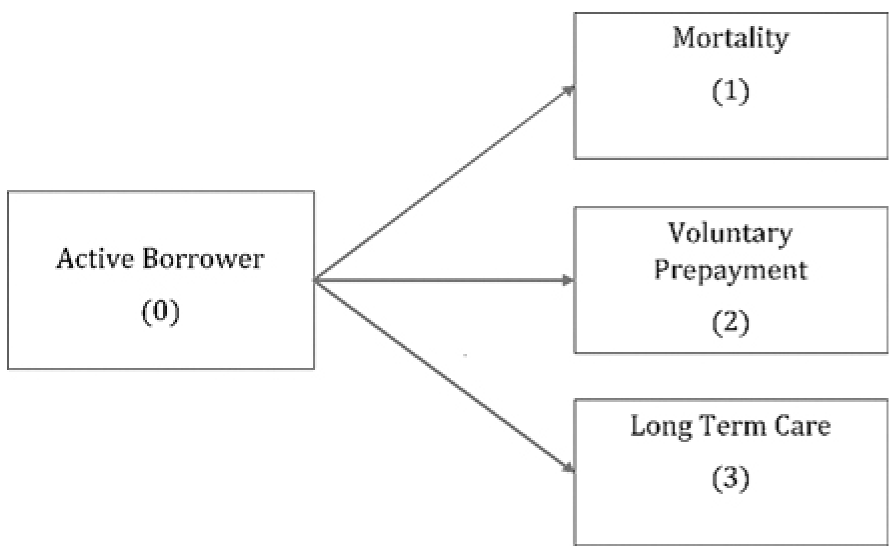

Figure 5.1 Discrete probabilities of ERP termination

This suggest that the principal borrower aged *x* moves out of the active state, thereby causing the ERM contract to terminate. All the three states of decrements are mutually exclusive, in this regard we denote the probability of failing due to any given decrement by $\widehat{{}_tq_x}$ which is the sum of the individual mutually exclusive probabilities of transition between the three states. For the three-state decrement model in Figure 5.1,

$$\widehat{{}_tq_x} = {}_tq_x^{(1)} + {}_tq_x^{(2)} + {}_tq_x^{(3)} \tag{5.5}$$

The probability of remaining in the active state is $\widehat{{}_tp_x} = 1 - \widehat{{}_tq_x}$. On this basis, the probability that any contract will be terminated within *t* years due to a specific decrement depends on *x* being in the active state before the failure year. More specifically, *x* survives $t - 1$ years before failing in the *t*-th year. The probability that an ERP contract issued to *x* terminates due to decrement *d* within *t* years is

$$\begin{aligned} {}_tq_x^{(d)} &= \sum_{k=0}^{t-1} {}_kp_x^{(\tau)} q_{x+k}^{(d)} \\ &= {}_{k|}q_x^{(d)} \end{aligned} \tag{5.6}$$

This can be extended to the form:

$${}_{t|u}q_x^{(d)} = \sum_{k=t}^{t+u-1} {}_kp_x^{(\tau)} {}_uq_{x+t}^{(d)} \tag{5.7}$$

which denotes the probability that a borrower aged *x* terminates a contract due to decrement *d* between time $x + t$ and $x + t + u$. In instances where the loan is given to a living couple, the loan will remain active as long as one of the couple survives. One common assumption is to use for a borrowing couple a 95% adjustment factor of the base mortality table for the male and female.

Knapcsek and Vaschetti (2007) calculate the joint *cumulative* probability of death after *t* years for a couple *x*, and *y* with the formula

$$_{t}q_{xy} = {_{t}p_{x}} \times {_{t}p_{y}} \tag{5.8}$$

where $_{t}p_{x}$ is the cumulative probability of death by year *t* for *x*. The same goes for $_{t}p_{y}$. There is also a possible correlation built-in as couples can take care of each other and survive longer.

In this regard the ERP loan contract survives *t* years and fails within the next *u* years due to some decrement *d*. In a multiple decrement setup, the ERP value is the present value of weighted cash flows where the weight is equal the multiple decrement probability.

5.2.5 Long-Term Care Risk Modelling for ERPs

Morbidity is defined as the movement of people into long-term care. This is defined as the inability to carry out at least two activities of daily living ("ADLs"). The ADLs test the borrower's ability to care for themselves in their own home and include the capacity to feed, clothe and wash themselves, among others. There is very little data available on the movement of people into long-term care as a result of their inability to perform ADLs and making it difficult to accurately predict the rate of morbidity – which will affect the timing of the underlying cash flows entering the transaction.

The people who have contracted a reverse mortgage have a greater incentive to remain in their property. Future governmental policies may benefit the owners. The actuarial market practice in the U.K. calculates morbidity as a percentage of the mortality rate.

TABLE 5.2 The adjustment factors for deriving the morbidity rates.

Age	Males(%)	Females(%)
≤ 70	2	3
$(70, 80]$	4	12
$(80, 90]$	5	13
$(90, 100]$	4	8

Notes: The figures are collected in discussions with various market professionals as of 2018. The authors first used this table while they were working on the IFoA research project on equity release mortgages in the UK see (Tunaru and Quaye, 2019).

5.2.6 Interest Rate Risk Modelling for ERPs

Interest rate risk interferes with ERP valuations in multiple ways. First any discounting cash-flow requires a discount rate that is taken from yield curves, either under the physical measure for risk management and stress analysis purposes or under the risk-neutral measure for valuation.

Risk-free rates are needed for discounting but the interest rate applied on the ERP loan, called roll-up rate, are also of interest. The latter depend on funding rates and other liquidity and regulatory measures and they are expected to be higher than risk-free rates.

5.2.6.1 Risk-free interest rates

The risk-free rate is essential for risk-neutral valuation because the risk-free rate is defined as the expected growth rate of asset prices under a risk-neutral pricing measure, or martingale measure. The martingale condition for discounted asset price processes implicitly makes use of the risk-free rate. For some time the risk-free rate was associated with the yield of government treasury bonds. Then, with the introduction of the floating rates and the development of interest rate derivatives markets, market makes used LIBOR, the short-term borrowing rate of AA-rated financial institutions, as the risk-free rate. The most known interest rate derivative is a vanilla swap whereby LIBOR is exchanged for a fixed rate, on a periodic basis until some given maturity. LIBOR was associated with the risk-free rate because the valuation of interest rate swaps was direct and the reference LIBOR interest rate acted also as the discount rate. For most important banks the LIBOR-swap curve was used to construct the daily risk-free discount curve.

This was widely accepted until the subprime crisis of 2007 that highlighted the role of funding. When banks were squeesed for liquidity and collateral payments were used to manage counterparty credit risk, banks started being suspicions of each other that they did not declare their liquidity needs in face of possible bankruptcy. The TED spread, defined as the difference d between three-month U.S. dollar LIBOR and the three-month U.S. Treasury yield rate, is quite small historically in normal market conditions. However, this spread increased to between 1% and almost 5% during the global financial crisis between October 2007 and May 2009.

There are two main reasons why LIBOR lost its place as the proxy for the risk-free rate after the global financial crisis. First, there was the LIBOR scandal that suggested that this rate could have been tampered with to help major banks, but more to the point some of their traders, make additional profits. On of the authors of this book argued at the time that the accusations, while having some element of truth, were disproportionately portrait. The LIBOR construction by design was removing marginal inputs and retained only the middle values before averaging those into the daily LIBOR. The only way to truly tamper with that market was if a lot more than half of the participants in the daily quotation system would be part of the rigging ring. The second reason, was the dispute between academics and practitioners over what rate to use for discounting when pricing derivatives. The famous paper by Hull and White (2013) was hotly debated between academics and practitioners.

Currently, the overnight indexed swap (OIS) rates are used to determine the discount factors applied to value derivatives contracts. Overnight indexed swaps are interest rate swaps exchanging a fixed rate for a floating rate, where the latter is computed as the geometric mean of a daily overnight rate. The payment on the floating side should replicate the cumulative interest that accrues from rolling over a sequence daily loans at the overnight rate. In the US the OIS is the effective federal funds rate or the secured overnight funding rate (SOFR), in the EU it is the Euro Overnight Index Average (EONIA), in the UK it is the Sterling Overnight Index Average (SONIA), and, in Japan it is the Tokyo overnight average rate (TONA).

Table 5.3 Panel A presents descriptive statistics results that are, for the most part, within the same order of magnitude. In contrast to CAD and AUS, the volatility of the GBP 10-year swap rate is comparatively larger. In addition, the Jarque and Bera (1980) test yields a high test statistic value, which is sufficient evidence to reject the null hypothesis and show that the three sampled countries' 10-year swap rate deviates from a normal distribution. The notable skewness and kurtosis, which further point to a deviation from normality, corroborate this finding. The 10-year swap rate time series is most likely stationary as the ERS unit-root[1] test findings in Panel C reject the null hypothesis that it has a unit-root in each of the three sampled cases. In the context of ERP valuation with stochastic interest rate modelling, this is a desired feature. Using the Fisher and Gallagher (2012) test, we also examine whether the 10-year swap rate time series contains temporal dependencies. Here, the null hypothesis states that there is no significant autocorrelation in the 10-year swap rate time series, up to 20 lags. We reject this null, suggesting that the 10-year swap rate time series of the sample of three countries exhibits considerable autocorrelation.

TABLE 5.3 Statistical properties of the 10-year swap rate

	United Kingdom (GBP)	Canada (CAD)	Australia (AUS)
Panel A: Descriptive statistics			
Mean	0.000	0.000	0.000
SD	0.033	0.022	0.023
Min	-0.190	-0.181	-0.239
Max	0.234	0.157	0.138
q_{25}	-0.015	-0.012	-0.010
q_{75}	0.015	0.012	0.011
Panel B: Distributional test			
Skewness	-0.102**	-0.019	-0.539***
	(0.015)	(0.644)	(0.000)
Kurtosis	5.926***	4.254***	8.394***
	(0.000)	(0.000)	(0.000)
JB	5027.245***	2588.266***	10241.332***

	United Kingdom (GBP)	Canada (CAD)	Australia (AUS)
Panel A: Descriptive statistics			
	(0.000)	(0.000)	(0.000)
Panel C: Unit root and autocorrelation test			
ERS	-27.657***	-7.683***	-14.693***
	(0.000)	(0.000)	(0.000)
$Q(20)$	24.203***	17.871**	59.160***
	(-0.003)	(-0.042)	(0.000)
$Q^2(20)$	1625.056***	1710.382***	2532.938***
	(0.000)	(0.000)	(0.000)
Panel D: Unconditional correlation			
	GBP	CAD	AUS
GBP	1.000	0.622	0.551
CAD	0.622	1.000	0.576
AUS	0.551	0.576	1.000

Notes: This Table reports the summary statistics, and results for normality test, unit-root, and autocorrelation tests using daily 10 year interest rate swap time series from United Kingdom, Canada, and Australia. The sample period is from 1 January 2010 to 1 March 2023. The Mean (Mean), Standard Deviation (SD), Minimum (Min), Maximum (Max), 25th quantile (q_{25}), 75th quantile (q_{75}) of each series is reported in Panel A. The distributional analysis results reported in Panel B include the Kurtosis (Kurtosis), Skewness (Skewness) and those for Jarque and Bera (1980) normality test (JB). The unit-root test (ERS) using the Elliott et al. (1996) and the Fisher and Gallagher (2012) weighted portmanteau test $Q(20)$ and $Q^2(20)$ respectively provided in Panel C. The unconditional sample correlation between the three sample time series are reported in Panel D. Data on the sampled countries are from Bloomberg.

There is a wide market consensus that, since posted collateral pays OIS, the correct discount rate to compute cash-flows for collateralised derivatives is the OIS curve. Kenyon and Stamm (2012) discuss under what circumstances the OIS curve is applicable for discounting cash-flows for valuation of derivatives. One very important point they insist on is for derivatives to be fully hedged, the collateral that is posted is in cash and in the same currency as the derivative contract payoff and last but not least, the collateral posted by one party to be postable to another counterparty a principle called rehypothecation. Furthermore, on page 64 Kenyon and Stamm (2012) state that "pricing costs of non-fully hedged trades using the risk-neutral measure can be questioned…. since they are not hedged their funding volumes will be paid in the real (aka historical) measure even if the rates are linked to LIBOR." For the purposes of ERP valuation, it seems that some of these assumptions (if not all) are not met and under the current market principles one cannot talk about a fully-hedged ERP in the UK, while in the US the insurance and reinsurance may provide full protection to reverse mortgage issuers but then the problem is only shifted to reinsurers.

Another problem with risk-rates for ERP calculus is the long maturities involved. The cash-flows that need to valued could go even to more than 40 or 50 years. Discount rates for such long maturities are obtained by extrapolation.

Table 5.3 describes in parallel the descriptive statistics and some standard tests for the 10-year swap rates in the UK, Canada and Australia. As expected there is relatively high correlation among those rates.

5.2.6.2 Negative interest rates

Short maturities are also highly relevant for cash and risk management of ERPs, and recent negative rates observed in some developed economies, have thrown a new problem into the ERP calculus territory.

In recent times negative nominal rates have been observed on the yield curve for some important economies. For example, for the euro yield curve the rates have been negative between June 2014 and June 2022 for maturities up to 5 years.

Figure 5.2 Historical evolution of the Euro yield curve

Notes: This Figure plots the daily time series evolution of the 1-year, 5-year, and 10-year maturity government bond, nominal, in the Euro area. The sample period is from 9 September 2004 to 12 September 2023. The sample period roughly covers normal times and advances during crisis period.

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[1]The unit-root suggests that the given time series data has a stochastic trend, so that it does not have a tendency to revert to a constant mean over time.

Source: https://data.ecb.europa.eu/data/datasets/YC

5.2.6.3 Interest rates for long maturities

Interest rates may impact ERP loans a lot more severe than they would influence coupon-bearing bonds or forward mortgages. Boehm and

Ehrhardt (1994a) show that the differential in interest rate risk between the reverse mortgages and the standard fixed income instruments could be very large, quite often reaching several orders of magnitude greater. Furthermore, interest rates for very long maturities such as 40 to 60 years are very difficult to calibrate because of lack of information at this end of the yield curve.

Even the 10-year spot interest rate could become problematic to gauge. Hanson and Stein (2015) show that a 100 basis point increase in the 2-year nominal yield on a Federal Open Markets Committee announcement day could determine a 42 basis point raise in the 10-year forward real rate. This is an important finding because it is contradicting standard macro-models relying upon the idea that monetary policy events cannot move by themselves real rates over a horizon longer than that over which all prices in the economy can readjust.

A common problem that insurance companies and pension funds have is the requirement to value liabilities that have maturities farther into the future than the longest maturity bonds trading in fixed-income markets. In order to discount these long maturities liabilities for all practical purposes yield curve extrapolations produced by models calibrated on existing observed yields are needed. Christensen, Lopez, and Mussche (2022) propose using the dynamic Nelson-Siegel (DNS) yield curve family of models to generate these extrapolations for risk-free yield curves. Comparing this exercise for yield curves that are representative for Switzerland, Canada, France, and the U.S. they find only some small biases in extrapolated long bond yields based on the DNS model.

An ERP product at inception is likely to depend on quite long maturities, maybe beyond 10 years. It is not easy to find out the right maturity at the long end. Furthermore, even 10-year interest rates sometimes can be negative. In Figure 5.3 the 10-year swap rate, that is used by many financial trading desks as a proxy for the risk free rate for the ten year maturity, was negative for Japan in 2016 and 2019, and it was also negative for EURO zone in 2019, 2020 and 2021.

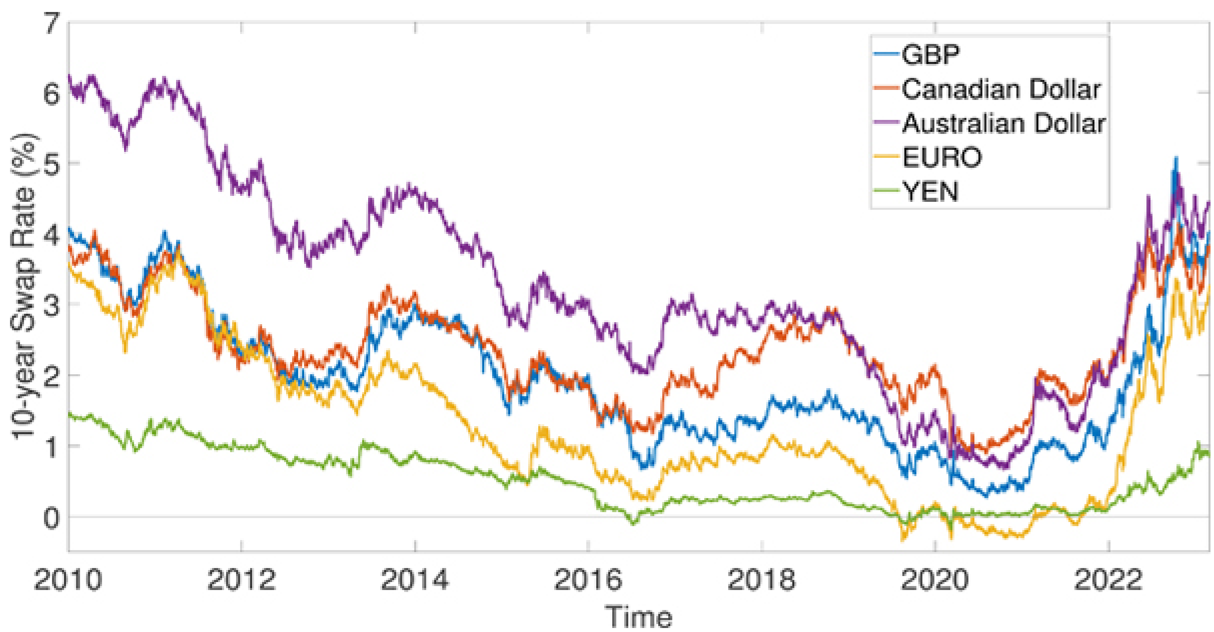


Figure 5.3 Historical evolution of the 10-year swap rate

Notes: This Figure plots the daily time series evolution of the 10-year interest rate swap benchmark across selected countries including the Eurozone. The sample period is from January 2010 to February 2023. The sample period roughly covers the past 13 years, being enough to substantially account for normal times and advances during crisis period.

The above figure also suggest that the interest rates were decreasing for almost a decade but the shock given by the covid-19 pandemic has triggered a new regime with interest rates increasing abruptly. Some of the rates such as in the case of GBP reached rapidly 5% after they were 1% not long ago. If the 10-year swap rate is taken as a proxy for discounting, one can easily see how the roll-up rate R fixed at the issuance of the loan could become lower than the risk-free discount rate *later on*.

5.2.7 House Price Risk for ERPs

Since 1870 until the middle of the 20th century, historical data suggest that real house prices stayed stable in many countries worldwide, see Jorda, Schularick, and Taylor (2015). House prices in most developed economies stayed relatively constant in real terms from the 19th century to the 1960s (Knoll, Schularick, and Steger, 2017). Glaeser (2013) provided evidence that the United States has been historically a nation of property speculators, where local and regional boom-bust periods induced significant social costs and financial instability. However, these changes in house prices frequently compensated each other in the long run. Based on a repeated sales index covering 86 properties in New York City's borough of Manhattan over a hundred years, Wheaton, Baranski, and Templeton (2009) showed that in every decade property prices rised by 20 to 50 percent and then declined likewise, such that in the late 2000s real estate in New York was worth in real terms almost the same as at the turn of the 19th century.

The evidence points out that, since 1870, in Australia house prices grew faster than income; in the U.S. and European countries like Belgium, Sweden and Germany house prices did not grow as fast as income, and for Canada, Japan and the United Kingdom, house prices grew more or less in line with income (Jorda et al., 2015).

There is evidence that the volatility of real house prices had started rising significantly only after the 1970s, see discussion in Fabozzi, Shiller, and Tunaru (2020). One main reason could be the increase in residential land prices following World War II (Knoll et al., 2017). Between late 1980s and the 2007 subprime crisis the rate of growth of real house prices outpaced significantly the rate of income growth. But this increase was more heterogeneous geographically. Metcalf (2018) studied changes in the real median house prices for the core-based areas in the United States between

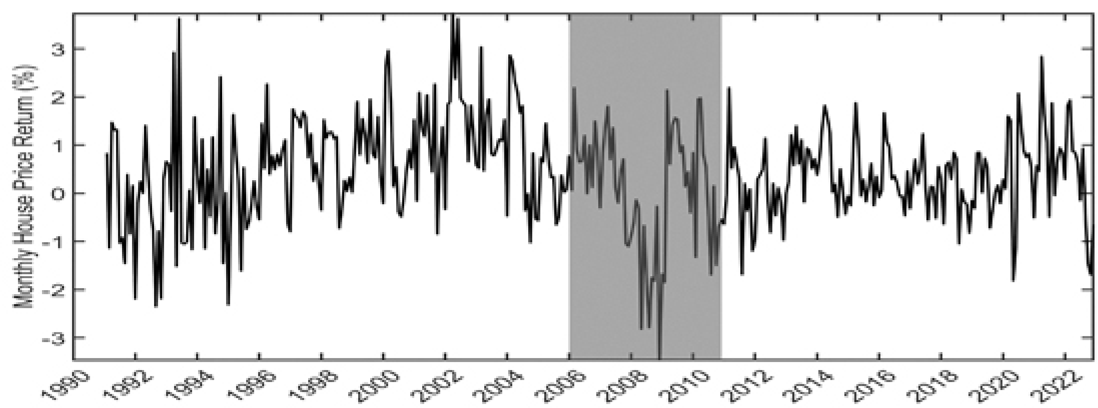
1996 and 2016. This varied from 16% in Atlanta-Sandy Springs-Roswell to 75% in New York-Newark-Jersey City and a maximum of 168% in San Francisco-Oakland-Hayward. This phenomenon led to many house owners having housing equity representing a large proportion of their personal wealth.

The total wealth tied up in real estate is extremely high in all developed economies. Post 1870 in most developed economies, the banking sector has gradually switched from business loans to mortgage loans, particularly after World War II. In western countries. total mortgage loans outstanding have increased (on average) from about 20% of annual GDP at the beginning of the 20th century, increasing to 70% of GDP by 2010 (Jorda et al., 2022). The value of US real estate in possession of households and nonprofits (thus not counting property owned by firms) is roughly $30 trillion (Federal Reserve Board of Governors, 2019), a similar size to the value of the entire US equity market. The estimated value of all developed real estate worldwide, including residential, commercial and agricultural land is $217 trillion (Research, 2016).

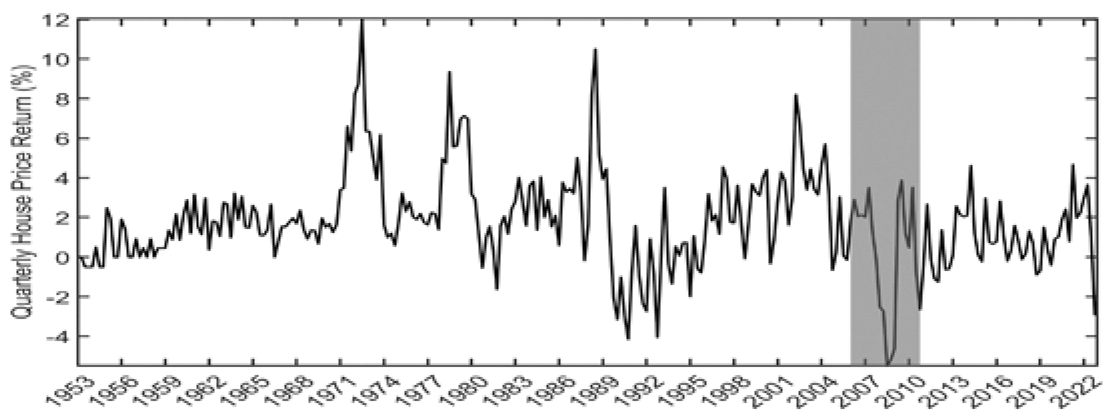
The main risk embedded in ERP is the collateral-effect channelled by house price risk. Therefore, the behaviour of the main stakeholder in this financial market depends on their understanding of house price risk. The house price risk determines the NNEG risk which is managed through two channels, by charging a portion of the interest rate risk to cover this potential fall and by insisting on a low LTV. LTVs are in general age-dependent, with lower LTVs for “younger” borrowers and higher LTVs for “older” borrowers, the difference reflecting the expectation of the lender of exit rates. There are lenders who are fine to give larger amounts of cash to borrowers that can prove that they are in poor health.

Although the most common assumption regarding the house price dynamics is the geometric Brownian motion. This is assumed for a real-estate index, for which data is available. Thus, for reverse mortgages, basis house price risk house price risk is introduced reflecting the difference in the evolution of the house price index and the price of the particular house that is the collateral in a given loan.

The graphs in Figure 5.4, Figure 5.5, Figure 5.6, Figure 5.7 illustrate the series of monthly and quarterly returns for some of the best known house price indices in the US and the UK. The frequency of the index used in the analysis may give very different parameter estimates for house price volatility. The series capture the market downfall at the beginning of the 1990s and also in the aftermath of the global financial crisis.



(a) Monthly price returns

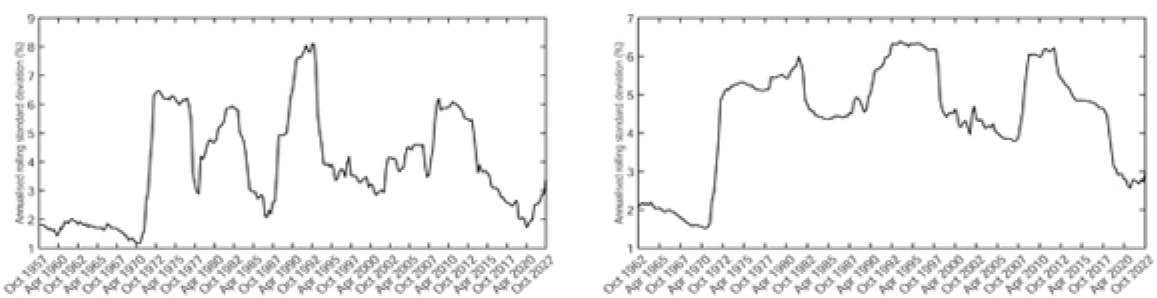


(b) Quarterly price returns

Figure 5.4 Historical evolution of Nationwide UK average house price.

Notes: This Figure depicts the historical evolution of log-returns of the Nationwide UK average house price time series. Sub-plot (a) is a plot of the monthly average house price return time series and sub-plot (b), the quarterly average house price returns. The monthly time series is from January 1991 to December 2022 and the quarterly series is from 1952Q4 to 2022Q4. The Nationwide house price database provides the following updates in relation to the quarterly time series: 1952 - 1959Q4 Simple average of purchase price. 1960Q1 - 1973Q4 - weighted average using floor area (thus allowing for the influence of house size). 1974Q1 - 1982Q4 - weighted averages using floor area, region and property type. 1983Q1 - Development of new house price methodology. A statistical “regression” technique was introduced under guidance of “Fleming and Nellis” (Loughborough University and Cranfield Institute of Technology). This was introduced in 1989 but data was revised back to 1983Q1. For 1993 - Information about neighbourhood classification (ACORN) used in the model were significantly updated following Census 1991 publication - regular updates since but typically for new postcodes.

Source: https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series



(a) 20-quarter rolling SD (annualised) (b) 40-quarter rolling SD (annualised)

Figure 5.5 Historical evolution of rolling volatility for Nationwide UK house price indices.

Notes: The historical evolution of the rolling standard deviations (SD) of the log-returns for the nationwide UK average house price time series is depicted in this figure. The quarterly average house price log-returns from 1952Q4 to 2022Q4 are used to compute the rolling standard deviations. The evolution of standard deviations, computed using a rolling 5-year and 10-year window, are respectively shown in sub-plot (a) and (b). The window size is fixed in all scenarios as we slide down to the tail end of the time series. The Nationwide house price database provides the following updates in relation to the quarterly time series: 1952-1959Q4 Simple average of purchase price. 1960Q1-1973Q4 - weighted average using floor area (thus allowing for the influence of house size). 1974Q1-1982Q4 - weighted averages using floor area, region and property type. 1983Q1- Development of new house price methodology. A statistical "regression" technique was introduced under guidance of "Fleming and Nellis" (Loughborough University and Cranfield Institute of Technology). This was introduced in 1989 but data was revised back to 1983Q1. For 1993 - Information about neighbourhood classification (ACORN) used in the model were significantly updated following Census 1991 publication - regular updates since but typically for new postcodes.

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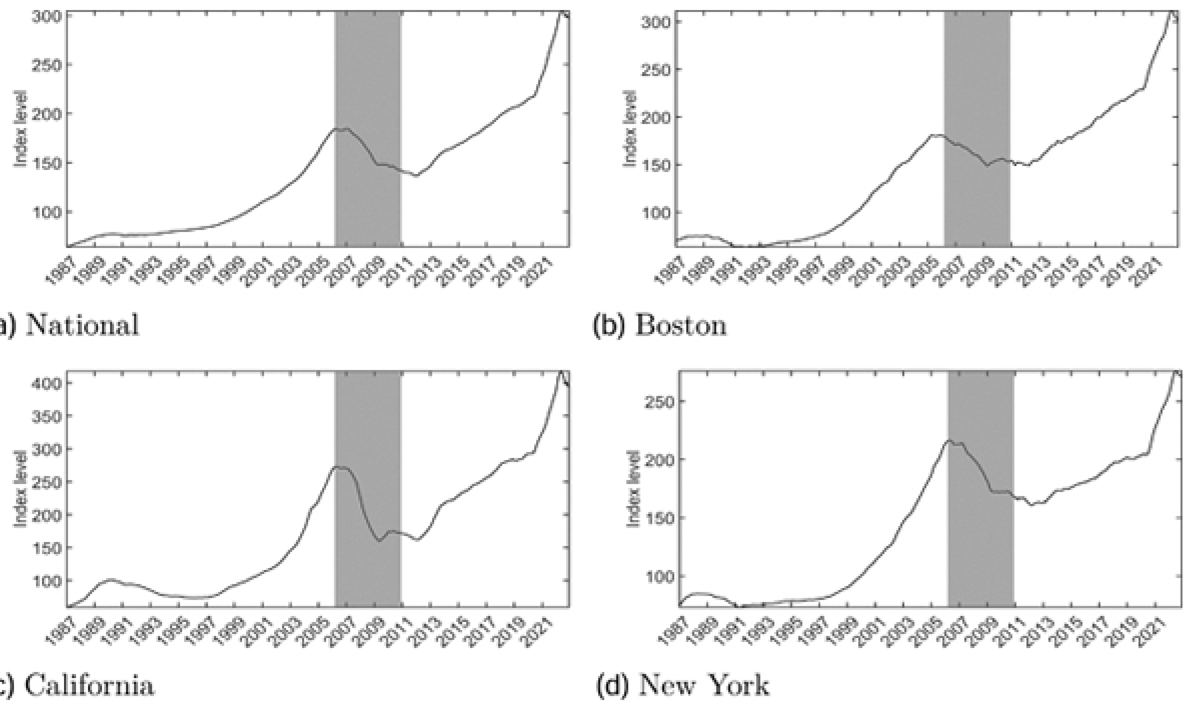


Figure 5.6 S & P Case-Shiller US house price index levels

Notes: The S & P Case-Shiller US home price index levels' time series evolution is shown in this figure. The National index is displayed with the time series plots for California, New York, and Boston. The region shaded red represents the period of the sub-prime crisis. The time series data is monthly, from January 1987 to December 2022, and seasonally adjusted with January 2000 = 100.

Source: S & P Dow Jones Indices LLC, S & P/Case-Shiller U.S. National Home Price Index, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CSUSHPISA, 23 September 2023.

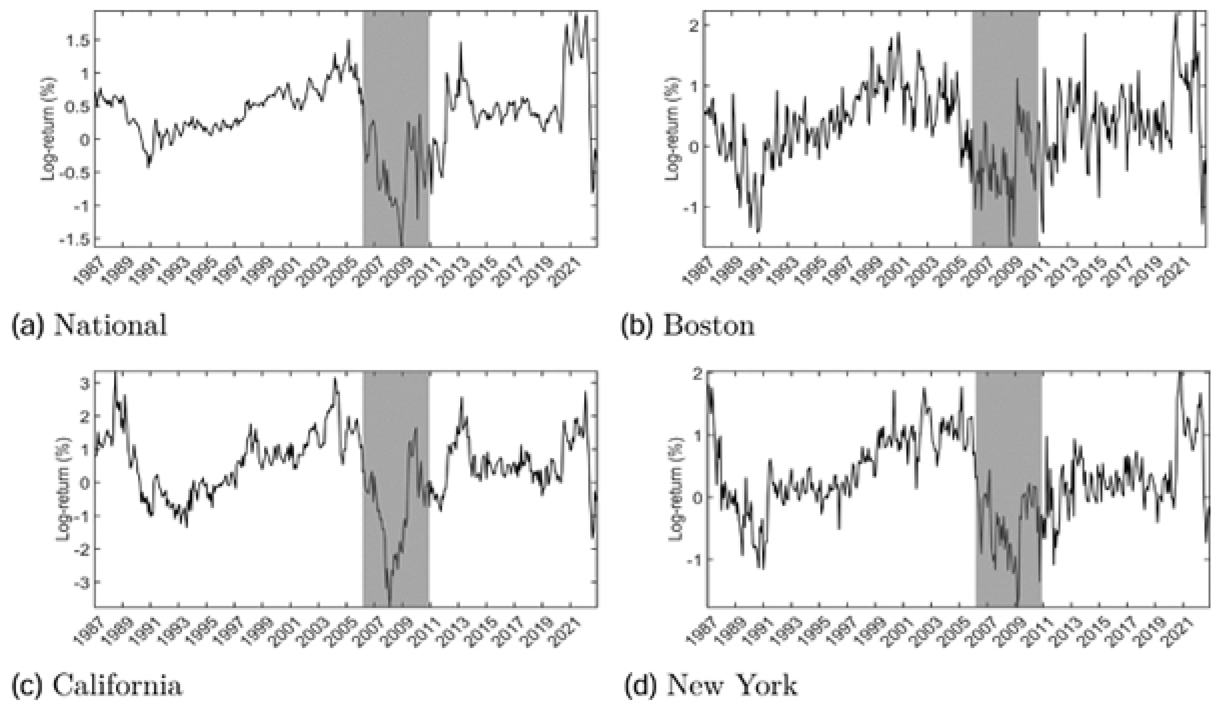


Figure 5.7 Log-return of S & P Case-Shiller US house price index levels

Notes: The log-returns of the S & P Case-Shiller US home price index levels' time series evolution is shown in this figure. The National index is displayed with the time series plots for California, New York, and Boston. The region shaded red represents the period of the sub-prime crisis. The time series data is monthly, from January 1987 to December 2022, and seasonally adjusted with January 2000 = 100. *Source*: S & P Dow Jones Indices LLC, S & P/Case-Shiller U.S. National Home Price Index, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CSUSHPISA, 23 September 2023.

Considering the Nationwide House Price index in Figure 5.4 we observe that the length of the series goes in back for longer for the quarterly series. This could have serious implications for the uncertainty of estimates of volatility of house price returns. A similar conclusion is drawn when looking at Figure 5.5. The calculations of annualised volatility using 5-year rolling window and 10 year rolling window show a different range for estimated volatilities. With the 5-year estimation window the volatility estimate peaks at about 8% whereas when using the 10-year estimation window the maximum is just below 6.5%. Up to the start of the second quarter of the 1970s, volatility is minimal; it then rises in the 1980s and 1990s. There is also evidence of the mid-1990s crash, which happened about 1993–1994. Following the crash, volatility held steady and reached a new peak in October 2007.

The graphs in Figure 5.6 illustrate the evolution of the S & P Case-Shiller house price index, at the national level and also for some important economic cities or regions such as Boston, New York or California. The price series are seasonally adjusted and reported at the monthly level. They have a very similar price display, peaking in 2007, then falling abruptly until some time point in 2012, then surging ahead again almost linearly until 2020 and then increasing very abruptly in the post-covid time and then falling recently.

The house price risk is measured in ERP calculus by the volatility, defined as standard deviation of returns per unit of time. The graphs in Figure 5.7 show the evolution of returns over the same time period. The returns series for National and for California look very similar but one should notice that the peaks and lows are larger for California. The highest monthly return for California is about 3% whereas for National is just below 2%; likewise for negative returns California has experienced a monthly return close to -4% while for National is just about -1.5%. Boston and New York have similar peaks with the National.

The view of many economists on the property bubble in the U.S. one year prior to the eruption of the subprime crisis was one of reassurance that house prices will not collapse (Himmelberg, Mayer, and Sinai, 2005). At the same time, the second edition of Irrational Exuberance (Shiller, 2005) contained a clear warning that “Significant further rises in these markets could lead, eventually, to even more significant declines. The bad outcome could be that eventual declines would result in a substantial increase in the rate of personal bankruptcies, which could lead to a secondary string of bankruptcies of financial institutions as well. Another long-run consequence could be a decline in consumer and business confidence, and another, possibly worldwide, recession.” Shiller (2006) presented ample historical evidence that house prices at the time were far from the norm suggested by historical patterns.

Bertus, Hollans, and Swidler (2008) demonstrated that investors exposed to house price risk in Las Vegas could have potentially used the Chicago Mercantile Exchange futures to reduce risk by more than 88% from 1994 to 2006 (one year prior to the subprime crisis).

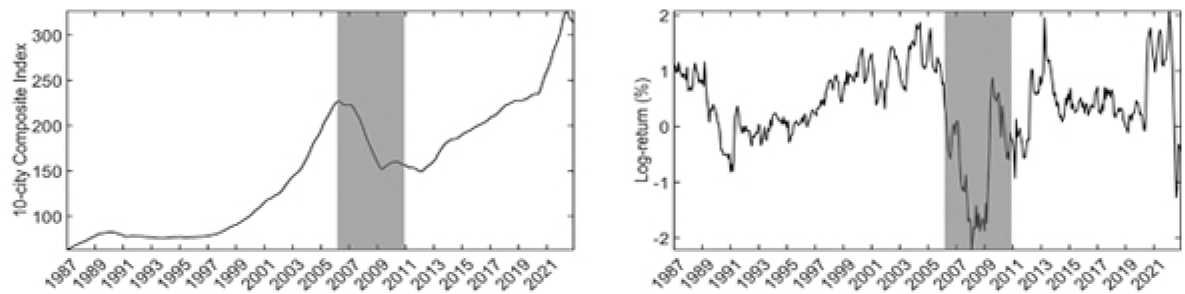
Looking at house price evolutions in general, there is a general conclusion that house prices go up and up for long periods followed by short periods of abrupt price realignment or even price crash. Some more advanced methods to detect real-estate house pricing bubbles are discussed in Fabozzi et al. (2020). The series of returns for UK seems to be more stable than for US but this could be just a characteristic dependent on the period of investigation.

The period highlighted in red in Figure 5.6 accounts for the house price crash correction over the global financial crisis. One could argue that if investment bankers and regulators would have taken Shiller's warning about the high level of house prices more seriously, the landing in the aftermath of the global financial crisis would have been softer. One other important aspect that can be observed on the graphs in Figure 5.6 is the almost linear increase in nominal house prices between 2012 and 2020. This followed, quite surprisingly actually, by an even steeper increase in house prices during the covid pandemic.

A much needed solution for the problems related to real-estate risk is to have fully developed real-estate derivatives markets. The current status is better for commercial real-estate, see Fabozzi, Stanescu, and Tunaru (2013); Geman and Tunaru (2012); Fabozzi and Tunaru (2017). It is still surprising that the largest spot markets in the world do not have corresponding derivatives markets to allow risk sharing in an elegant and transparent manner. These issues are discussed in more depth in a series of papers by Fabozzi, Shiller, and Tunaru (2009, 2010, 2012); Fabozzi et al. (2020).

For modelling purposes, one should focus on the time series of returns rather than on the series of nominal prices. The graphs in Figure 5.7 depict the geometric or logarithmic returns of the returns S & P Case Shiller indices. The graphs seem to indicate a mean-reversion feature for returns towards a long-run mean that is just slightly positive. Secondly, they also suggest that volatility changes over time so a model that explicitly specify dynamics equations for conditional volatilities may be at an advantage. It is rather surprising to see high positive returns during 2021, the year when the covid-19 pandemic was in full swing. This could be explained by a sudden spike in demand for houses from professionals living and working until then inside large cities. The downward massive drop in 2022 should also be noted, as the economy was coming out of the covid. The nominal house prices in the New York area seem to have an evolution very similar to Boston, suggesting a good pairing for hedging purposes of house price risk.

The graphs in Figure 5.8 refer to an index spanned by a portfolio of house prices in 10 cities in the US. The house price evolution between 1990 and 2023 shows that over long-term house prices tend to increase. One possible explanation is that they absorb inflation. The lower graph reminds us why for modelling purposes we need to think of returns and not prices. The returns series here seems to suggest that a mean-reverting type of process or a process with non-independent increments may be more appropriate for modelling purposes. In addition, it seems that the volatility of house prices does not increase with time, providing some preliminary empirical evidence against the use of geometric Brownian motion for modelling house prices.



(a) House price index (b) House price index return

Figure 5.8 10-city Composite Case-Shiller US national house price index.

Notes: The time series data is monthly, from January 1987 to December 2022, and seasonally adjusted with January 2000 = 100. *Source*: S & P Dow Jones Indices LLC, S & P/Case-Shiller U.S. National Home Price Index (SPCS10RSA), retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CSUSHPISA, 23 September 2023. The index is monthly and seasonally adjusted with January 2000 = 100.

Table 5.4 provides the descriptive statistics for a wide international panel of countries between the first quarter of 1960 and last quarter of 2022. By enlarge the mean returns were positive but small (with only one country showing negative mean return). The standard deviations that are sample estimates of volatilities ranged between 0.70% for IDN (India) and MEX (Mexico) and 6.80% for TUR (Turkey).

TABLE 5.4 Summary statistics on global house price index

Country	Mean(%)	SD(%)	Min(%)	Max(%)	$q_{25}(\%)$	$q_{50}(\%)$	$q_{75}(\%)$

Country	Mean(%)	SD(%)	Min(%)	Max(%)	$q_{25}(\%)$	$q_{50}(\%)$	$q_{75}(\%)$
AUS	2.00	2.10	-3.10	9.60	0.50	1.80	3.20
AUT	1.20	2.00	-9.00	5.70	0.20	1.40	2.30
BEL	1.40	1.60	-3.30	8.00	0.40	1.30	2.10
BGR	1.20	3.40	-11.50	8.70	-0.10	1.20	2.60
CAN	1.60	2.30	-6.50	8.90	0.40	1.60	3.00
CHE	0.90	1.90	-4.80	7.90	-0.30	0.70	1.70
CHL	2.10	1.60	-3.80	6.30	1.20	2.00	2.90
CHN	1.20	1.60	-2.90	5.20	0.20	1.00	2.10
COL	2.80	2.90	-4.00	16.10	1.30	2.30	3.90
CZE	1.40	1.90	-3.70	6.70	0.00	1.30	2.30
DEU	0.80	1.10	-4.70	3.60	0.00	0.80	1.60
DNK	1.40	2.60	-7.70	11.50	0.20	1.20	2.70
ESP	2.10	2.90	-5.10	10.80	0.30	1.60	3.70
EST	1.90	5.40	-24.30	12.90	0.40	2.40	4.00
FIN	1.30	2.40	-7.20	10.50	0.20	1.00	2.20
FRA	1.40	1.60	-3.30	5.40	0.20	1.50	2.50
GBR	2.00	2.40	-5.70	12.30	0.70	1.80	3.10
GRC	0.80	2.10	-4.20	4.90	-0.70	1.00	2.50
HRV	1.00	2.30	-3.40	8.50	-0.70	0.70	2.40
HUN	1.60	2.50	-3.30	6.00	-0.40	2.00	3.20
IDN	1.00	0.70	-0.10	4.50	0.50	0.70	1.20
IND	2.60	2.80	-4.80	12.70	1.10	1.70	4.30
IRL	1.90	3.30	-7.40	15.00	0.00	2.30	3.80
ISL	2.10	2.20	-5.30	9.20	1.10	1.90	3.20
ISR	1.40	2.10	-4.50	5.80	-0.20	1.40	2.50

Country	Mean(%)	SD(%)	Min(%)	Max(%)	$q_{25}(\%)$	$q_{50}(\%)$	$q_{75}(\%)$
ITA	1.70	3.50	-3.00	19.60	-0.30	0.90	2.20
JPN	1.20	2.30	-3.10	10.20	-0.40	0.70	2.80
KOR	0.80	1.90	-7.00	6.60	-0.20	0.50	1.60
LTU	1.30	4.10	-21.60	11.40	0.50	1.60	3.20
LUX	1.60	1.50	-2.50	5.00	0.60	1.60	2.50
LVA	1.10	5.30	-20.40	11.40	0.50	1.60	3.80
MEX	1.60	0.70	0.20	3.30	1.20	1.70	2.00
NLD	1.40	2.40	-5.60	10.00	0.40	1.40	2.60
NOR	1.70	2.10	-4.60	8.90	0.60	1.60	2.90
NZL	2.20	2.30	-4.10	11.00	0.70	1.80	3.40
POL	1.60	3.90	-4.80	21.30	-0.20	0.80	2.20
PRT	1.20	1.70	-3.20	6.60	0.10	1.00	2.40
ROU	0.20	2.20	-7.30	4.40	-0.80	0.90	1.40
RUS	3.00	3.80	-14.20	11.80	0.90	2.70	5.30
SAU	-0.40	1.30	-3.50	2.00	-1.10	-0.10	0.40
SVK	1.50	3.00	-6.90	8.80	-0.10	1.50	3.10
SVN	0.70	2.30	-5.70	4.60	-0.40	1.00	2.20
SWE	1.60	1.80	-5.60	6.10	0.70	1.60	2.70
TUR	5.20	6.80	-0.20	31.10	2.00	3.00	4.10
USA	1.30	1.30	-3.00	4.60	0.70	1.40	2.00
ZAF	2.30	2.50	-3.00	10.20	0.60	1.70	3.70
OECD	1.40	1.10	-2.20	4.40	0.80	1.30	2.10
EA	1.10	1.00	-1.40	4.70	0.50	1.10	1.70
EA17	1.40	1.40	-1.70	7.80	0.50	1.20	1.90

Country	Mean(%)	SD(%)	Min(%)	Max(%)	$q_{25}(\%)$	$q_{50}(\%)$	$q_{75}(\%)$

Notes: This Table reports the summary statistics and stylised facts on the nominal growth rate of international house prices across 47 countries. The data set is from (OECD, 2023). The data set is quarterly, starting from 1960Q1 and ending in 2022Q4. Aside the end date, each country has a different start date. We report the Mean (Mean), Standard Deviation (SD), Minimum (Min), Maximum (Max), 25th quantile (q_{25}), 50th quantile (q_{50}), 75th quantile (q_{75}) of each index return time series.

The average return for OECD is 1.40% while its corresponding standard deviation is 1.10%. It should also be noted the large negative minimum returns of -24.30% for EST (Estonia), -21.60% for LTU (Lithuania) and -20.40% observed for LVA (Latvia). On the other side, there were 31.10% for TUR, 21.30% for POL (Poland) and 16.10% for COL (Colombia).

Chen, Carbacho-Burgos, Mehra, and Zoller (2013) developed an equilibrium model for real house prices that can be used to forecast the Case-Shiller Home Price Index. In their paper they draw some interesting conclusions based on the data covered and the results of the fitted model. Here are some selection of conclusions that can be useful in the ERP area. For the Case-Shiller index (CSI), on average, a 1% increase in real per capita income in a metro area in the California, Mountain, and Pacific Northwest regions produces an increase in real house prices of about 70 basis points. In other words, there are 7% more house sales when income increases by 10%. Income seems to be less important in the Florida and South areas. Distressed sales, which are sales where homeowners cannot afford the mortgage payments—have a direct impact on house prices because they are usually sold at a discount. The REO sales to third parties is argued by Chen et al. (2013) to be a good indicator for house price dynamics. Some data evidence from RealtyTrac pointed out that the average pre-foreclosure sale (short sale) is discounted by 19%, while an the sale of a real estate owned house is discounted by more than double that amount.

In addition, Chen et al. (2013) show some evidence that the high the unemployment rate, user cost, construction costs, and foreclosures lead to slower real price growth, while high population growth and greater mortgage availability leads to increasing real price growth. There are other issues that perhaps should be considered more closely when dealing with house price dynamics. Ngai and Tenreyro (2014) found evidence of seasonality in house prices. The predictability and the magnitude of seasonal changes in house prices are not easy to deal with by current models of house prices.

Another important point refers to the long-term trend of house prices. Shiller (2006) showed that the real price of house prices in the US had an uptrend if data is considered only since 1913 but there is no long-term uptrend if data prior to 1913 years are taken into account. The real house prices even since 1913 is flat except for two episodes: the home price boom that followed World War II and the house price boom before the global financial crisis.

Table 5.5 reports further descriptive statistics and some further tests on the nature of house price return time series. We present results for Jarque and Bera (1980) normality test (JB), unit-root test (ERS) using the Elliott et al. (1996) ADF-GLS test and the Fisher and Gallagher (2012) weighted Portmanteau test $Q(20)$ and $Q^2(20)$ respectively. The standard errors for the various tests are reported in brackets.

TABLE 5.5 Distributional & time-series properties of global house price in

Country	Skewness	Kurtosis	JB

Country	Skewness		Kurtosis		JB		
AUS	0.603***	(0.001)	0.902**	(0.026)	19.571***	(0.000)	-
AUT	-1.520***	(0.000)	6.360***	(0.000)	186.363***	(0.000)	
BEL	0.491***	(0.004)	1.956***	(0.000)	41.926***	(0.000)	
BGR	-0.960***	(0.002)	3.268***	(0.001)	41.901***	(0.000)	
CAN	-0.114	(0.485)	1.465***	(0.002)	19.328***	(0.000)	-
CHE	0.640***	(0.000)	1.854***	(0.000)	44.435***	(0.000)	
CHL	-0.349	(0.177)	1.639**	(0.016)	10.709***	(0.005)	-
CHN	0.196	(0.527)	0.801	(0.145)	1.691	(0.429)	
COL	1.147***	(0.000)	3.170***	(0.000)	87.383***	(0.000)	-
CZE	0.487	(0.108)	0.698	(0.169)	3.471	(0.176)	
DEU	-0.506***	(0.003)	1.713***	(0.001)	34.626***	(0.000)	
DNK	0.082	(0.617)	1.957***	(0.000)	33.736***	(0.000)	-
ESP	0.451***	(0.009)	0.443	(0.170)	8.672**	(0.013)	
EST	-1.846***	(0.000)	7.390***	(0.000)	199.053***	(0.000)	
FIN	0.340**	(0.043)	2.713***	(0.000)	68.447***	(0.000)	-
FRA	-0.121	(0.459)	-0.085	(0.969)	0.581	(0.748)	
GBR	0.848***	(0.000)	3.030***	(0.000)	109.541***	(0.000)	-
GRC	-0.353	(0.131)	-0.688*	(0.061)	4.128	(0.127)	
HRV	0.694**	(0.017)	0.688	(0.160)	7.001**	(0.030)	
HUN	-0.074	(0.796)	-1.048***	(0.003)	2.891	(0.236)	
IDN	2.139***	(0.000)	6.783***	(0.000)	217.004***	(0.000)	
IND	0.831**	(0.012)	2.330***	(0.008)	18.434***	(0.000)	
IRL	0.060	(0.712)	1.497***	(0.002)	19.836***	(0.000)	-
ISL	0.082	(0.734)	1.903***	(0.007)	13.687***	(0.001)	-
ISR	-0.049	(0.820)	-0.052	(0.857)	0.059	(0.971)	

Country	Skewness		Kurtosis		JB		
ITA	2.585***	(0.000)	8.256***	(0.000)	830.224***	(0.000)	-
JPN	1.085***	(0.000)	1.538***	(0.001)	73.702***	(0.000)	
KOR	0.254	(0.194)	2.617***	(0.000)	43.517***	(0.000)	
LTU	-2.633***	(0.000)	13.564***	(0.000)	582.215***	(0.000)	-
LUX	0.05	(0.862)	-0.06	(0.781)	0.035	(0.983)	
LVA	-1.666***	(0.000)	4.454***	(0.000)	85.086***	(0.000)	
MEX	0.082	(0.761)	0.107	(0.573)	0.112	(0.946)	
NLD	0.294*	(0.078)	2.365***	(0.000)	51.981***	(0.000)	-
NOR	0.105	(0.524)	1.636***	(0.001)	23.815***	(0.000)	-
NZL	0.815***	(0.000)	1.794***	(0.001)	51.373***	(0.000)	-
POL	2.954***	(0.000)	11.233***	(0.000)	469.816***	(0.000)	-
PRT	0.221	(0.271)	0.486	(0.190)	2.483	(0.289)	
ROU	-1.138***	(0.001)	1.764**	(0.022)	18.647***	(0.000)	
RUS	-0.720***	(0.007)	4.110***	(0.000)	67.967***	(0.000)	
SAU	-0.451	(0.228)	-0.207	(0.863)	1.211	(0.546)	
SVK	-0.217	(0.424)	0.564	(0.211)	1.479	(0.477)	-
SVN	-0.682**	(0.025)	0.104	(0.570)	4.831*	(0.089)	
SWE	-0.812***	(0.000)	2.133***	(0.000)	62.860***	(0.000)	-
TUR	2.644***	(0.000)	6.205***	(0.000)	141.244***	(0.000)	
USA	-0.460***	(0.007)	1.622***	(0.001)	30.561***	(0.000)	-
ZAF	0.656***	(0.000)	0.482	(0.134)	18.480***	(0.000)	
OECD	0.380**	(0.024)	0.678*	(0.064)	9.082**	(0.011)	-
EA	0.289	(0.116)	0.763*	(0.061)	6.499**	(0.039)	-
EA17	1.292***	(0.000)	3.270***	(0.000)	151.984***	(0.000)	-

Country	Skewness	Kurtosis	JB

Notes: This Table reports the normality test statistics and stylised facts on the nominal growth rate o
The data set is quarterly, starting from 1960Q1 and ending in 2022Q4. Aside the end date, each count
test (JB), unit-root test (ERS) using the (Elliott et al., 1996) ADF-GLS test and the (Fisher and Galla
errors for the various tests are reported in brackets.

Using nominal house prices gathered by the Bank for International Settlements (BIS) (see https://www.bis.org/statistics/index.htm.) Thomas (2021) analysed the worst recorded housing market downturns for many countries and historical periods. His conclusions are highly informative when thinking about the parameter estimation uncertainty and sensitivity analysis that should be part of any good risk management, and we revise them here for convenience of any academics and practitioners covering ERP.

The largest house price drops recorded in the UK have not been as bad as those observed in some other countries: peak-to-trough falls of 12% during 1989–93 and 18% during 2007–09. Those *two* market falls were followed by much stronger recoveries at 10 years than in any of the other countries.

The reported statistics represent national indices, but analysts should be aware that the prices of individual houses which are relevant for NNEG mar experience even worse decreases locally. There is also considerable regional variation. The BIS figures are drawn from national indices published by the Office of National Statistics, which are based on Land Registry records of actual transaction prices for all purchases.

Other institutions such as Nationwide or Halifax publish regional sub-indices, based on their proprietary samples of purchases funded by a mortgage. The Halifax House Price regional sub-indices registered rises for Scotland and the north of England during the 1989–93 national slump, and a substantial regional fall of 35% from peak to trough for East Anglia.

During the 2007–09 slump, the largest regional fall was 26% for Greater London.

Hong Kong 1997–2003 and Ireland 2008–2013 represent exceptionally severe price falls, where the peak-to-trough falls over about 6 years exceeded 50%, but also experiencing strong recoveries shortly thereafter. Based on the above Thomas (1996) inferred that it is sensible to assume that a national house price index cannot drop beyond 50%.

5.3 THE SERVICE FLOW RATE (RENTAL YIELD)

Shiller (2006) pointed out that a genuine long-term downtrend in real rents. Based on the Bureau of Labor Statistics data real housing rents have been in decline in the US, experiencing a fall to about half in total since the Consumer Price Index was introduced in 1913. The CPI rent series analysed is adjusted for changes in the number of bedrooms, bathrooms, utilities and facilities provided, or changes in services expected of the renter.

Many papers assume that real house prices can be derived or are in equilibrium the present discounted value of future rents. Shiller (2006) argues that if we assume that rents are a random walk, then prices should closely track rents but if rents are driven by a mean reverting process then one might expect house prices "to track rents but to be less volatile than rents." Furthermore, Shiller (2006) emphasizes that the reality is very different.

> "Not only do real home and rent prices fail to track each other, but the rent-price ratio has shown a remarkable downtrend since 1913."

The rental yield is the rent-to-price ratio[2] of the collateral property. The NNEG calculation deals with the net rental yield thereby accounting for the impact of other related cost, for example maintenance cost and interest payments. The rental yield agreeably plays central role when detecting housing market disruptions. Essentially, the user cost of the property should equal the rationally expected return (rent) on owning the property at equilibrium (Himmelberg et al., 2005; Poterba, 1992). Thus the alternative nearest cost of ownership should be rental price. Per the contract specification, the ERM borrower strictly requires owner-occupancy of the underlying property. This stipulation does not however trivialise the role of the rental yield in the valuation process. We may engage the intuition behind the use of rental yield in ERM valuation by considering any one of the following:

[2] One could also consider the inverse i.e. price-to-rent ratio.

- the best estimate value of living in the property or what it would have cost to rent an equivalent property.
- the reasonable income one would agree to receive in order to defer ownership of the property for an equivalent horizon of the contract lifetime.
- opportunity cost of capital[3] i.e. the lost income that the ERM borrower would have received if she had invested the capital in an alternative investment.

The linkage between house prices and rents are relevant to both house owners and real estate investors. For the latter category, rental yields are a fundamental component of housing returns. Bracke (2015) employs a proprietary dataset with tens of thousands of housing sale and rental transactions in Central London during the 2006–2012 period. He calculates the rental yields for 1,922 properties that were both sold and rented out within six months, between 2006 and 2012, and he finds that rental yields are lower for larger houses and houses better positioned, being more central. Their study supports the user cost formula advocated by J. Poterba (1984).

There is some international evidence that rental yields are lower for more expensive houses, see Smith and Smith (2006) and Bracke (2015) for US, Hill and Syed (2012) for Australia and Hwang, Quigley, and Son (2006) for Korea. It should be mentioned though that the methodologies followed and type of data analysed differ across studies. The second important observation is that the relationship between rental yields and property values is inverse and it goes asymptotically flat for houses with larger values.

J. Poterba (1984) proposed a model leading to a no-arbitrage formula for the rental yield g

$$g = r_f - \alpha + m + \pi \tag{5.9}$$

where r_f is the risk-free rate, α is the expected appreciation, m reflects the maintenance cost and property depreciation, and π is a risk premium. Note that there could be differences in gross rental yields driven by the expected length of tenancies.

The rental yield could also be estimated with empirical data from the housing market. On this basis, we can consider the average rental yield or a property-specific rental yield. The average rental yield, although a good starting point, may introduce obscurities to cross-sectional variation within the NNEG portfolio. In reality, related user cost may vary across collateral properties in the loan-by-loan ERM contract valuation. Despite this, the loan insurer can aim towards diversification benefit in the portfolio to manage this risk.

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[3]A prudent analysis would account for risk differentials in the available opportunities, taxes, transaction cost, and other expenses.

When considering empirical estimation of the rental yield, it is worth noting that a more precise/representative estimation could be challenging depending on the proportion of buy-to-let properties and its spatial distribution within the country. The joint effect of these two factors may result in the lack of spatial homogeneity of buy-to-let activities, thereby increasing the idiosyncratic component of rental yield.

5.3.1 Rental Yield Computational Challenges

Any additional income produced by the collateral house needs to be adjusted for in any contingent claim calculations under the risk-neutral (market valuation) approach. For the majority of buyers, houses play the role of a consumption asset and not that of an investment asset. There is no evidence that rental yields are driving future house prices so the *expected* house prices at various future long horizons cannot be determined with growth models in the same way expected share prices may be determined with growth models linked to dividends. The PRA (see PRA CP13-18, paras 2.12-2.15) arrive at the 2% value for the *net rental yield* calculated as the gross rental yield (5%) minus maintenance costs, management costs, voids, with central estimate for net rental yield as 2% but 1% permitted as a *minimum* value.

The more precise calculations are challenging because, the buy-to-let percentage of a houses portfolio is relatively small and it varies geographically, with London and South-East as the main areas. Hence, the idiosyncratic component of rental yields is quite large. This spatial lack of homogeneity of buy-to-let activity, together with the fact that less than 20% of a housing portfolio may be considered to be associated with rented properties, makes it very difficult to consider rental yields as the main drivers of house prices.

5.3.2 Estimating Rental Yield with Rental Income Data

Under the risk-neutral valuation, the dynamics for both GBM and ARMA-EGARCH models depend locally on the risk-neutral drift $r - g$, where r is the risk-free rate and g is the rental yield. The latter parameter plays the role of dividend yield in equity share price models. The dynamics of the GBM as a continuous-time model with drift adjusted for dividend yield is based on the assumption that all dividends are reinvestable immediately in the equity stock. This cannot be the case for the standard house owner because of a lack of granularity (fungibility) of real estate housing markets. As with the dividend yield, when g increases $r - g$ decreases and therefore the risk-neutral distribution translates to the left. This effect increases the put options. *Ceteris paribus*, one may increase NNEG values by taking higher values for rental yield and decrease NNEG values by decreasing rental yields.

The rental yield data is coming out of the Office of National Statistics. This data covers private property rental yields but this pool of properties represents a minority stake in the total pool of properties in the UK. Furthermore, it is not clear whether the properties that will form the collateral in ERM loans are impacted at all by rental yields since lenders will not accept tenancy involved in ERM. In addition, while a house price index may be assumed to get some rental income, the ERM borrowers do not have access to this income flow. The rental yield concept may introduce an undesirable asymmetric future valuation view between the borrower and the lender on the same collateral house.

The Office for National Statistics has been gathering data on rental yields for about 10% of all properties rented out. From their data we have calculated the monthly sterling rental values average for England taking into account the weights and income given by property type.[4]. The monthly rental yield for England is then calculated by dividing the average sterling rental sum to the average property price in England in that month. In addition, we also calculated *proxy average* quartiles estimates for rental yields using weighted averages of lower, median and upper quartile of monthly sterling rental figures.[5] Figure 5.9 displays monthly series, average, proxy median and proxy lower and upper quartiles for England. The mean average monthly rental yield over this period is 0.4315% (5.1776% annualised) while the mean proxy upper quartile is 0.48% (5.76% annualised). Note that this rental yield corresponds only to the pool of properties rented out.

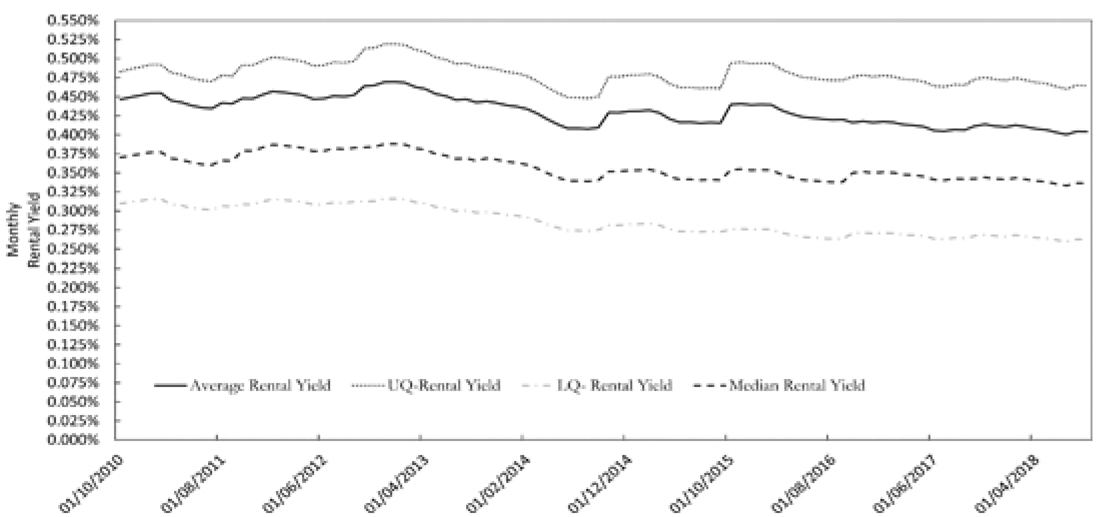

Figure 5.9 Monthly series, average, proxy median and proxy lower and upper quartiles for England between Oct 2010 and Sep 2018. Source: Author's calculation based on data from the Office for National Statistics.

There seems to be a lot of variation in the evolution of rental yields over time, with a large drop observed at the end of 2009 and first half of 2010. There is also great variation across regions in terms of rental yields[6] evolution that needs to be managed idiosyncratically similar to the same issue for volatility.

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[4]We left out the rents coming from room only.

[5]The proxy quartiles do not represent actual quartiles since weight averaging the medians will not necessarily produce the median, for example. We produced these proxies to have a rough idea of distribution of rental yields.

According to the Office for National Statistics, there were about 26.4 million households in the UK in the 2012 (following 2011 census) out of which approximately 5 million are rented out properties.[7] Hence less than 20% of properties are rented out. This means that a rough calculation would give a total rental yield, weighted by the 20% representing the actual renting market, of 1.03% ($5.1776\% \times 20\%$) per annum.

An even more precise calculation should take into account the *net rental yield* which is calculated as the rental yield net of running costs. The latter is calculated taking into account three elements.[8] First, the voids, defined as the number of months per year the property stays unrented. The usual rule of thumb is to assume one month's loss of gross rental income per annum, so the sterling pound average rental income will be multiplied by 11/12. Then, letting agent's fees in the range 10-15% of the rental income plus VAT (12%-18% including VAT) at the current rate of 20%. We can take the mid-value of 15% that needs to be deducted from the resulting sum after applying the voids. The third component refers to maintenance costs that are typically around 15% of the gross rental income, inclusive of any VAT.

Hence, agents' fee and maintenance cost together will erode the rental income by 30%. The average net rental yield then following from the above calculations will give an annualised net rental yield of 0.66%. In this study we used an average value of 1% as representative for 2018 in the UK for baseline scenarios, and we considered higher and lower values (including negative) for sensitivity scenarios discussed later on.

There are few other important points regarding the relationship between RM and net rental yield. By contractual terms, the collateral houses in the RMs cannot be rented out. This implies a rental yield of 0. The rental yield was calculated from the rental income that is representative across the properties in the index. If more than 20% of properties become available for renting the rents are likely to decrease because of supply and demand. It is not clear what will happen with the house prices then, so we cannot say either way what will be the effect overall on the house market.

Recall that the 20% of the houses that produce rental income is not just a sample of from the total population of houses that produce rental income. It is the full subset of the population of houses in the UK. Hence, the 80% remaining will not have one house that will pay rent. Since we are trying to determine the dynamics of the data-generating process, at the moment, any house price index will have to adjust rental income over the entire population. Likewise, if 80% of the houses will produce that rental income then we would multiply $5\% \times 0.80$ to get the relevant rental yield, and if all houses are rented out producing 5% rental income then 5% is the rental yield on the index.

However, while those issues are important in themselves, our modelling is using a data-generating process for a house price index. We envisage that the NNEG valuations obtained in this way are only "indicative", say for a house that has exactly the same price as the index. The data-generating process, say GBM, requires the additional income part to be taken into account at the risk-neutralisation stage. Rental yield is needed for GBM but also for any other model employing the conditional Esscher martingale measure for risk-neutral pricing purposes.

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[6] The Global Property Guide 2018, projects lower gross annual rental yield in London. It estimates gross rental yield for houses in prime central London to be around 3.2%. https://www.globalpropertyguide.com/Europe/United-Kingdom#rental-yields

[7] Personal communication with Rhys Lewis from the Office of National Statistics.

[8] The values for these elements were selected upon consultation with specialists in the field.

5.3.3 Other Risks

5.3.3.1 Dilapidation Risk

This risk is mainly related to the home equity moral hazard discussed by Shiller and Weiss (2000b). After getting the ERP loan the owners of the collateral house may stop repairing it when it is needed and therefore impairing the expected market value sale of the house at termination.

Repeat-sales house price indexes (HPI) quantify house price changes using the price adjustment between sales of the same house. One major assumption underpinning the construction of repeat-sale indexes such as those in the S & P Case-Shiller index family is that the quality of the house does not change over time. This strong assumption leads to a bias that has been reported by some studies (Palmquist, 1980; Goetzmann and Spiegel, 1995; Nowak and Smith, 2020). The methodology behind Case-Shiller index accounts for a part of the time-varying attribute bias, using mainly filters and a robust weighting method. In a recent paper, Nowak and Smith (2020) use machine learning techniques to identify a lexicon employed by real-estate listing agents about individual houses that is then embedded into the repeated-sales regression model. They also showed that the direction, magnitude, and source of the time-varying attribute bias varies with the market cycle and with the respective metropolitan statistical areas (MSAs). The quality adjusted S & P Case-Shiller HPIs for the eight most representative MSAs in the United States can be therefore reconstructed with the novel technique. Importantly, Nowak and Smith (2020) show that "the Case-Shiller HPIs are biased downwards by as much as 7% during the recent financial crisis and upwards by as much as 20% after the crisis." The bias is directly related to the homeowners' (dis)incentive to keep the house in good order during housing market booms and busts. The new quality-adjusted approach accounts for whether the house was renovated (extensive margin) and how well the house was repaired (intensive margin). Remarkably, the quality-adjusted repeated sales methodology advanced by Nowak and Smith (2020) does not require to either state ex-ante a list of controls or to have location-specific knowledge for every housing area. This new technology opens a new avenue for research that may lead to solve the idiosyncratic house price bias that is so relevant for the ERP.

In recent years legislation has been improved and borrowers are directly responsible for maintaining the house in good condition. This is a legal requirement in many countries. Furthermore, many ERP programmes in many countries consider home improvement as one of the main reasons for elderly borrowers to enter into ERP agreements. Thus, on a par comparative basis one may argue that many of the collateral houses in the ERP loans are much improved immediately once the loan is taken and therefore in the long run things may be possible even out.

However, dilapidation risk is directly linked to house price idiosyncratic risk in the sense that each loan and its collateral property is individually treated. In countries like UK, the regulator does not allow to take advantage of portfolio diversification and risk management calculations regarding things like the NNEG should be carried out individually.

One way to deal with dilapidation risk and/or idiosyncratic risk is to apply a haircut to the expected sale value of the house at termination of the contract. This haircut could be calibrated in-house based on previous experience and auditors and independent advisors could investigate that the procedure is carried out in line with observed values.

5.3.3.2 Prepayment Risk

Very little is known about the values of the prepayment rate for reverse mortgages. In the U.S. in the early days of the HECM programme a flat prepayment rate of 0.3 times the mortality rate of the youngest borrower in the family was used. Empirical evidence suggest that roughly 20% of total HECM terminations in Fiscal Years 2000– 2014 were attributed to loan refinancing (Jiang and Miller, 2019). Furthermore, according to the United States Government Accountability Office, 2019 report, in Fiscal Years 2014–2018, about 28.5% of loan terminations were attributed to refinancing, repayment, or borrower moving/title conveyance.

In Korea a prepayment rate of 0.2 times the 2010 mortality rate for females was chosen based on Korean demographic data.

Prepayment risk in this case is driven more about the willingness to pay or to stay locked in the loan or not. This risk is high mainly when refinancing rates are significantly lower. It also depends on the type of ERP and legislation covering the respective loan.

The academic literature is relatively sparse on this topic. Surrender decisions are driven by the macro-economic environment, such as house price increases and interest rate decreases (Rodda et al., 2004; Davidoff and Welke, 2017; Jiang and Miller, 2019). It is important to notice that, as opposed to the traditional forward mortgage, when house prices boom there is a higher incentive to refinance the loan, with more home equity becoming available to be released by entering into a new contract or down-sizing. The second important incentive is given by interest rate decreases and since ERP loans are mainly on fixed rate roll-up rates, the borrower can lower the ERP loan rate through refinancing.

It should also be remarked that some prepayment decisions are driven by non-economic factors and may not appear from a rational choice. For instance, divorce or changing countries may lead to borrowers needing to move and leave the property. A notable article on these topics is Shi and Lee (2021) who proposed a model that captures both macro-economic and non-economic drivers in surrender decisions.

5.4 COMBINING ALL RISK FACTORS INTO ONE MODELLING FRAMEWORK

5.4.1 Computational Considerations for ERPs

There are several risks that must be considered jointly for valuation and risk management of ERPs. Because of practical considerations interest rate risk and mortality risk are considered to evolve independently of house price risk. More advanced models would require allowing dependencies among risks. With advances in data science Monte Carlo (MC) simulation seems to be the general panacea for these problems. A procedure for valuation or risk management would follow more or less the following steps

1. Calibrate an interest rate model (Hull-White, Vasicek, CIR, Ho-Lee, LIBOR model, market model, OIS models etc.)
2. Calibrate a mean-reversion model for a representative house price index HPI.
3. There could be correlation between the two above.
4. Decide on a prepayment model.
5. Calculate the mortality-morbidity migration table for the borrower(s) or get projections from a stochastic mortality model.
6. Simulate the loan termination time for each MC scenario. This is the earliest of the time of death or move into care and the prepayment time.
7. Calculate the payoff to the lender, taking care of the no-negative equity agreement, and discount back to present time using either the risk-free yield curve or some risky-adjusted curve.
8. Average the results and obtain the MC valuation of price and/or other risk management measures.

The procedural steps outlined above are easy to follow and various models can be build in blocks. In our experience it is easier to use a grid or lattice spanned by the periodic information available. At the moment of writing this book, house prices are updated in a more meaningful way at monthly level at best, with quarterly and annually more common. Mortality rates again are entered into the calculation on an annual basis.

5.4.2 Modelling Issues

5.4.2.1 General Considerations

There are various assumptions that originators of reverse mortgage programmes have made over time. For the HECM programme Szymanoski (1994) discusses critically those assumptions. The first assumption states that the loan end time is independent of interest rate and house prices. It should be noticed that lower interest rates are convenient to borrowers since their outstanding balance will grow at a lower rate. There is no sudden ramp-up interest rate charge in monthly payments under the design of HECM. Refinancing is not an incentive due to the transaction costs and crystallisation of payments to be made to the lender. When property prices decline, say through a recession, this motivates borrowers to keep the reverse mortgage alive.

The second assumption refers to the concept of *mutuality* which describes a mechanism for giving back to borrowers excess revenues as dividends in the case when those borrowers as a vintage have been profitable to the insurer. The idea is to incentivise the mortgage seller to be risk averse at the beginning of the program, and in order to balance it out, pay dividends back to borrowers at the back end. The problem as Szymanoski (1994) remarked is that in the case of reverse mortgages, the borrowers cannot receive the dividends because they will die or move into care.

The third assumption relates to the nature of the reverse mortgage originator. If this is a government-sponsored enterprise then the lenders should only break even. Thus, another important assumption is that the government backed insurers should be only *risk-neutral*. This last assumption is a bit more tricky since the break-even point is determined based on covering the expected losses on interest rate and house prices.

Earlier models used to price HECM and other reverse mortgages used static mortality tables. Thus, the trends in mortality rates for some vintages as well as more extreme mortality jumps observed in society were largely ignored. Chen, Cox and Wang (2010) combined a generalised Lee-Carter model with asymmetric jump effects, with an ARIMA-GARCH model for a house price index to evaluate the non-recourse provision of reverse mortgages. They demonstrated that on that basis, the HECM program in the U.S. is viable. However, somehow surprisingly they considered the interest rate to be fixed.

Furthermore, the housing prices were modelled with a geometric Brownian motion, see Tsay, Lin, Prather, and Jr. (2014), in contradiction with serial correlations and possible stochastic volatility effects revealed in the literature. Szymanoski (1994) argued that the dynamics of house prices is well represented by a geometric Brownian motion. This is in contradiction with the findings of Case and Shiller (1989) and a large body of empirical evidence. Using a geometric Brownian motion for house prices is wrong for several reasons. First of all the well-documented serial correlation of returns of property prices is not captured. Secondly, the variance for a GBM increases infinitely with the time horizon. Last but not least, a GBM will not be able to produce a property crash since all paths are continuous. A GBM is used to model house prices in the reverse mortgages literature mainly for convenience.

Wang, Huang, and Lee (2014) developed an analytical formula for calculating the loan-to-value (LTV) ratio in an adjusted-rate reverse mortgage (RM) with a lump sum payment. In their model, interest rates are modelled jointly with the adjustable-rate RM, and the housing price follows a jump diffusion process with a stochastic interest rate. Assuming that the loan interest rate is adjusted instantaneously with the short rate given by a CIR model, they show that the LTV ratio is independent of the term structure of interest rates, even when the housing prices follow an exponential Lévy process. They raise concerns about the viability of the HECM (Home Equity Conversion Mortgage) at high levels of housing price volatility. Interestingly, when the loan interest rate is based on LIBOR they suggest that the LTV ratio is insensitive to the parameters characterising the short rate process.

Shao, Hanewald, and Sherris (2015) consider that there are only two main risks that insurers selling ERMs face, real-estate risk and longevity risk. They investigated the joined effect of real-estate price risk and longevity risk on the pricing and risk profile of reverse mortgage loans. Their stochastic multi-period model was based on a new hybrid hedonic/repeat-sales pricing model and a stochastic mortality model with cohort trends (the Wills-Sherris model). They concluded that using an aggregate house price index and not considering cohort trends in mortality may lead to an underestimation of total risk in reverse mortgage loans.

5.4.2.2 Some Generic Formulae

The value of an ERP at any time t_0 can be conceptualised as the value V_{t_0} of a coupon free bond with maturity τ_x and face value $\Gamma \times H_0 \times e^{R\times\tau_x}$ less the cost at t_0 to cover the possible negative equity between t_0 and τ_x. For the former the value would be easier to calculate *if we knew* τ_x but in reality we do not. The termination time τ_x is a random stopping time depending on the joint mortality risk, morbidity risk and prepayment risk.

Since $\tau_x \in \{t_0+1, t_0+2, \ldots, T, \ldots\}$ using a probability conditioning argument we can see that

$$V_{t_0} = \sum_{t \geq t_0+1} df(t_0, t) \times [p_t \times \Gamma \times H_0 \times e^{R\times(t-t_0)}] \tag{5.10}$$

where p_t is the probability that the loan is settled in the period $(t-1, t]$ and $df(t_0, t)$ is the discount factor for the period $[t_0, t]$. It is standard to assume that even if the borrower dies or moves into care or prepays at any time during a time period $(t-1, t]$ the loan it will settle at the end of the period t. It is also evident now why working with an annual frequency is practically more useful. Remark that series on the right side of the above formula (5.10) is conceptually infinite but for practical purposes is finite. There is a finite time T such that $p_t = 0$ for all $t \geq T+1$. In that case

$$V_{t_0} = \sum_{t \geq t_0+1}^{t=T} df(t_0, t) \times [p_t \times \Gamma \times H_0 \times e^{R\times(t-t_0)}] \tag{5.11}$$

The same conditioning argument applies to the computation of the non-negative equity cover. Hence

$$NNEG_{t_0} = \sum_{t \geq t_0+1} p_t \times nneg_{t_0}(t-1, t) \tag{5.12}$$

where $nneg_{t_0}(t-1,t)$ is the value at time t_0 of the NNEG payoff if the loan is settled at the end of period $(t-1,t]$. Once again computationally it is easier to compute $NNEG_{t_0}$ if a maximum term for the existence of the loan is given. Then,

$$NNEG_{t_0} = \sum_{t \geq t_0+1}^{t=T} p_t \times nneg_{t_0}(t-1,t) \tag{5.13}$$

One may argue that the discount factors in 5.10 or 5.11 are computed from the risk-free discount curve, but the discount factors applied to get the $nneg_{t_0}(t-1,t)$ *should be risk-free only when there are derivatives contracts traded on house prices*. In other words, risk-free discount factors would work in the US because of the existence of Case-Shiller index futures whilst discount factors using risk-adjusted rates would be more appropriate in the UK at the time of writing this book. One should emphasize that, before the global financial crisis, there were some important players such as RBS and Merrill Lynch who were market-makers in forwards on Halifax house price index. At that time, even for the UK, risk-free discounting would have been appropriate.

Combining 5.10 and 5.12 provides the value of the ERP loan at current time t_0. This would be equal to

$$ERP_{t_0} = V_{t_0} - NNEG_{t_0} \tag{5.14}$$

The formula (5.14) points out that an ERP loan could be reconceptualised as a portfolio of a bond and a put option on the collateral house price with same time to maturity as the bond and the exercise price equal to the face value of the bond.

Another important aspect that needs to be mentioned is that the probabilities $\{p_t\}_{t\geq 0}$ employed in 5.10 and 5.12 are taken as given. If they are not known and a stochastic mortality model is used, taking advantage of the assumption that mortality/morbidity risk and house price risk are usually considered to be independent, we can use a Monte Carlo approach for valuation. To this end, the stochastic mortality model will be used to generate a large number of mortality pathways, then the formulae 5.10 and 5.12 are evaluated along each pathway and the results averaged to get a final valuation.

5.4.2.3 The Ortiz-Stone-Zissu Model

One of the simplest models around for pricing reverse mortgages is the model introduced by Ortiz, Stone, and Zissu (2013). Denoting by H_t the house price at time t, by h the rate of inflation, by K_t the value of the reverse mortgage at time t, by $\Gamma \in (0,1)$ the percentage of the property value that is financed under the loan or the LTV, and by r the interest rate charged on the loan, the model assumes that the house price will increase/decrease at the rate of inflation/deflation as described by the equation

$$H_t = H_0(1+h)^t \tag{5.15}$$

The interest accrues over time and it is added to the outstanding loan balance as follows

$$K_t = \Gamma H_0 (1+R)^t \tag{5.16}$$

Importantly it is assumed that $R \geq h$. The two parts of the model represented by the evolution equations (5.15) and (5.16) represent two curves that start from different initial points, at time zero the value of the reverse mortgage reverse mortgage is always smaller than the value of the house, but with time the two curves may cross-over. The cross over point can be determine by requiring that

$$H_0(1+h)^t = \Gamma H_0 (1+R)^t \tag{5.17}$$

The crossover point determines the negative equity territory. It is important to realise that there is a secondary crossover point between the house price curve and he funding curve, which is very important from the issuer perspective. Another direct observation is that the excess spread takes at least 15 years to become more substantial. This implies that the mortality trends of various mortgagor vintages are very important, as well as extreme mortality rates that bring the termination point closer to the issuance point.

This model assumes that the inflation rate and interest rate are exogenous to the system and only δ determines actively the time t^* of the cross over. Solving (5.17) for t^* one gets

$$t^* = \frac{\ln(\Gamma)}{\ln(1+h) - \ln(1+R)} \tag{5.18}$$

This function is decreasing and concave as a function of δ.

Tunaru (2017) pointed out that this model implies no possibility of a property price crash. Furthermore, interest rates were kept constant and there is no consideration about the determination of the stochastic termination event related to mortality risk.

5.4.2.4 Modelling House Price Correlations

One of the most difficult problems in ERP calculus is dealing with house price correlations. In a sense, for jurisdictions such as the UK, the NNEG calculations must be computed and applied on a loan-by-loan basis. This may entice the risk manager to ignore the house price correlations.

However, if there is one important lesson from the recent global financial crisis, is that assuming assets in a portfolio have more or less an independent price evolution can lead to catastrophic results when all the bad conditions occur at the same time. Thus, even if calculations can or should be performed on a loan-by-loan basis, both the risk manager at the lending institution and the regulator should perform frequent analyses of the impact in *change* of correlations across house prices. Therefore, not only we should be aware of correlations across house prices but also should be aware that these correlations may change over time.

Pu et al. (2014) proposed modelling house price correlations using an N-dimensional price vector for the N houses in the ERP loans portfolio that is supposed to follow the dynamics given by a multidimensional GBM stochastic process described as follows

$$dH_t^{(i)} = \mu H_t^{(i)} dt + \sigma H_t^{(i)} dW_t^{(i)} \tag{5.19}$$

Following an earlier idea of Miao and Wang (2007), who argued that the total level of volatility for real-estate can be decomposed into a systematic volatility component and a idiosyncratic volatility component, Pu et al. (2014) worked on the error term to decompose it as

$$dW_t^{(i)} = \rho_i dZ_t + \sqrt{1-\rho_i^2} dB_t^{(i)} \tag{5.20}$$

Here $\{Z_t\}_{t\geq 0}$ is a GBM that accounts for the systematic component and $\{B_t^{(i)}\}_{t\geq 0}$ is a GBM describing the idiosyncratic shock of the i-th house. The two components are considered to be independent for each house and also across houses.

Let $H_t = \sum_{i=1}^{N} H_t^{(i)}$ be the total price of all houses at time t in the portfolio. From the independence assumption for all processes involved and assuming additionally for simplicity that $\rho_i \equiv \rho$, we can further derive

$$\sum_{i=1}^{N} dH_t^{(i)} \quad = \mu \sum_{i=1}^{N} H_t^{(i)} dt + \sigma \sum_{i=1}^{N} H_t^{(i)} dW_t^{(i)} \tag{5.21}$$

$$dH_t \quad = \mu H_t dt + \sigma H_t \left(\rho dZ_t + \sum_{i=1}^{N} \frac{H_t^{(i)}}{H_t} \sqrt{1-\rho^2} dB_t^{(i)} \right)$$

(5.22)

Denoting by $\theta = \sqrt{\rho^2 + (1-\rho^2) \sum_{i=1}^{N} \left(\frac{H_t^{(i)}}{H_t} \right)^2}$ and defining the Brownian process

$$\widetilde{W}_t = \frac{1}{\theta} \left[\rho Z_t + \sum_{i=1}^{N} \frac{P_t^{(i)}}{P_t} \sqrt{1-\rho^2} B_t^{(i)} \right]$$

(5.23)

it follows that, starting from $H_0 = \sum_{i=1}^{N} H_0^{(i)}$,

$$dH_t = \mu H_t dt + \sigma^* H_t d\widetilde{W}_t$$

(5.24)

where $\sigma^* = \theta\sigma$. It is not difficult to see that $\theta < 1$, because $\rho \in (0,1)$ and the sum of squares of positive weights is always lower than the sum of weights itself which is equal to one. Therefore, *under this model*, the volatility of the entire portfolio containing collateral houses will always be less than the volatility of each individual house price process. This demonstrates, again under this model, that there would be a benefit to allow portfolio diversification.

This computations also led some researchers to think that the volatility of an individual house should always be larger than the volatility of a house price index. This idea, as demonstrated mathematically above, is correct if one assumes that house prices are driven by GBM type processes.

However, basic portfolio theory also taught us that, under general assumptions about asset returns and without restrictions to particular stochastic data generating processes, that the volatility of a portfolio is less than the sum of volatilities of individual components. For example, the volatility σ_{AB} of a portfolio of two assets A and B is less than the sum of the volatilities σ_A and σ_B. But while we know that $\sigma_{AB} < \sigma_A + \sigma_B$ there is nothing theoretical in general terms requiring that $\sigma_{AB} < \sigma_A$ and $\sigma_{AB} < \sigma_B$.

Coming back to our calculations, if $\frac{H_t^{(i)}}{H_t} = \frac{1}{N}$ for all house prices then one can easily see that

$$\theta = \sqrt{\rho^2 + (1-\rho^2)\frac{1}{N}} \tag{5.25}$$

and

$$\widetilde{W}_t = \frac{1}{\theta}\left[\rho Z_t + \frac{1}{N}\sum_{i=1}^{N}\sqrt{1-\rho^2}B_t^{(i)}\right] \tag{5.26}$$

Comparing this scenario with the previous heterogeneous scenario it can be observed that the expected house price growth is the same. Hence, under this simplified model, the heterogeneity of houses that are collateral in the reverse mortgage loan portfolios only impacts the volatility of the future house prices, not their expectation.

It is important to see in this context that when the portfolio size gets larger it shows that $\lim_{N\to\infty} \theta = \rho$. Other extreme cases for this model imply that when $\rho = \pm 1$ then $\theta = 1$ and $\sigma^* = \sigma$. For the special case when $\rho = 0$ we can see that $\theta = \sqrt{\frac{1}{N}}$, which is the minimum value for a given N. Therefore, when house prices are assumed uncorrelated the idiosyncratic risk of the underlying property portfolio can be diversified the most.

5.5 SUMMARY

There are several risk drivers that have to be taken into account when performing the valuation and risk management of ERPs. House price risk is by far the most prominent risk of all of them. While in other asset classes the financial innovation and modelling is more advanced in the industry than in academia, for real-estate finance it is the other way round. It is very difficult to find out models that fit well the historical dynamics of house prices. Furthermore, risk-neutralisation for this asset class is more difficult than for other asset classes such as equity, foreign exchange or commodities. The main reason for that is the lack of derivatives products that would allow hedging house price risks in a similar vein with other asset classes. A thorough discussion on the evolution of real estate derivatives can be found in Fabozzi et al. (2020).

One of the less talked risks associated with ERP valuation is model risk. House price volatility is time and time again mis-estimated. One possible reason is the heuristic implication that the volatilities of each individual house prices are all less than the volatility of the basket of properties. While this may become true under some very stringent assumptions, there is no general proof that this is always the case for any collateral house in an ERP.

Interest rates could also pose some challenges for ERP calculus. The rate for risk-free discounting has changed over the years and interest rates are occasionally subject to sudden changes by central banks. Furthermore, it is becoming less and less clear whether ERP calculus in some countries such as the UK should be using risk-free discounting or whether they should use historical date for risk-adjusted discounting under the physical measure.

Mortality rates and interest rates are frequently assumed to be independent of the house prices. House prices do have a link with interest markets through mortgage rates. When house prices overheat, central banks do take measures to increase interest rates in order to cool off the house price overheating. The covid pandemic showed us also that mortality rates can become interlinked with house prices when such extraordinary events occur. A future pandemic could see clustering of deaths well out of the national historical mortality charts together with a house price fall and economic recessions.

Portfolio diversification is not allowed by regulators. This could be a double edge sword. On one hand regulator is right in asking lenders to have provisions on a loan-by-loan basis so that if an extreme crisis occurs, the loans do not cause systemic risks. On the other hand, being conservative for a long time may lock in capital and stifle market development to the required needs. One possible solution would be to allow lenders to manage their portfolio of loans and subject them to stress tests.

Building the discount curves for valuation and risk management should take into account many aspects regarding the characteristics of the asset class, funding, regulation, market agreements etc. One of the best sources to help any analysts to disentangle the various interwined risks and to side step many of the pitfalls in this area is Kenyon and Stamm (2012).

For real estate finance and for real-estate derivatives in particular, one of the authors of this book published several papers over the years. The articles describe the state of affairs at several points in time and they are written with a view of “what could happen” rather than what happens.

Relevant literature on ERPs tend to address different facets of the ERP risk-management process. Ma and Deng (2013) presented an actuarial based model for pricing the Korean ERP with constant monthly payments and also with graduated monthly payments indexed to the growth rate of consumer prices. They found that any shock to house prices may impact younger borrowers more severely. The Ma and Deng (2013) sensitivity test results on contract maturity (termination) showed how call option values decrease with maturity, suggesting higher ERM values at younger ages. The study also reported a positive relationship between house price volatility and call option value. Thus higher volatility will impact younger borrowers more. Younger borrowers include borrowers within lower age range profile, i.e. 55 - 60 years (Li, Hardy, and Tan, 2010; Ma and Deng, 2013).

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CHAPTER 6

NNEG Calculus

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6.1 INTRODUCTION

THE NON-NEGATIVE-EQUITY-GUARANTEE (NNEG) condition stipulates that any excess of the accrued ERM loan amount above the market value of the collateral property after the exit event will be borne by the lender. The NNEG clause is common to the United Kingdom (UK) market, where it is also subject to other prudential regulatory conditions. ERM loan contracts in Australia, Japan, Canada, and South Korea have embedded NNEG conditions which slightly differ by age profile of the target market. The immediate risk is the event that the market value of the collateral property at termination is below the accrued loan balance. This risk stays with the loan issuer who is usually an insurer.

For some time the NNEG option embedded in ERMs has been modelled using the well-known Black-Scholes model. The simplicity of application of this model for determining the value of a put option has lured many academics, who focused on other aspects of ERM, into employing the geometric Brownian motion for house prices. Examples in ERM literature include Chinloy and Megbolugbe (1994b); Ma and Deng (2006); Pu et al. (2014); Tsay et al. (2014). The geometric Brownian motion for house prices was also used in the context of securitisation of ERMs, again based on convenience of closed-formula, as in Wang, Valdez, and Piggott (2008).

The review paper by Hosty et al. (2008) "sanctified" for a period of time in the UK ERM market the usage of Black-Scholes model with the implicit lognormal model calibrated to the Nationwide House price index. Likewise the Department of Housing and Urban Development (HUD) used the same framework for NNEG for their Home Equity Conversion Mortgage (HECM) program introduced in 1989. The assumption that the house price return is a random walk was backed by studies such as Kau et al. (1992,1993,1995), Cunningham and Hendershott (1984) and Kau et al. (1993) and it implied that the house price returns have no memory so predicting future values is meaningless.

However, numerous studies that tested the random walk hypothesis in housing markets provided very strong evidence against it. Case and Shiller (1989) rejected the weak-form efficiency in the US housing market and pointed out the positive autocorrelation effects in both the changes in house prices and after-tax excess returns. Hosios and Pesando (1991), Ito and Hirono (1993) got similar results for the Toronto and Tokyo housing markets respectively. The Institute of Actuaries (2005) also finds that there are positive autocorrelations in the Nationwide House Price Index in the UK. Tunaru (2017) confirms the positive autocorrelation short term and negative autocorrelation long-term, internationally.

The point about autocorrelations in a house price index actually is that it suggests that the price series has some memory and there is the possibility of speculative price bubbles, as well as mean reversion, to occur, as discussed by Szymanoski (1994). In addition, house prices may also experience jumps. Using Chicago Mercantile Exchange futures price data, Mizrach (2008) found evidence of jump risk in a 315-day sample. Using the US national average new home price returns for single family mortgages from January 1986 to June 2008, Chen et al. (2010b) identify 14 times that the monthly housing price changed more than 10% per month.

Recent studies accept that house price time-series exhibit serial correlation that invalidates the GBM assumption, (see Kogure, Li, and Kamiya, 2014). Li et al. (2010) considered the Nationwide House Price index and they remarked that, for this property index, a) there is a strong positive autocorrelation effect among the log-returns, b) the volatility of the log-returns varies with time, c) a leverage effect is present in the log-return series. All these three properties invalidate the use of the GBM for house prices. The characteristics of large movements in house prices presented in Sun and Tsang (2019) also contradict a GBM process as a data-generating process for house prices.

Without a liquid market in NNEG insurance there is no benchmark market price. Using the Black-Scholes formula in pricing NNEG will inflate ceteris paribus, however, for sure, the value of the NNEGs through the volatility channel since under geometric Brownian motion the volatility grows with the square root of time, so the time value for long-term derivatives such as NNEG gets inflated.

Recent academic research noticed this financial economics obstacle and they proposed alternative solutions that avoid this problem. Examples include Lee et al (2012) who proposed a jump-diffusion model, Chen et al. (2010) and Lee et al. (2012) used an ARMA-GARCH model, the approach by Sherris and Sun (2010) and refined by Alai et al. (2014), Shao et al.

(2012) and Cho et al. (2013) who used a VAR model based on economic scenarios; Wang et al. (2014) aimed for a model capable to generate housing price jumps so they selected exponential Levy processes for house prices; mean-reverting models were discussed by Fabozzi et al. (2012) and Tunaru (2017). All these models depart substantially from Black-Scholes, not only theoretically but also numerically.

The evidence enumerated above points out that there are two schools of thought, one orbiting around the Black-Scholes formula and another emerging on the basis of more appropriate modelling of house prices. There are advantages and disadvantages with both schools of thought. Ideally a model should have the simplicity of Black-Scholes but covering the time series features observed in house prices. Since ultimately the exercise of the NNEG is determined by house prices it seems logical to compare through extensive simulation exercises the two approaches.

6.1.1 Risk-Neutral Approach

Hosty et al. (2008) describes a risk-neutral (also called in insurance markets “market consistent”) approach where a lognormal model is calibrated to the Nationwide Average House Price with a house price volatility taken at 11%, a value obtained by upgrading the 5% p.a. to a higher value (11%) based on the desmoothing procedure described in Booth and Marcato (2004) for *commercial* properties. Although not named clearly, the data-generating process tacitly assumed by Hosty et al. (2008) for NNEG calculus is a GBM process with the drift calculated on a risk-neutral basis as a difference between the yield on government stock less a rental yield calibrated from the IPD residential property index.

Since that milestone paper, other papers considered various other models, all using risk-neutral pricing as a valuation principle. Here is a list of models (not necessarily complete) that priced the NNEG using the risk-neutral approach: Hosty et al. (2008), Kogure et al. (2014), Alai, Chen, Cho, Hanewald, and Sherris (2014), Ji, Hardy, and Li (2012), Lee, Wang, and Huang (2012), Wang et al. (2014), Li et al. (2010), and Chen et al. (2010).

6.1.2 Real-World Pricing

When a very good econometric model is identified that fits well the house price data so that forecasting is robust, an alternative can be considered based on the physical or real-world measure. The disadvantage of this procedure is that it requires an issuer risk premium specified exogenously or calibrated from a different market.

Hosty et al. (2008) uses the same GBM model, under a real-world measure, with a drift specified on a best estimate approach (mean value). Remark that for GBM model the volatility should be the same under risk-neutral approach and real-world approach. They recognise some of the shortcomings of this model such as the fact that values of house price index in a future period is independent from preceding periods. At the same time they hint that a mean-reversion approach may also be appropriate.

Examples of real-world pricing methodologies are Chinloy and Megbolugbe (1994a), Ortiz et al. (2013), Lew and Ma (2012) and Ma and Deng (2013). Many of these papers follow a deterministic approach to house price growth, which can be quite misleading and dangerous.

Based on our research and extensive reading in the area, our *current* view is that there are two clear avenues emerging regarding to NNEG calculation. The first one is based or conditioned on the existence of a derivatives market on residential house prices. One such example is the existence of a tradeable futures contract on house prices in the US, contingent on the S & P Case-Shiller index. In this case, all issues related to the econometrics side of house prices is absorbed in the futures price and for the valuation of an NNEG one can confidently use the Black' 76 formula. This is robust, efficient and easy to calculate with basic tools in an Excel spreadsheet. It also qualifies as market consistent.

The second avenue is defined for situations where there are no derivatives traded on house prices, neither on a house price index, and even less so for actual house prices that are collateral in the ERP. This is at the moment the case for the UK. Our view is that in this instance it is theoretically better to use a more traditional actuarial approach and determine the NNEG value under the physical measure. Essentially this becomes a forecasting exercise. As we demonstrate again and again in this book, for this second case, there could be substantial differences in both directions between a market consistent valuation and a real-world valuation, starting with the same model. It is quite likely that applying a market consistent approach in this second case may lead to either a false sense of security or a valuation that is too conservative which may stifle the market development of ERP. These differences may become further inflated when the interest rates are experiencing a sudden shock and move to different levels.

6.1.3 Navigating through NNEG Models

Here we offer a concise list of main points related to models applied to NNEG valuation.

The risk-neutralisation of predictive distributions obtained from discrete time econometric models is obtained through several approaches. The main steps are as follows

1. Fit an econometric model to a time series of a house price index; VARDCC/GARCH as in Kim and Li (2017), ARMA-EGARCH was fitted by Chen et al. (2010), Li et al. (2010), Kogure et al. (2014), Yang (2011), Lee et al. (2012); a VAR model was fitted by Shao et al. (2015) and Alai et al. (2014).

2. Obtain a predictive distribution for the required horizon under the real-world (econometric) measure.

3. Risk-neutralise the predictive distribution. The actual risk-neutralisation step was done in the literature with several methods:

    - the Esscher transform, see Chen et al. (2010), Li et al. (2010), Lee et al. (2012).
    - the Wang transform (Wang et al., 2014). Li (2010) pointed out that for this transform market price of risk is selected based on subjective choices and parameter uncertainty is difficult to gauge while Tunaru (2015) discussed situations when this transform may introduce arbitrage.
    - the (Bayesian) entropy (Kullback-Leibler) approach; used in Kim and Li (2017) and Kogure et al. (2014).
    - stochastic discount factor approach as detailed in Alai et al. (2014) and Shao et al. (2015), following Ang and Piazzesi (2003) and Ang, Piazzesi, and Wei (2006).

4. Apply other models for risks such as mortality, long-term care and prepayment that are orthogonal to house price risk.

5. Value the target contingent claim.

One should note that other continuous-time models such as the mean-reverting process in Fabozzi et al., (2012) or in (Knapcsek and Vaschetti, 2007), a jump-diffusion process as in (Wang et al., 2014), (Lee et al., 2012) or (Knapcsek and Vaschetti, 2007), and the Levy process also in (Wang et al., 2014), are specified under the real-world measure before switching to a risk-neutral measure.

6.2 NNEG CALCULUS

Calculation of the NNEG is relevant in prudential regulation to protect against systemic failure. The NNEG calculation could be done on a loan-by-loan or portfolio basis. When used, the loan-by-loan approach does not directly provide for any form of diversification benefit in the risk management process. A simplifying assumption in NNEG calculation is to consider that the termination time, which is a random variable, is independent of interest rate and house prices. There are two types of interest rates to consider in the NENG valuation. The initial loan is accumulated at the *roll-up* rate while rational investors will discount cash flow in the economy at the risk-free interest rate. A lower service rate is convenient for the borrower since the outstanding loan balance will grow at a lower rate. The service rate could be floating or fixed within the design.

Below is an illustration of the NNEG computation principles. This can be adapted to any data generating process imposed on the evolution of underlying property prices. The first definition is the loan process $K = \{K(i,t)\}_{(i,t)\in\mathscr{S}}$, with initial value given by:

$$K(0,1) = H_{0,1} \times \Gamma, \tag{6.1}$$

where Γ is a real-valued constant representing the loan-to-value ratio and $H_{0,1}$ is the initial house price, both at the inception of the contract. We denote the cartesian product of these sets by $\mathscr{S} = \mathscr{I}^{(t)} \times \mathscr{T} = \{(i,t) \mid i \in \mathscr{I}^{(t)} \text{ and } t \in \mathscr{T}\}$, with the assumption that $(I_{t-1}, t-1) = (0,t)$ for any $t = 1, \ldots, T$. Note that the time index $\mathscr{T}$ can be viewed as a subset of $\mathscr{S}$ if we use the notational convention that $t := (I_t, t)$ for any $t \in \mathscr{T}$. This is a discrete-time framework in which the time index $\mathscr{T} = \{t \mid t = 0, \ldots, T\}$, where T refers to a certain period, usually representing the number of quarters/years that span the lifetime of the contract. For any time period $(i,t) \in \mathscr{S}$, we assume that the loan process evolves deterministically according to the following equation:

$$K(i,t) = K(i-1,t) \exp\left(R_{i,t}^{(l)}\right), \tag{6.2}$$

where $R_{i,t}^{(l)}$ is the roll-up interest rate on the loan over the period $(i-1,t)$ to (i,t). In practice, for computational ease and without the loss of generality, we assume constant roll-up rate and risk-free rates for every period, so that $R_{i,t}^{(l)} = R$ and $r_{i,t} = r$.

By denoting the maturity time point by (I_M, T_M), with $1 \leq T_M \leq T$ and $1 \leq I_M \leq I_{T_M}$, then the terminal NNEG value is calculated as:

$$F := F\left(H_{I_M,T_M}\right) = \max\left[K(I_M, T_M) - H_{I_M,T_M}, 0\right] \tag{6.3}$$

where $K(I_M, T_M)$ is the accumulated balance of the loan and H_{I_M,T_M} is the house price, at time point (I_M, T_M).

The NNEG value at time $(0,1)$, for a loan with maturity (I_M, T_M), or equivalently, time to maturity $\tau_M = \sum_{k=1}^{T_M - 1} I_k + I_M$ periods, is given by:

$$\Pi_{0,1}(\tau_M) = E^{\mathbb{Q}}[\exp(-r\tau_M) \cdot F], \tag{6.4}$$

where $\mathbb{Q}$ is the risk-neutral measure required under market pricing principles. The value of the loan at current time is given by:

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}[\min\{K(I_M, T_M), H_{I_M, T_M}\}] \tag{6.5}$$

This formula can be rearranged as:

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}[(K(I_M, T_M) - \max(K(I_M, T_M) - H_{I_M,} \tag{6.6}$$

The value of the risky loan can be further decomposed as a risk-free bullet bond with face value equal to the accumulated ERM loan balance at time (I_M, T_M) and a short position in a put option contingent to the nominal collateral house price and with a strike equal to the accumulated loan balance:

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}[K(I_M, T_M)] - \Pi_{0,1}(\tau_M).$$

$$(6.7)$$

The value of the loan can be also expressed as the value today of future house possession minus a call option on the value of the house with the strike price $K(I_M, T_M)$:

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}\left[H_{I_M,T_M}\right] - \exp(-r\tau_M) \cdot E^{\mathbb{Q}}\left[\max\left(H_{I_M,}\right.\right.$$

6.3 MODELS FOR HOUSE PRICES

6.3.1 The Geometric Brownian Motion

The model of choice for house price dynamics was for many years the geometric Brownian motion. This was mainly for computational convenience because it could be easily manipulated analytically. Regarding ERM modelling authors that employed the GBM for house prices are Hosty et al. (2008); Kau, Keenan, and Epperson (1992); Huang, Wang, and Miao (2011); Ji (2011), Ji et al. (2012); Pu et al. (2014). Some, like Szymanoski (1994), even advocated that the dynamics of house prices is well represented by a GBM.

From an empirical and also theoretical standpoint using a GBM for house prices is not a great idea. Empirically Case and Shiller (1989) and other researchers pointed out that house prices exhibit serial correlation and hence they are incompatible with evolution of GBM. More empirical evidence is discussed amply by Tunaru (2017).

Employing a GBM for house prices is wrong for several reasons. First of all, the well-documented serial correlation of returns of property prices is not reproducible by a GBM which has independent increments. Secondly, the variance for a GBM increases infinitely with the time horizon while it is well-known that the returns of house prices exhibit mean reversion. Last but not least, a GBM will not be able to produce a property crash since all paths are continuous.

Here we will study the NNEG pricing under a GBM assumption for house price dynamics *because of computational convenience*. This is not something that we recommend, quite the contrary!

The GBM dynamics is specified under the real-world measure as follows

$$dH_t = \mu H_t dt + \sigma H_t dW_t \tag{6.8}$$

For simplicity we denote by $K = L_0 e^{RT}$ the exercise price of our NNEG put option at maturity T.

6.3.1.1 Risk-neutral world GBM pricing

Under risk-neutral world the dynamics changes only in the drift to

$$dH_t = (r - g) H_t dt + \sigma H_t dW_t \tag{6.9}$$

where g is the rental yield[1].

The Black-Scholes formula behind the NNEG put option is

$$Put(H_0, K, T) = e^{-rT} E^Q \left(\max \left[K - H_T, 0 \right] \right)$$

(6.10)

where Q is the risk-neutral measure implied by the Black-Scholes model. Then

$$Put(H_0, K, T) = Ke^{-rT}N(-d_2) - H_0e^{-gt}N(-d_1)$$

(6.11)

where $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (r - g + 0.5\sigma^2)T\right]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

6.3.1.2 Real-world GBM pricing

Under this method securities are priced using real-world probabilities derived from the historical information and a risk-neutral (funding rate) discount rate.

This would be valued under real-world measure as

$$Put(H_0, K, T) = e^{-r^*T}E^P\left(\max\left[K - H_T, 0\right]\right)$$

(6.12)

where r^* should be the risk-adjusted interest rate reflecting the premium charged for investing in this market.

Using the usual trick that

$$\begin{aligned} E^P\left(\max\left[K - H_T, 0\right]\right) &= E^P\left((K - H_T)1_{\{H_T<K\}}\right) \\ &= E^P\left(K1_{\{H_T<K\}}\right) - E^P\left(H_T1_{\{H_T<K\}}\right) \\ &= KP(H_T < K) - E^P\left(H_T1_{\{H_T<K\}}\right) \end{aligned}$$

(6.13)

[1] We consider rental yield here in order to be able to compare GBM-rn as used by some insurers with other approaches. We do not necessarily agree that $g \neq 0$.

One can show with standard calculations that

$$P\left(H_T < K\right) = N\left(-\frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu - 0.5\sigma^2)T\right]\right)$$

and

$$E^P\left(H_T 1_{\{H_T<K\}}\right) = H_0 e^{\mu T} N\left(-\frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu + 0.5\sigma^2)T\right]\right)$$

Thus

$$Put(H_0, K, T) = e^{-r^*T}\left[KN\left(-d_2\right) - H_0 e^{\mu T} N\left(-d_1\right)\right]$$

(6.14)

where $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu + 0.5\sigma^2)T\right]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

6.3.1.3 Black 76 Model

Some argued that the “correct” approach is to use the Black (1976) formula for pricing the NNEG. Under this model pricing the NNEG would be done with the formula

$$Put = e^{-rT}[K_T N(-d_2) - F_T N(-d_1)]$$

(6.15)

with

$$d_1 = \frac{\ln(F_T/K_T) + 0.5\sigma^2\tau}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

where r is the risk-free rate of interest, K_t is the strike price for period T calculated as $K_T = L_0 e^{R\times T}$ (here L_0 is the initial loan value) and F_T is the *forward* house price for year T, that *also* has the formula

$$F_T = H_0 e^{(r-g)T}$$

(6.16)

where g is the house rental rate and H_0 is the current house price.

Unfortunately, Black (1976) cannot be applied in the current context for the NNEG market since there is no futures house price contract currently traded in the UK. The introduction of such a futures contract would complete the market and many of the current challenges in valuing house price contingent claims would be easily solved. One of the major impediments in launching the property futures is exactly the development of a simple and flexible modelling approach for this asset class. In our opinion, the formula (6.15) simply does not apply here and this is unrelated to the Gaussian distribution assumption.

Furthermore, the forward contract on a house price cannot be calculated as in (6.16), simply imitating the no-arbitrage formula for a stock paying dividend, where the dividend yield is replaced by the net rental rate. That formula cannot work because currently we cannot shortsell the value of a house. Hence, the no-arbitrage principle does not apply here to lock in the forward price as in the case of corporate stock.

Black 76 model is in essence the GBM model (Black-Scholes) applied to the futures contract as underlying. It can be easily proved that when we assume that the house price process is GBM then the futures price process contingent on the house prices has the same volatility. However, a constant volatility for futures is in contradiction with Samuelson's effect saying that the volatility of futures contracts increases as the futures contract approaches its maturity.

Last but not least, Black (1976) provides an option pricing formula using futures values and not forward values. This distinction is important in the context of stochastic interest rates and when the futures/forward underlying asset price is correlated with the interest rates. The futures and forward prices are identical when the interest rates are constant but even then we cannot circumvent easily the problem explained above, because of the impossibility of shortselling.

Computationally Black 76 and the GBM risk-neutral will give the same results, so mnemonically we will call this approach the GBM-rn/Black76.

6.3.2 The ARMA-EGARCH family

More recently, house prices in the NNEG literature have been modelled based on the broadly speaking GARCH family of models, (see e.g., Li et al., 2010; Kim and Li, 2017; Tunaru and Quaye, 2019). The progressive research on volatility confirms the existence of different components of volatility and likewise, benefits from separately modelling these components (Ding and Granger, 1996; Engle and Lee, 1999; Alizadeh, Brandt, and Diebold, 2002; Ghysels, Santa-Clara, and Valkanov, 2004; Engle and Rangel, 2008; Engle, Ghysels, and Sohn, 2013). A different, but equally important, approach for NNEG pricing is proposed by Alai et al. (2014) and Shao et al. (2015) who uses a stochastic discount factor approach linked with macroeconomic variables. Ideally one would like to merge the two subclasses and use the best features in each class.

We shall denote by $Y_t = \ln\left(\frac{H_t}{H_{t-1}}\right)$ the log-return of the house price index at time t.

6.3.3 ARMA-EGARCH Model

6.3.3.1 *Model specification under real-world measure*

This model is based on a submodel for log-returns and a submodel for conditional volatilities. Hence, as in Li et al. (2010), first we specify an ARMA(m,M)

$$Y_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j} + \epsilon_t \tag{6.17}$$

where $\epsilon_t \sim N(0, h_t)$; and then, for the conditional variance h_t the EGARCH(b,B) model is specified

$$\ln(h_t) = k + \sum_{i=1}^{b} \alpha_i \ln(h_{t-i}) + \sum_{j=1}^{B} \beta_j [|\widetilde{\epsilon_{t-j}}| - E|\widetilde{\epsilon_{t-j}}|] + \sum_{j=1}^{B} \gamma_j \widetilde{\epsilon_{t-j}}$$

(6.18)

with $\widetilde{\epsilon_t} = \frac{\epsilon_t}{\sqrt{h_t}}$ is the standardised innovation at time t, see Li et al. (2010) for more details.

It follows then that, under the real-world measure $\mathbb{P}_t$

$$\boxed{Y_t | \mathscr{F}_{t-1} \sim N(\mu_t, h_t)}$$

(6.19)

where $\mu_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j}$.

The ARMA and EGARCH exact specifications are selected based on goodness-of-fit diagnostic statistics.

6.3.3.2 *Risk-neutralisation of ARMA-EGARCH*

Let T be the longest possible maturity for the ERM product; as an example, for a 65 years old if we consider 100 the longest survivor age then $T = 35$ and let $\mathbb{P}$ be the probability measure associated with the information set $\mathscr{F}_T$. Consider $\mathbb{P}_t$ be the projected measure $\mathbb{P}$ on the smaller information set $\mathscr{F}_t$. Following Buhlman, Delbaen, Embrechts, and Shiryaev (1996), Siu, Tong, and Yang (2004) and Li et al. (2010), for a given sequence of constants $\lambda_1, \lambda_2, \ldots, \lambda_t, \ldots$ the conditional Esscher distribution $\widetilde{\mathbb{P}}_t$ is defined computationally through

$$F_{\widetilde{\mathbb{P}}_t}(y;\lambda_t|\mathscr{F}_t)=\frac{\int_{-\infty}^{y}e^{\lambda_t x}dF_{\mathbb{P}_t}(x|\mathscr{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t}|\mathscr{F}_t)}$$

(6.20)

The key to the risk-neutralisation under the conditional Esscher measure is to observe that the moment generating function of Y_t given $\mathscr{F}_{t-1}$ under $\widetilde{\mathbb{P}}_t$ is calculated from

$$E_{\widetilde{\mathbb{P}_t}}(e^{zY_t};\lambda_t|\mathscr{F}_{t-1})=\frac{E_{\mathbb{P}_t}(e^{(z+\lambda_t)Y_t}|\mathscr{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t}|\mathscr{F}_t)}$$

(6.21)

Because $Y_t|\mathscr{F}_{t-1}\sim N(\mu_t,h_t)$ so then it can be proved that[2]

$$E_{\widetilde{\mathbb{P}_t}}(e^{zY_t};\lambda_t|\mathscr{F}_{t-1})=e^{(\mu_t+h_t\lambda_t)z+\frac{1}{2}h_t z^2}$$

(6.22)

The risk-neutral-measure is identified from the local martingale condition by finding those λ_t^q such that

$$E_{\widetilde{\mathbb{P}_t}}(e^{Y_t};\lambda_t^q|\mathscr{F}_{t-1})=e^{r-g}$$

(6.23)

with r the risk-free rate and g the rental yield[3]. This gives the risk-neutralising constants

$$\lambda_t^q = \frac{r - g - \mu_t - \frac{1}{2}h_t}{h_t}$$

(6.24)

Combining things together gives the sequence of risk-neutral measures Q_t such that

$$E_{\widetilde{\mathbb{Q}_t}}(e^{zY_t}; \lambda_t^q | \mathscr{F}_{t-1}) = e^{(r-g-\frac{1}{2}h_t)z+\frac{1}{2}h_t z^2}$$

(6.25)

[2]We have corrected some typos in their formulae.

[3]In the absence of market prices for forwards/futures or total return swaps on property, the martingale condition here is linked directly to say that, locally, the discounted asset price is driftless. It is not clear from the previous literature why g was also used here.

which shows that the risk-neutralisation effect is to keep the same type of normal distribution but change by translation the parameters. Thus, under $\mathbb{Q}_t$, we have that

$$\boxed{Y_t | \mathscr{F}_{t-1} \sim N(r - g - \frac{1}{2}h_t, h_t)}$$

(6.26)

For pricing the NNEG we need to calculate the following risk-neutral expectation, see (Li et al., 2010),

$$e^{-r(k+0.5+\delta)}E^{\mathbb{Q}}[\Big(L_0e^{R(k+0.5+\delta)} - H_{k+0.5+\delta}\Big)^+]$$

(6.27)

For simplicity let us denote by $\tau = k + 0.5 + \delta$ which is the known maturity given by the termination of the ERM and $K = L_0 e^{R(k+0.5+\delta)}$ is the accrued balance at τ which is known. Hence the option above is a put option on H_τ. Now, a correct approach will have to take a path-dependent approach and build recursively the chain of conditional volatilities (variances) to the required maturity. For example, for maturity τ, the house price H_τ can be calculated as

$$H_\tau = H_0 \exp(\sum_{i=1}^{i=\tau} Y_i)$$

using Monte Carlo simulation based on (6.26) for the risk-neutral measure and based on (6.19) under the real world measure. We shall refer to this Monte Carlo simulation approach[4] as the ARMA-EGARCH risk neutral (ARMA-EGARCH-rn) for the former and the ARMA-EGARCH real world (ARMA-EGARCH-rw).

6.3.3.3 Criticism of GBM-rn/Black 76 Model

Some authors insist for the application of the geometric Brownian motion as a data generating process for the house prices and/or for the pricing of the European put options contingent on house prices, (see Buckner and Dowd, 2018; Dowd, Buckner, Blake, and Fry, 2019; Dowd, 2018; Hosty et al., 2008). Here are some of the main critiques and reservations about the

GBM-risk neutral methodology and also of the Black76 option pricing model.

- The GBM as a data generating process for house prices is totally inappropriate because it ignores serial correlation and stickiness of prices, as well as clustered volatility and downward jumps.
- Thus, GBM may forecast inflated values of the house price, because the volatility/variance associated with this process increases with time and it is not mean-reverting. This can be very dangerous for both risk-neutral (market consistent) or real-world valuations, making the NNEG valuations very small because of the overshooting in house prices.

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[4]A similar procedure applies for the AR(I)MA-GARCH family of models.

- The assumptions needed to apply the Black-Scholes or Black76 are not satisfied in financial economics terms for house prices and for an economy where there no futures contracts traded on house prices.
- Furthermore, Black-Scholes or Black76 depend heavily on the risk-neutral house prices growth rate *taken* as $r - g$, where g is the rental yield. Hence, NNEG value calculated based on either model may be inflated if $r - g$ stays very small (even negative), say low interest rates and higher rental yield, and may be undervalued if $r - g$ is relatively large, say high risk-free rates or small negative rental yield.
- If valuation must be done following the principle of risk-neutral valuation (market consistency) this does not mean that the GBM model is the appropriate data generating process. Just because it is a

widely known and easier to apply analytically model does not mean its implications are harmless.

- In Tunaru and Quaye (2019) we discuss at large that is possible to find models that are more suitable as data generating processes for the house prices than the more common geometric Brownian motion. For the suitable model we can price the NNEG under risk-neutrality or under the physical measure. Other models, continuous-time mean-reverting processes as highlighted by Knapcsek and Vaschetti (2007) or jump diffusion models) or discrete time (any other model from ARMA-GARCH-type family), may offer also a viable approach.
- Another common problem with the application of GBM-risk neutral and Black76 model is taking a constant risk-free rate operating from one year ahead to a long maturity (45 years). A risk-free yield curve, with different rates possibly for different maturities, may be more appropriate.
- From a regulatory perspective, it would be wrong in our opinion, to allow the geometric Brownian motion to be used as a data generating process for house prices and also to use the Black76 option pricing model as a benchmark, if there is no open market for forwards/futures on house prices. These models may induce false security in good time, are over-dependent on the level of risk-free rates and require the estimation of a parameter– rental yield–, that is difficult to connect with real-data.
- The regulator may try to consider model-free boundaries for NNEG values but these boundaries need to be determined clearly from first financial economics principles. However, if the bounds are not sharp,

even this idea may lead to serious overestimation or underestimation of NNEG values.

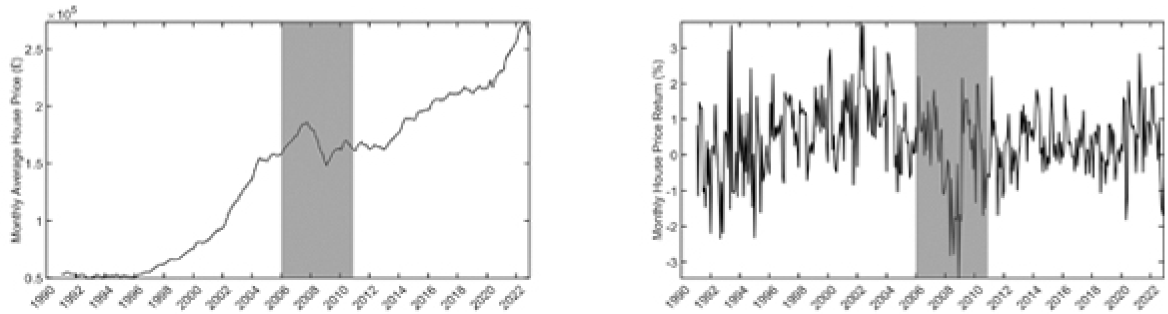
- It should be also noted that the Black76 model would be appropriate indeed for the US market where there is an opportunity to trade futures contracts on the Case-Shiller index family. However, given the way the reverse mortgage market is organised in the US, negative equity risk is managed with insurance and reinsurance products and it does not need an NNEG type of valuation.

6.3.4 Parameter Estimation under the Real-World Pricing Measure

6.3.4.1 GBM data generating process

For the GBM process specified in (6.8) we estimate the model parameters on the monthly log-return series of the Nationwide seasonally adjusted house price index between 1991 and 2022. The reason for using this historical time series rather than the quarterly Nationwide series going back to 1974 was that, for the monthly series, our sample size is almost double the sample size of the quarterly series. Models from the GARCH family require a relatively long sample in general to be able to fit reliably their parameters.

The Nationwide monthly average national house price time series and its returns are illustrated in Figures 6.1a and 6.1b. We also present the quarterly house price time series and its returns in Figures 6.2a and 6.2b. The descriptive statistics of the two time series are reported in Table 6.1. From these time series we keep an out of sample of 48 months (16 quarters) 2019-2022 for a comparative forecasting exercise. The evolution of house prices in the UK seems to be very similar to the evolution of nominal house prices in the US, as evidenced by the graphs of S & P Case Shiller indices, nationally and regionally. One can also remark that the house price correction over the global financial crisis period is of a lesser magnitude than it was experienced in the US. In addition, the long-run level seems to be a bit higher than zero by comparison to US, too.

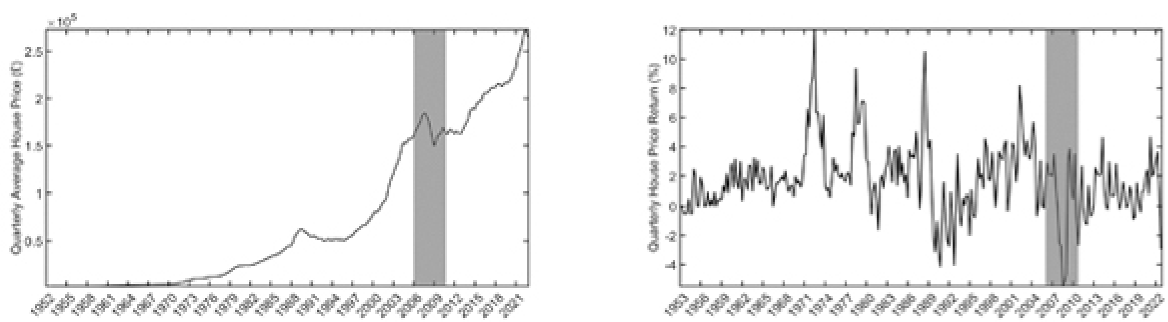


(a) Average House Price (Monthly) (b) Log-returns (Monthly)

Figure 6.1 Monthly Nationwide UK National Average House Price

Notes: The figure plots the historical evolution of UK Nationwide average house price time series. The frequency of the time series data is monthly, starting on January 1991 and ending on December 2022. We calculate the month t log-return as $\ln(H_t/H_{t-1})$.

Source: https://www.nationwidehousepriceindex.co.uk/download/uk-monthly-index



(a) Average House Price (Quarterly) (b) Log-returns (Quarterly)

Figure 6.2 Quarterly Nationwide UK National Average House Price

Notes: The figure plots the historical evolution of UK Nationwide average house price time series. The frequency of the time series data is quarterly, starting on October 1952 and ending on December

2022. We calculate the quarter t log-return as $\ln(H_t/H_{t-1})$. *Source*: https://www.nationwidehousepriceindex.co.uk/download/uk-monthly-index

TABLE 6.1 Summary statistics of Nationwide UK house price time series

Descriptive	Monthly House Price		Quarterly House Price	
	Price	Log-return	Price	Log-return
Mean	140,787.97	0.42%	72,312.85	1.77%
Median	159,548.00	0.45%	44,433.55	1.58%
Minimum	49,602.00	-3.46%	1,853.48	-5.49%
Maximum	273,751.00	3.73%	273,135.11	12.03%
SD	65,324.87	1.09%	77,836.56	2.41%
Skewness	-0.05	-0.11	0.89	0.57
Kurtosis	-1.23	0.64	-0.62	2.31
5% percentile	51,242.50	-1.49%	1,977.36	-2.07%
95% percentile	243,879.75	2.10%	216,084.75	6.33%

Notes: The table reports the summary statistics of Nationwide UK house price index. The time series data of monthly observations is from January 1991 to December 2022 for the summary statistics calculations reported in columns two and three. The calculations in columns four and five are based on quarterly time series observations from October 1952 to December 2022. SD is the standard deviation of the time series.

Source: https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series

Historically, the monthly log-return of house prices in the United Kingdom has varied between -2% and 3.5%. Throughout the subprime mortgage period, monthly house price log-returns dropped to levels around -3% and -3.3%; they eventually recovered to their historical level after 2010. Similarly, the quarterly house price log-returns of Nationwide UK dataset historically varies between -2% and 4%, reaching an ultimate high of 12%.

Table 6.1 provides some comprehensive empirical evidence that supports an argument for distributional differences between the quarterly and monthly average house price time series and their corresponding log-returns. The log-return of the monthly time series data has a negative skewness value of -0.11, indicative of a left-skewed distribution, and a low kurtosis value as compared to the quarterly log-return time series, which has a right-skewed distribution (positively skewness value 0.57) with a relatively high kurtosis value of 2.31. The quarterly time series exhibits a higher degree of variability compared to the monthly time series. The is due to the longer time span we have for the quarterly time series. This empirical overview simply confirms that the NNEG or ERP valuations produced with monthly house price series will be different from those produced with quarterly house price series.

We proceed to fit the geometric Brownian motion (GBM) model separately for the log-returns of monthly and quarterly average house price time series. For the GBM model, after discretising at monthly frequency, we use three methods to estimate the parameters from historical data, the maximum likelihood estimation (MLE), method of moments (MM) and generalised method of moments (GMM).

In Table 6.2 we report the estimation results for drift and volatility parameters comparatively, using data from Nationwide and for Halifax, the two main families of indexes for housing in the UK. For Nationwide we noticed that the MLE and MM methods give very close results. However, the GMM method gives a slightly lower volatility and almost half the drift rate when compared to the other two models. This indicates that the potential of model risk is real.

TABLE 6.2 Parameter estimates for the GBM process applied to the Nationwide and Halifax indexes.

Method of Estimation	Nationwide - Monthly		Nationwide - Quarterly		Halifax	
	μ	σ	μ	σ	μ	σ
Maximum Likelihood Estimation (MLE)	5.00%	3.78%	7.06%	4.81%	5.80%	3.96%
Generalised Method of Moments (GMM)	5.29%	6.89%	7.21%	5.30%	6.45%	2.27%
Method of Moments (MM)	5.08%	3.78%	7.18%	4.82%	5.88%	3.96%

Notes: The table reports the parameter estimates for the GBM process applied to the Nationwide and Halifax house price indexes. We use three different estimation methods. They are, the Maximum Likelihood Estimation (MLE), the Generalised Method of Moments (GMM), and the Method of Moments (MM). We use the entire time series in each estimation exercise. The monthly time series from Nationwide spans from January 1991 to December 2022, and the quarterly series is from October 1952 to December 2022. The data from Halifax is monthly from January 1983 and December 2014.

Source: https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series

The volatilities quoted in other studies are much larger, around 10%. A possible explanation is that they used quarterly data for a longer period going back to 1974 and stopping in 2006, before the subprime crisis.

We have recalculated the house price inflation across regions in the UK and we report the summary statistics in Table 6.3. The time series data is quarterly, starting on the first quarter of 1974 and ending on the last quarter of 2022. The volatility of the national index is 9.34% per annum (i.e. $4.67 \times \sqrt{4}$) over the extended period.

TABLE 6.3 Comparison of Nationwide house prices sample descriptive statistics across regions in the UK, 1974–2022.

Region	Mean (annualised)	Skew	Excess Kurtosis	SD (annualised)
North	6.39%	0.71%	-1.85%	6.29%
YorksHside	6.46%	0.75%	-0.10%	6.16%
NorthWest	6.85%	0.75%	-1.28%	5.28%
EastMids	6.94%	1.10%	1.45%	5.79%
WestMids	6.83%	1.21%	2.43%	5.72%
EastAnglia	7.10%	0.50%	-0.62%	6.32%
OuterSEast	7.26%	0.21%	-1.83%	5.74%
OuterMet	7.37%	0.12%	-1.83%	5.47%
London	7.83%	0.08%	-2.50%	6.08%
SouthWest	7.29%	0.68%	-0.43%	5.57%
Wales	6.64%	0.77%	0.18%	6.29%
Scotland	6.26%	0.02%	-1.99%	4.71%
NIreland	6.68%	-0.20%	-0.88%	7.43%
UK	6.90%	0.38%	-1.19%	4.67%

Notes: The table reports the summary statistics of the UK Nationwide house price index at regional level. The time series data is quarterly observations starting from January 1974 to October 2022.
Source: https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series

The annual average growth in different regions over the period is relatively similar, with an overall average of 6.90% per annum. The lowest region is Scotland with 6.26% and the highest is London with 7.83%. The Nationwide series is biased towards the South-West. An earlier version of these results is reported in Table A.1 of Appendix A using data from quarterly time series data from 1974 to 2018.

Here we conducted a sensitivity analysis with respect to the estimation of the parameters of the geometric Brownian motion when applied to house price time-series in the United Kingdom. Table 6.4 reports the estimates for rate of growth of house prices μ and volatility σ, under GBM, across various regions, and using three methods of estimation.

TABLE 6.4 Estimation of annualised drift and volatility parameters from Nationwide quarterly time series 1974–2022 for the entire UK and also across regions, using three methods of estimation maximum likelihood estimation (MLE), method of moments (MM) and generalised method of moments (GMM)

Region	MLE		MM		GMM	
	μ	σ	μ	σ	μ	σ
Period: 1974-2022						
North	6.15%	6.11%	6.34%	6.13%	6.36%	6.42%
Yorks The Hamber	6.22%	6.00%	6.41%	6.01%	6.43%	6.32%
North West	6.66%	5.14%	6.79%	5.15%	6.82%	5.56%
East Mids	6.73%	5.60%	6.88%	5.61%	6.91%	5.98%
West Mids	6.61%	5.53%	6.77%	5.54%	6.79%	5.88%
East Anglia	6.85%	6.16%	7.04%	6.17%	7.06%	6.46%
Outer S East	7.04%	5.61%	7.20%	5.63%	7.22%	5.99%
Outer Met	7.16%	5.36%	7.30%	5.37%	7.32%	5.74%
London	7.57%	5.95%	7.76%	5.97%	7.78%	6.31%

Region	MLE		MM		GMM	
	μ	σ	μ	σ	μ	σ
Period: 1974-2022						
South West	7.08%	5.41%	7.22%	5.43%	7.24%	5.79%
Wales	6.39%	6.12%	6.59%	6.13%	6.61%	6.42%
UK	6.74%	4.57%	6.85%	4.58%	6.87%	5.11%
Period covered in Hosty et al. (2008) paper 1974-2006						
North	8.55%	6.64%	8.77%	6.67%	8.79%	6.89%
Yorks The Hamber	8.39%	6.47%	8.59%	6.49%	8.61%	6.73%
North West	8.95%	5.31%	9.10%	5.33%	9.13%	5.78%
East Mids	8.68%	6.04%	8.86%	6.07%	8.88%	6.36%
West Mids	8.56%	6.04%	8.74%	6.06%	8.76%	6.36%
East Anglia	8.68%	6.68%	8.90%	6.71%	8.92%	6.93%
Outer S East	8.82%	6.00%	9.00%	6.02%	9.02%	6.32%
Outer Met	8.86%	5.62%	9.02%	5.64%	9.05%	6.04%
London	9.21%	6.16%	9.40%	6.18%	9.42%	6.46%
South West	9.06%	5.73%	9.23%	5.75%	9.25%	6.12%
Wales	8.52%	6.35%	8.73%	6.38%	8.74%	6.63%
UK	8.69%	4.65%	8.81%	4.67%	8.83%	5.17%
Period: 2007-2022						
North	1.20%	3.83%	1.27%	3.86%	1.33%	4.81%
Yorks The Hamber	1.74%	4.10%	1.82%	4.13%	1.86%	4.84%
North West	1.88%	3.79%	1.95%	3.82%	2.00%	4.61%
East Mids	2.66%	3.86%	2.73%	3.89%	2.78%	4.65%
West Mids	2.55%	3.55%	2.61%	3.58%	2.66%	4.46%
East Anglia	2.97%	4.36%	3.08%	4.39%	3.12%	5.10%

Region	MLE		MM		GMM	
	μ	σ	μ	σ	μ	σ
Period: 1974-2022						
Outer S East	3.24%	4.15%	3.33%	4.19%	3.38%	4.95%
Outer Met	3.50%	4.26%	3.59%	4.29%	3.64%	5.02%
London	3.96%	5.06%	4.09%	5.10%	4.12%	5.52%
South West	2.80%	3.96%	2.89%	3.99%	2.94%	4.83%
Wales	2.00%	4.96%	2.12%	5.00%	2.15%	5.47%
UK	2.58%	3.63%	2.64%	3.66%	2.68%	4.51%

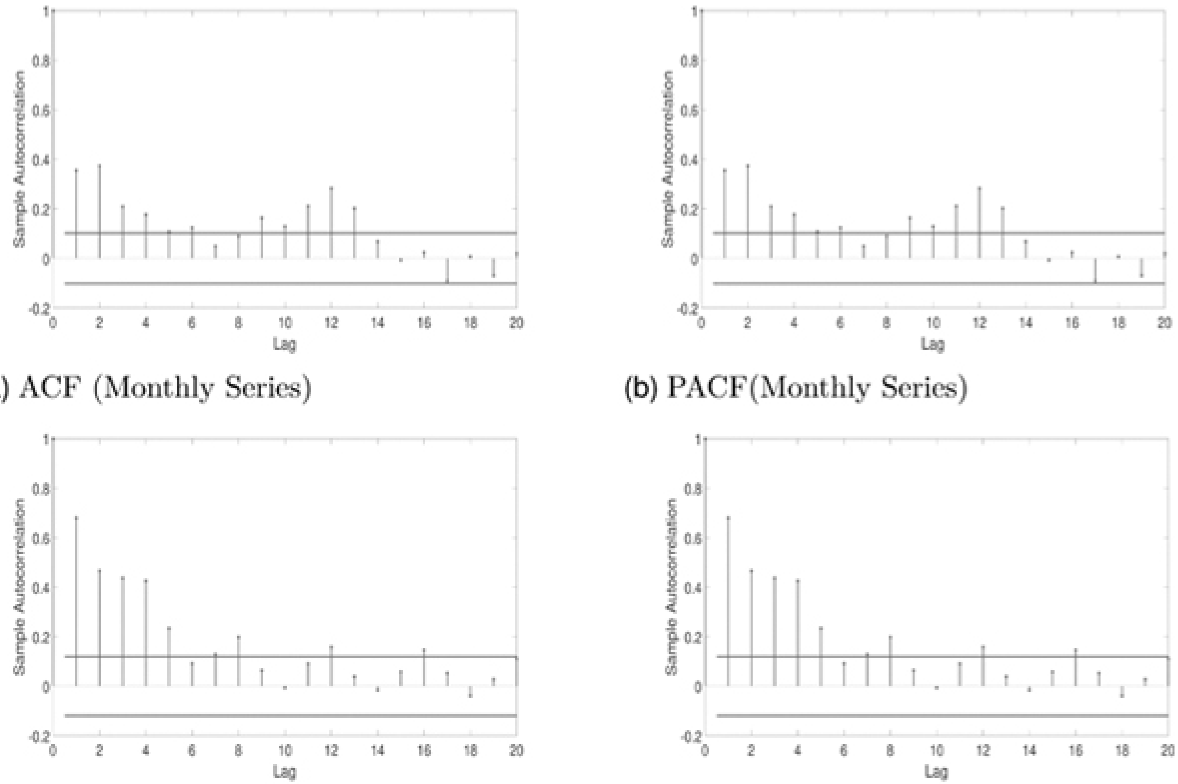
There is variability in the estimates of house price expected growth rate and volatility, across regions, depending on the period of estimation and method of estimation. Recall the estimates for monthly data of the same Nationwide series were slightly different as well. One can note that post subprime crisis the volatility is smaller than before the crisis and the expected growth rate decreased also substantially. Comparing the volatility value obtained in Table 6.4 for the period 1974-2006 for the entire UK we notice a discrepancy between our estimated figures (4.65%, 4.67% and 5.17%) and the reported figure of 8% in Hosty et al. (2008).

We have a close figure of 4.65% in Table 6.3 for the standard deviation of house prices and we conjecture that the value reported in Hosty et al. (2008) refers to this standard deviation. On this point, please note that the standard deviation for house price levels is not to be used as the σ volatility parameter in a GBM process and any additional increments due to idiosyncratic house price risk should be added on the returns volatility value and not to the standard deviation. This point is important since in the NNEG literature 10% volatility is taken as indicative for the UK and based on the results in A.2 we can see that this already a very conservative estimate.

6.3.4.2 *Estimating the ARMA-EGARCH model*

In Figure 6.1(b), the returns clearly fluctuate around a constant level that is close to zero. Small changes is common and consecutive exhibiting some degree of conditional heteroscedasticity. In line with good practice, we scale the monthly return time series by 100 and centre the percentage returns so that $100 \times (Y_t - \overline{Y}_t)$. The allows us to introduce some degree of numerical stability.

We also check for autocorrelation from the sample autocorrelation function, Figure 6.3a, and partial autocorrelation function, Figure 6.3b, in the scaled time series of house price returns. There is significant autocorrelation up to lag 5 and also at lag 7 in the ACF plot. The PACF plot also show significant autocorrelation at up to lag 2 and also at lag 7. We also test for the significance of autocorrelations at lag 5, using the Ljung-Box test of Ljung and Box (1978). The null hypothesis that all autocorrelations are zero up to and including lag 5 is rejected with a test statistic of 27.845. The results presented in those graphs clearly indicate that house prices are strongly characterised by serial autocorrelation.



(a) ACF (Monthly Series)

(b) PACF(Monthly Series)

(c) ACF (Quarterly Series)

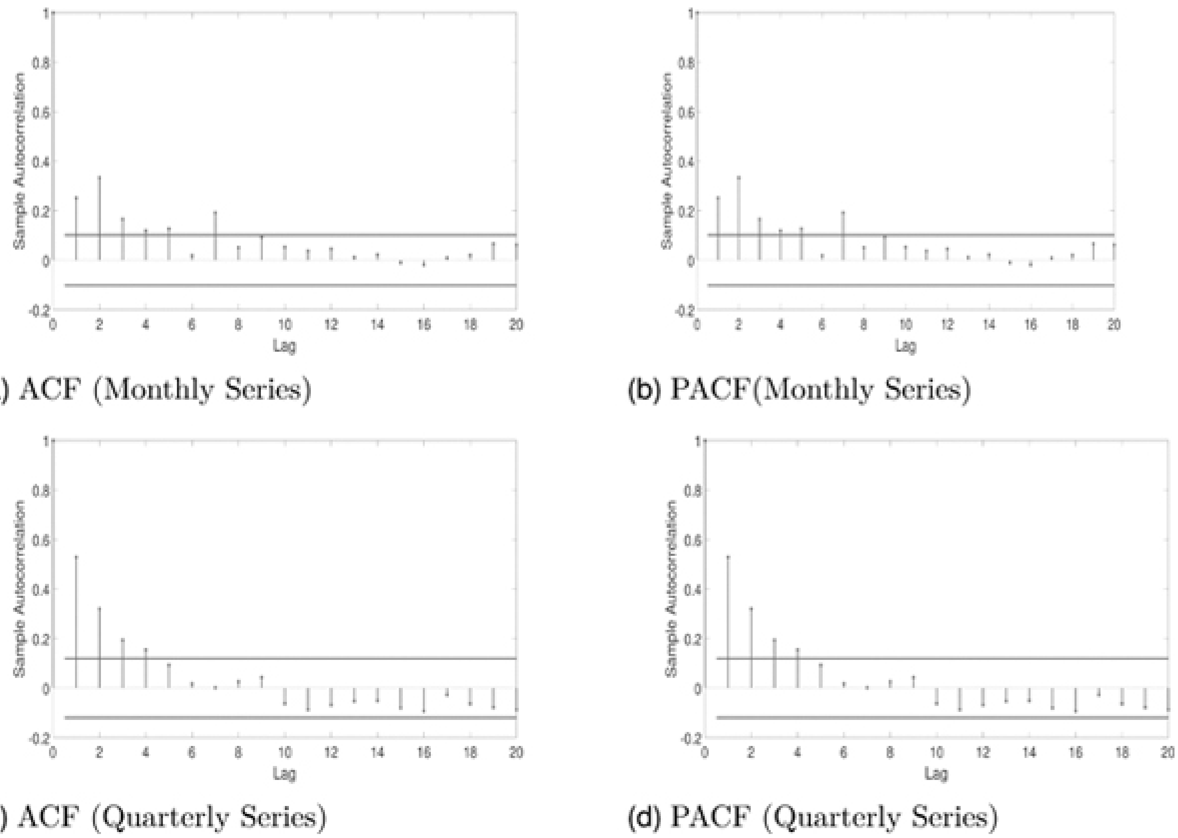
(d) PACF (Quarterly Series)

Figure 6.3 Checking for autocorrelation in house price returns

Notes: The figure plots the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) for the standardised house price returns. The frequency of the time series data is monthly for subplots (a) and (B), starting from January 1991 to December 2022. Subplots (c) and (d) are from quarterly house price time series starting from October 1952 to December 2022

Source: https://www.nationwidehousepriceindex.co.uk/download/uk-monthly-index

We additionally check for the presence of conditional heteroscedasticity using the ACF and PACF plots of the squared scaled returns time series. The ACF and PACF plots are respectively reported in Figures 6.4a 6.4b and Figure 6.4c 6.4d. Both plots depict significant serial dependence. This indicates that the house price returns are conditionally heteroscedastic. Using the Bollerslev, Engle, and Nelson (1994) ARCH test, we reject the null hypothesis of no conditional heteroscedasticity against the alternative of an ARCH model with four lags. The test statistic of the Engle ARCH test is 54.5792. This is further evidence that house prices returns time series depart from the usual properties of the Brownian motion and hence, better models are needed to represent the dynamics of house price data. One such class of models is the ARMA-EGARCH family of models.



(a) ACF (Monthly Series)

(b) PACF(Monthly Series)

(c) ACF (Quarterly Series)

(d) PACF (Quarterly Series)

Figure 6.4 Checking for conditional heteroscedasticity in squared returns of Nationwide house price time series

Notes: The figure plots the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) for the standardised squared house price returns. The frequency of the time series data is monthly for subplots (a) and (B), starting from January 1991 to December 2022. Subplots (c) and (d) are from quarterly house price time series starting from October 1952 to December 2022 *Source*: https://www.nationwidehousepriceindex.co.uk/download/uk-monthly-index

There are potential problems with stationarity of squared returns time series of house prices, as visualised on the autocorrelations graphs in Figure

6.4a, Figure 6.4d. Thus, the geometric Brownian motion would not be appropriate. Furthermore, a model considering conditional volatilities specifically would be highly desirable even if that means it will require more advanced option pricing techniques.

Another important aspect coming out of these figures is that the models may belong to the same family of models but they may have very different parameter estimated depending on the data that is used. As we pointed out already, for the UK case, the historical sample size is very different between the monthly and the quarterly frequency for the same house price index. Therefore, parameter estimation risk, model specification risk and data sample risk are all present in this context. A thorough risk management analysis would have to take into account all these considerations.

For the ARMA-EGARCH models we consider a forward model selection procedure. From all models that fit well data we select the model using an Occam's razor approach, looking for the simplest possible model that has significant parameters and provides a very good fit to the data. More specifically, the forward model selection procedure begins with the ARMA order (m, M), and EGARCH order (b, B) order respectively starting at (0,0) and (0,1) and ending at the order (4,4) and (1,1). We follow a similar approach in Li et al. (2010) where the selection criteria for the optimal specification is based on the ACF and the Ljung-Box test for standardised innovations and squared standardised innovation. The ARMA(4,4)-EGARCH(1,1) model, with the parameters provided in Table 6.5 (Panel A) is the optimal specification for the monthly time series while ARMA(4,3)-EGARCH(1,1) is the optimal specification for the quarterly time series data.

TABLE 6.5 Parameter estimates for ARMA-EGARCH models

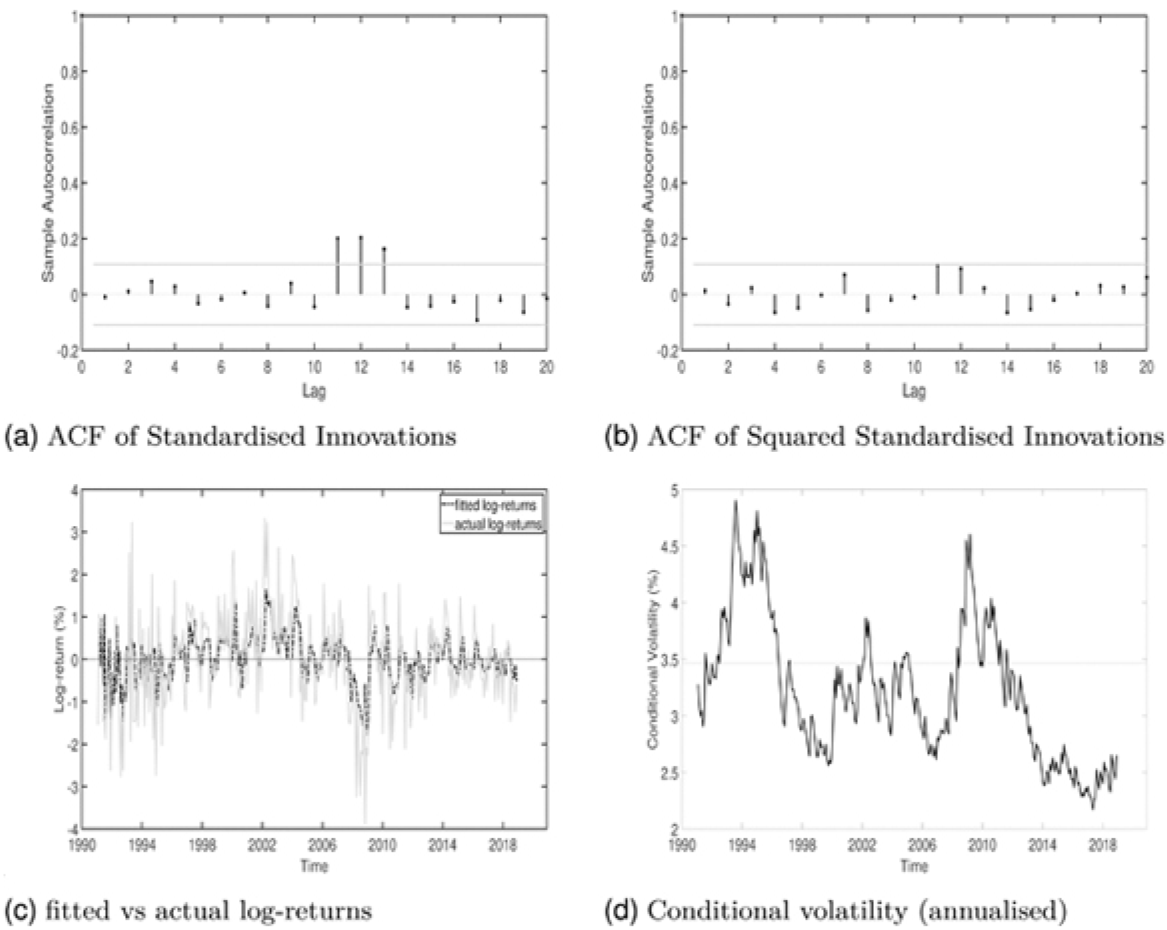
Parameter	Estimate	Standard Error	T-Statistic	P-Value
Panel A: ARMA(4,4)-EGARCH(1,1)				
In-sample Data: Monthly time series (1991 - 2018)				
c	0.0171	0.1424	0.1198	0.9046
ϕ_1	-0.7941	0.1493	-5.3188	0.0000
ϕ_2	-0.2001	0.1582	-1.2647	0.2060
ϕ_3	0.2072	0.1228	1.6878	0.0914
ϕ_4	0.5257	0.0723	7.2700	0.0000
θ_1	1.0900	0.1708	6.3812	0.0000
θ_2	0.7928	0.2193	3.6146	0.0003
θ_3	0.3198	0.1862	1.7182	0.0858
θ_4	-0.2963	0.1244	-2.3814	0.0172
k	-0.0034	0.0082	-0.4093	0.6823
α_1	0.9691	0.0260	37.2270	0.0000
β_1	0.1345	0.0523	2.5703	0.0102
γ_1	-0.0246	0.0219	-1.1233	0.2613
Panel B: ARMA(4,3)-EGARCH(1,1)				
In-sample Data: Quarterly time series (1952 - 2018)				
c	-0.3167	0.3465	-0.9141	0.3606
ϕ_1	-0.2624	0.0351	-7.4743	0.0000

Notes: The table reports parameter estimates for ARMA(4,4)-EGARCH(1,1) model over the monthly Nationwide house price time series between Jan 1991 and Dec 2022 in Panel A. The in-sample period for this estimation is from Jan 1991 to Dec 2018 and the out-of-sample period is from Jan 2019 to Dec 2022. Panel B similarly reports parameter estimates for ARMA(4,3)-EGARCH(1,1) model over the quarterly Nationwide house price time series between 1952 Q4 - 2022 Q4. The in-sample period is from 1952 Q4 to 2018 Q4 and the out-of-sample period is from 2019 Q1 - 2022 Q4. The time series data is from Nationwide data and resource website https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series

Parameter	Estimate	Standard Error	T-Statistic	P-Value
Panel A: ARMA(4,4)-EGARCH(1,1)				
In-sample Data: Monthly time series (1991 - 2018)				
ϕ_2	-0.2750	0.0318	-8.6556	0.0000
ϕ_3	-0.2837	0.0330	-8.5858	0.0000
ϕ_4	0.7216	0.0346	20.8850	0.0000
θ_1	0.9257	0.0310	29.9090	0.0000
θ_2	0.9597	0.0133	72.2400	0.0000
θ_3	0.9673	0.0323	29.9180	0.0000
k	0.1372	0.0496	2.7673	0.0057
α_1	0.8177	0.0627	13.0440	0.0000
β_1	0.4718	0.1121	4.2096	0.0000
γ_1	0.1475	0.0538	2.7422	0.0061

Notes: The table reports parameter estimates for ARMA(4,4)-EGARCH(1,1) model over the monthly Nationwide house price time series between Jan 1991 and Dec 2022 in Panel A. The in-sample period for this estimation is from Jan 1991 to Dec 2018 and the out-of-sample period is from Jan 2019 to Dec 2022. Panel B similarly reports parameter estimates for ARMA(4,3)-EGARCH(1,1) model over the quarterly Nationwide house price time series between 1952 Q4 - 2022 Q4. The in-sample period is from 1952 Q4 to 2018 Q4 and the out-of-sample period is from 2019 Q1 - 2022 Q4. The time series data is from Nationwide data and resource website https://www.nationwidehousepriceindex.co.uk/resources/f/uk-data-series

The goodness-of-fit usual checks presented in Figure 6.5 look good, the in-sample fit is comparatively good and the conditional volatilities series are in the expected range, varying in a mean-reverting fashion around the value of 0%. One can also notice that the fitted conditional volatilities varied between 0.65% (2.26% annualised) and 1.6% (5% annualised).



(a) ACF of Standardised Innovations

(b) ACF of Squared Standardised Innovations

(c) fitted vs actual log-returns

(d) Conditional volatility (annualised)

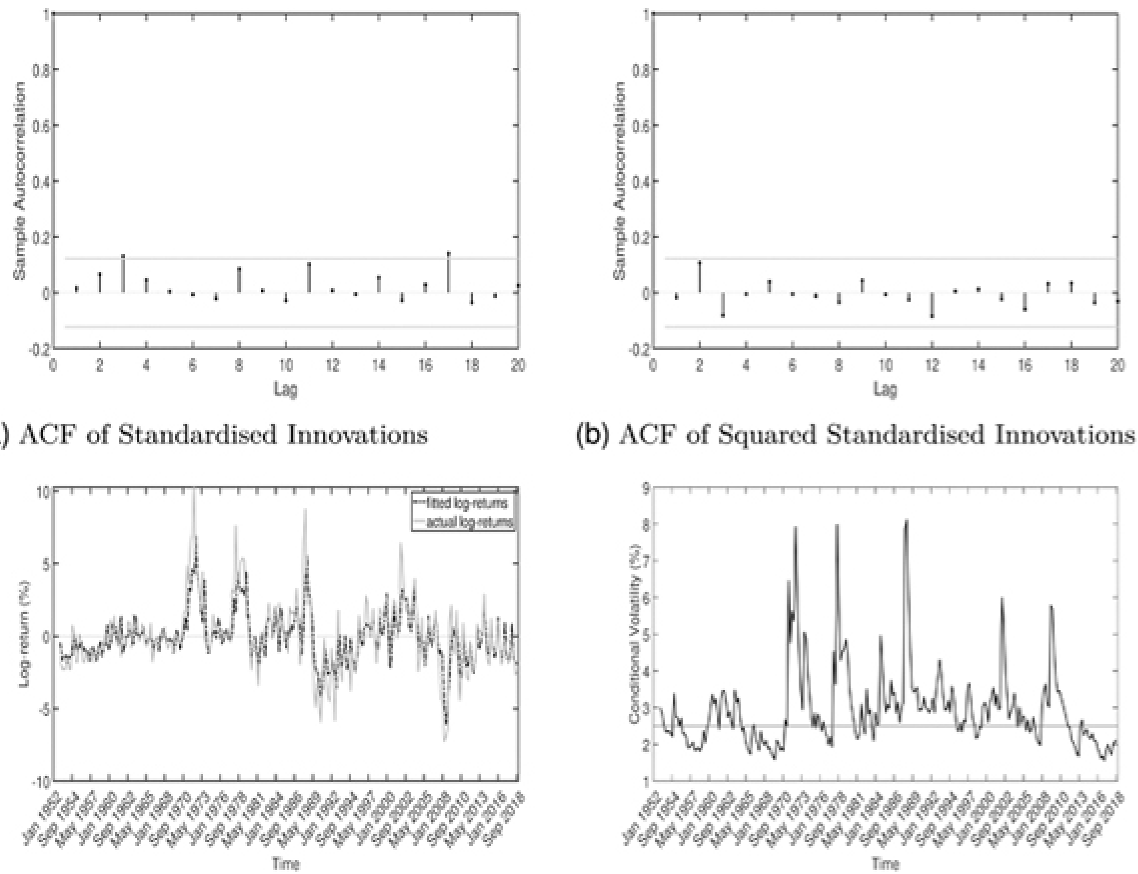
Figure 6.5 Goodness-of-fit: ARMA(4,4)-EGARCH(1,1) model for Nationwide monthly average house prices.

Notes: The time series data used for in-sample fitting is from January 1991 to December 2018. The out-of-sample data is from January 2019 to December 2022. The data set is from the monthly Nationwide national average house price series and the goodness-of-fit results are for the fitted model reported in Panel A of Table 6.5.

The results are of course sensitive to the historical sample period, the frequency of data and the model that is selected to represent the dynamics of the house prices. It is useful to consider different sample periods and frequencies and to be prepared to employ possibly some other model from the same family.

Furthermore, we remind readers the well known puzzle in financial econometrics that models that seem to fit well in sample are not necessarily fitting well out-of-sample. Therefore, when judging a model we should also think of checks and performance measures related to a forecasting exercise since NNEG calculation for example is a forward looking risk management exercise.

The goodness-of-fit usual checks presented in Figure 6.6 are again suggesting a good model, the in-sample fit is excellent and the conditional volatilities series are in the expected range, varying in a mean-reverting fashion around the value of 0.72% (monthly) or 2.5% on an annualised basis. One can also notice that this model produced in-sample conditional volatilities between 0.4% (1.5% annualised) and 2.3% (8% annualised).



(a) ACF of Standardised Innovations

(b) ACF of Squared Standardised Innovations

(c) fitted vs actual log-returns

(d) Conditional volatility (annualised)

Figure 6.6 Goodness-of-fit: ARMA(4,3)-EGARCH(1,1) model for Nationwide quarterly average house prices.

Notes: The time series data used for in-sample fitting is from 1952 Q4 to 2018 Q4. The out-of-sample data is from 2019 Q1 to 2022 Q4. The data set is from the quarterly Nationwide national average house price series and the goodness-of-fit results are for the fitted model reported in Panel B of Table 6.5.

It should also be remarked that very rarely the conditional annualised volatilities for the Nationwide quarterly series went above 5%. The extreme volatilities seem to be associated with well-known financial market crashes but they were very short-lived.

Table 6.6 reports the p-values for serial autocorrelation tests. For both monthly and quarterly datasets, and for both returns and squared returns, there is clear indication of serial correlation. Hence, this is further evidence against possible use of the geometric Brownian motion as a data generating process. It also suggests that a model embedding conditional volatilities explicit modelling would be useful.

TABLE 6.6 LM test for the presence of serial correlation

Lags	Time series of Residuals		Time series of Squared Residuals	
	Monthly	Quarterly	Monthly	Quarterly
1	0.864	0.777	0.809	0.723
2	0.969	0.649	0.674	0.522
3	0.853	0.060	0.808	0.164
4	0.873	0.128	0.571	0.258

Notes: The table reports the *p*-values for LM test for the presence of serial correlation from 1 to 20 lags. The test is conducted on the time series of residuals and the squared residuals respectively produced by the ARMA-EGARCH models specified in Table 6.5. The test is under the null hypothesis of no serial correlation. The implementation of the LM test follows from the Kevin Sheppard's MFE toolbox, using robust standard errors.

Lags	Time series of Residuals		Time series of Squared Residuals	
	Monthly	Quarterly	Monthly	Quarterly
5	0.884	0.150	0.526	0.125
6	0.856	0.225	0.756	0.174
7	0.910	0.326	0.405	0.245
8	0.879	0.252	0.310	0.331
9	0.892	0.299	0.376	0.372
10	0.895	0.355	0.446	0.454
11	0.135	0.277	0.412	0.552
12	0.006	0.324	0.103	0.537
13	0.000	0.392	0.134	0.606
14	0.001	0.400	0.116	0.659
15	0.001	0.411	0.068	0.710
16	0.001	0.440	0.079	0.443
17	0.001	0.125	0.079	0.323
18	0.003	0.133	0.121	0.295
19	0.003	0.121	0.073	0.253
20	0.004	0.144	0.075	0.279

Notes: The table reports the *p*-values for LM test for the presence of serial correlation from 1 to 20 lags. The test is conducted on the time series of residuals and the squared residuals respectively produced by the ARMA-EGARCH models specified in Table 6.5. The test is under the null hypothesis of no serial correlation. The implementation of the LM test follows from the Kevin Sheppard's MFE toolbox, using robust standard errors.

6.3.5 Forecasting Comparison

Ultimately a good model for house price returns should have good forecasting power, at least at short and medium horizon. We retained the out-of-sample period of 2018-2022, monthly, to compare the forecastability of various models.

In Table 6.7 we report some measures of forecasting accuracy such as root mean squared error (RMSE) and mean average error (MAE) as well as the Diebold-Mariano test for comparing GBM model under different estimation methods with the selected ARMA-EGARCH model, based on the out-of-sample data for monthly Nationwide index. The models in bold provide superior forecasting performance by comparison with the paired model. Clearly the ARMA(4,4)-EGARCH model outperforms the GBM specification, under any of the three parameter estimation method. Moreover, the GBM does better under the GMM method of parameter estimation.

TABLE 6.7 Comparing GBM model under different estimation methods with the selected ARMA-EGARCH model with Diebold Mariano test over the out-sample period January 2019 to December 2022.

MODEL	RMSE (%)	MAE (%)	
GBM-MLE	1.1809	0.9582	
GBM-GMM	1.2852	1.0398	
GBM-MM	1.2323	0.9994	
ARMA(4,4)-EGARCH(1,1)	0.9727	0.7631	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-4.5918	0.0000
GBM-MLE	GBM-MM	-4.4989	0.0000

MODEL	RMSE (%)	MAE (%)	
GBM-MLE	**ARMA(4,4)-EGARCH(1,1)**	2.3601	0.0225
GBM-GMM	**GBM-MM**	4.6653	0.0000
GBM-GMM	**ARMA(4,4)-EGARCH(1,1)**	2.9177	0.0054
GBM-MM	**ARMA(4,4)-EGARCH(1,1)**	2.6632	0.0106

In Figure 6.7 we illustrate the forecasting error for the out-of-sample Nationwide monthly time series for the last four years. It is evident that the ARMA(4,4)-EGARCH(1,1) outperforms the GBM house price forecasting. It is also observable that the GMM method of estimation gives better results than the MLE and MM estimation variants.

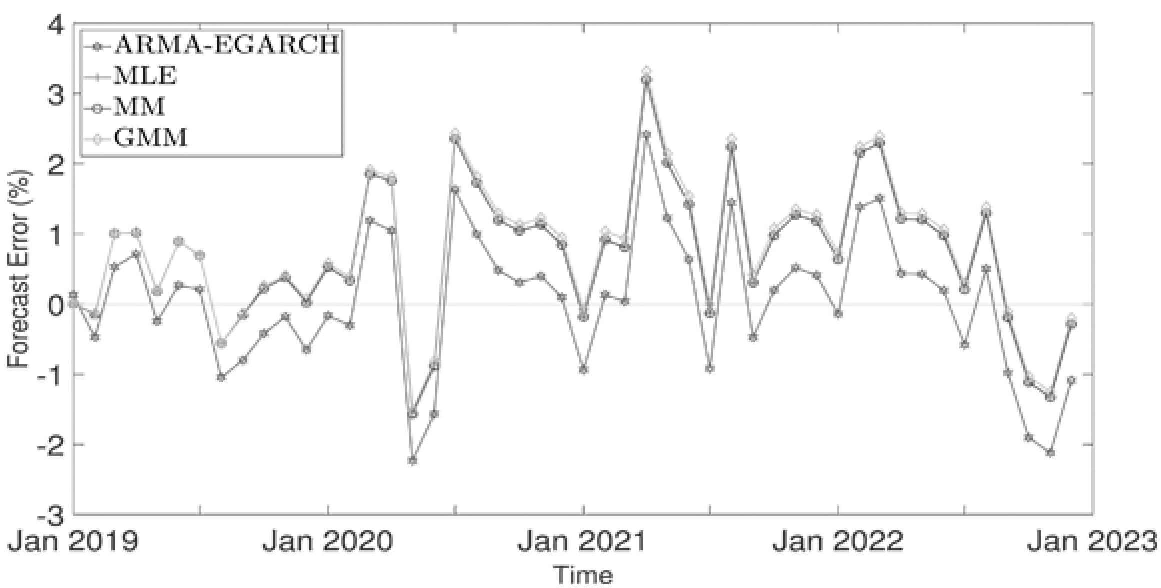


Figure 6.7 Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,4)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Jan 2019 to Dec 2022.

In Figure 6.8, we redo the same analysis for the forecasting error for the out-of-sample Nationwide monthly time series with four years out of sample. The ARMA(4,4)-EGARCH(1,1) outperforms the GBM house price forecasting. Moreover, now the MLE estimates for GBM dominates the MLE and GMM method, confirming that there is substantial parameter estimation risk even for such a simple model as GBM.

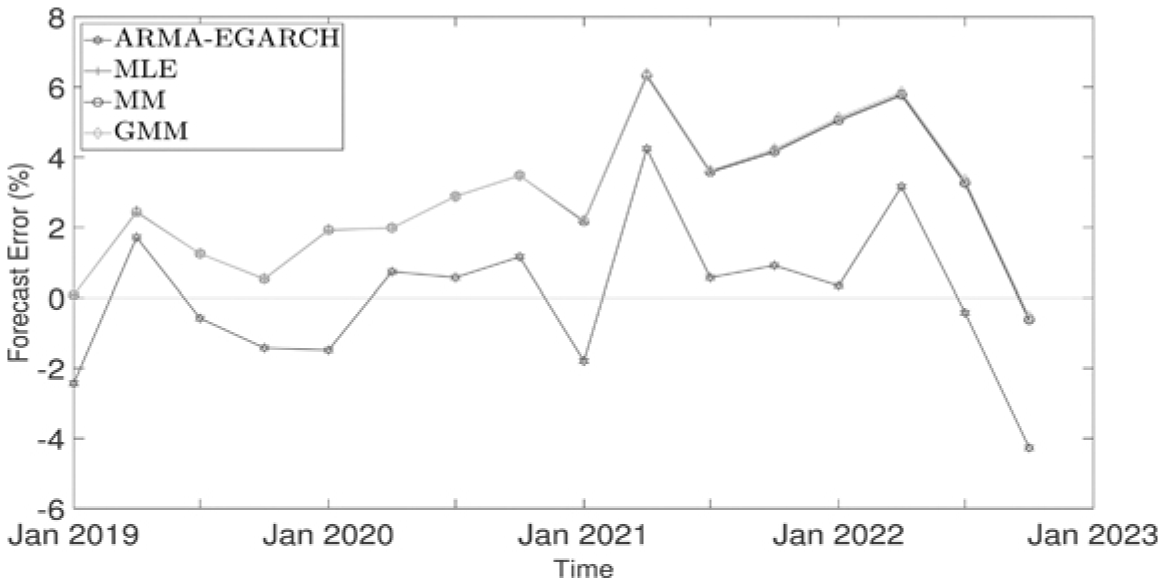


Figure 6.8 Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Quarterly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period 2019 Q1 to 2022 Q4.

In Table 6.8, we present the forecasting testing result based on monthly frequency and refitted models.[5] Even for this much longer period the forecasting under the ARMA-EGARCH model is net superior to the forecasting under GBM.

TABLE 6.8 Comparing forecasting (quarterly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample period 2019Q1–2022Q4.

MODEL	RMSE (%)	MAE (%)	
GBM-MLE	3.3534	2.8407	
GBM-GMM	3.3962	2.8689	
GBM-MM	3.3624	2.8473	
ARMA(4,3)-EGARCH(1,1)	2.0398	1.6143	
Diebold-Mariano Forecast Accuracy Testing			
MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-2.8874	0.0113
GBM-MLE	GBM-MM	-3.4585	0.0035
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	2.4088	0.0293
GBM-GMM	**GBM-MM**	2.7556	0.0147
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	2.4357	0.0278
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	2.4162	0.0289

[5]For ease of comparison we retained the GBM model with the three estimation methods and ARMA(4,3)-EGARCH(1,1) that again provides a good fit to the data in-sample.

For the NNEG put option pricing we are going to simulate forecasting pathways for the conditional variance series $\{h_t\}_{\{t\geq 0\}}$. The graphs in Figure 6.9 describe the conditional simulated pathways for variance series and returns series under ARMA(4,4)-EGARCH(1,1) model. For some months volatilities can spike up leading to potentially high local NNEG values.

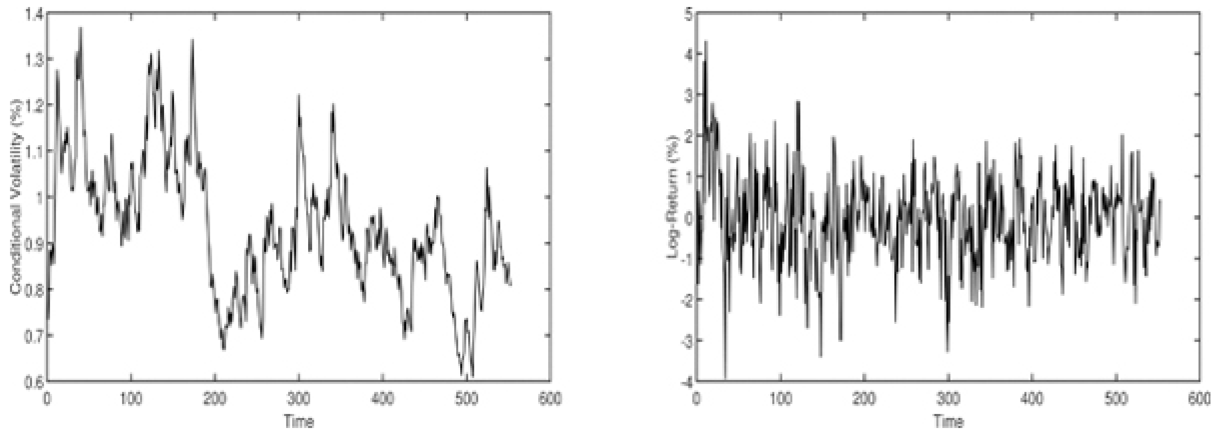


Figure 6.9 Simulated paths for the conditional volatilities and conditional returns under the ARMA(4,4)-EGARCH(1,1) model for 45×12 months ahead.

The simulated pathways are only some singular paths for explanation purposes. For any ERP calculus the analyst must consider many such simulated paths. The valuations will be finalised by averaging valuations performed on individual pathways. This methodology highlights the importance of recognising the stochastic nature of modelling house prices.

One word of caution. Models do not have sentimental influence on house prices. They try to capture the dynamics of this particular asst class. However, stepping back one step we should recognise that the economic environment may change dramatically over long periods of time, the people buying and selling houses may be from different generations and have very different set of preferences. It is very difficult, if not impossible, to identify a model that would perform well all the time.

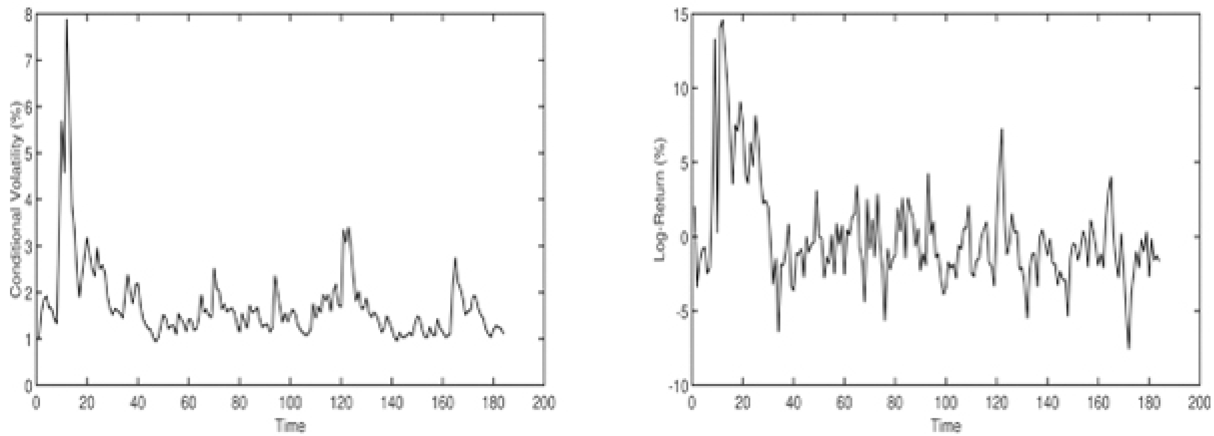


Figure 6.10 Simulated paths for the conditional volatilities and conditional returns under the ARMA(4,3)-EGARCH(1,1) model for 45×4 quarters ahead.

6.4 RISK-NEUTRAL VERSUS REAL-WORLD

6.4.1 Interest rate input data

Market data on the yield-to-maturity (YTM) of UK government bonds and treasuries as of 15 November 2023 (13.45 GMT) is shown in Table 6.9. One can notice that this yield curve is inverted and also has a slightly upward trending back end. Based on the information provided in Table 6.9, the risk-free interest rate utilised in the NNEG and ERM valuations will be determined. For simplicity of exemplifications, several times i this book we will use 5% as the risk-free rate to discount cash-flows paid at the end of the year ahead. More precise calculations would require working with different risk free rates corresponding to different maturities. In order to have risk-free rates for all maturities we need to construct the discount curve for all years using interpolation and even extrapolations of market rates. An even more precise methodology would insist on using stochastic interest rates which can be produced by various financial models that are calibrated on market rates.

TABLE 6.9 United Kingdom Yield to Maturity: 15 November 2023 13:45 GMT

Maturity	Yield	1 week ago	1 month ago
1 Month	5.27%	5.63%	5.63%
3 Month	5.32%	5.69%	5.69%
6 Month	5.35%	5.75%	5.75%
1 Year	4.98%	5.24%	5.24%
2 Year	4.61%	5.07%	5.07%
3 Year	4.32%	4.73%	4.73%
4 Year	4.12%	4.44%	4.44%
5 Year	4.21%	4.60%	4.60%
7 Year	4.06%	4.32%	4.32%
8 Year	4.14%	4.37%	4.37%
9 Year	4.11%	4.32%	4.32%
10 Year	4.21%	4.41%	4.41%
15 Year	4.52%	4.67%	4.67%
20 Year	4.60%	4.76%	4.76%
30 Year	4.62%	4.79%	4.79%

Notes: This Table reports the market yields on UK treasuries and bonds as of 15 November 2023. The market information was obtained from the Financial Times (FT) bonds market data which is available at https://markets.ft.com/data/bonds.

It is interesting to note that interest rates were raised recently following the covid-19 pandemic exogenous shock and the geopolitical turmoil. The revised NNEG valuations for an off-the run ERM may use much higher risk-free rates, very close and even higher than the roll-up rate that was fixed when the loan was issued and that it stays the same during the life of the loan. For example, if the risk-free yield curve was flat at 1% and the roll-up rate was 5% when an ERM is issued, later on the risk free rate may be still flat but increased at 5% or 6% but the roll-up rate comes as an input into revised calculations at the same 5%. This shows a design problem with the ERM loans in the UK market.

We will perform additional sensitivity tests around this benchmark risk-free rate of interest in order to further explore our valuation and look into how it relates to the NNEG and ERM values.

Figure 6.11 shows the yield curve with linear interpolation between market interest rates and also all the interest rates that had to be extrapolated all the way to 45 years maturity. Different methods of interpolation, such as log-linear or splines, could give different extrapolated points. A robust risk management analysis should take these considerations into account. Considering these rates and bearing in mind a funding rate or risk-free rate lying anywhere between 0.5% to 4.5%, one can see how the insurer/lender risk premium can vary quite dramatically with changes of one rate or the other.

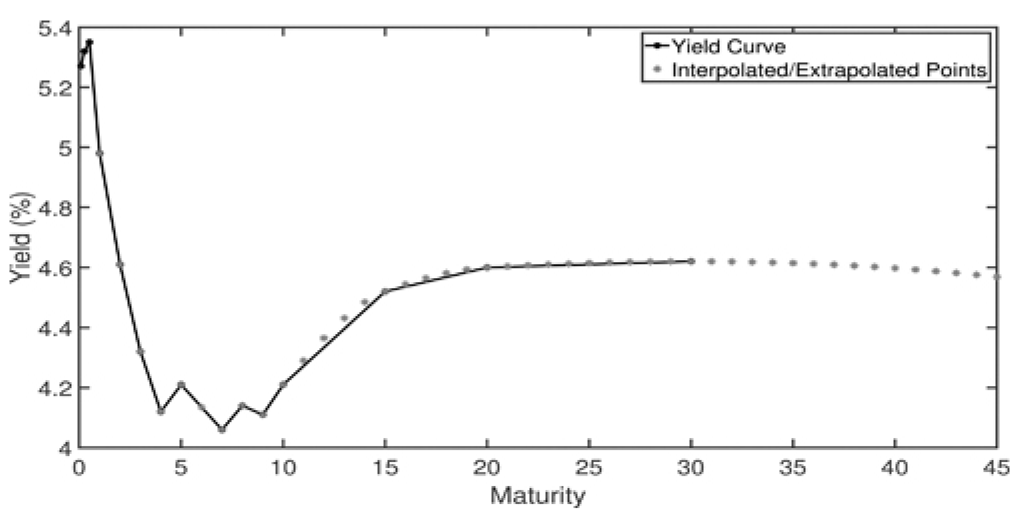


Figure 6.11 Yield curve interpolation/extrapolation for ERP valuation

Notes: This Figure depicts the interpolation and extrapolation of risk-free rate of interests that match the required maturities associated with the ERP and NNEG valuation. The interpolation and extrapolation exercise is based on the yield to maturity presented in Table 6.9

The negative equity risk or NNEG value could be inflated easily if the roll-up rates are set-up larger than "what they should be". The question is "what should they be" and equally important "who should decide" on that. At a first glance at least, it seems that the regulators can play a more proactive role in answering these questions and make sure that appropriate levels of roll-up rates are offered in the market to preserve stability.

Setting the benchmark roll-up interest rate that will be used for the loan provided under the terms of the ERM contract is also necessary. This flat rate was calculated using the UK Equity Release Council's (ERC) Spring 2022 report's available market data. The average roll-up interest rate for ERM contracts from 2016 to 2022 is shown as a time series in Table 6.10. There is significant fluctuation across the years but also within the years. The spread between the roll-up rate and the benchmark risk-free rate is much lower in 2023 than it was before in 2016 or 2017.

TABLE 6.10 Average equity release mortgage market roll-up interest rates

Panel A: Equity Release Council, Spring 2022 market report	
Date	Avg roll-up rates
Jan-16	6.20%

Notes: This Table reports the time series of annual average roll-up interest rate for the UK ERM market space for the month of January, from 2016 to 2022 in Panel A. The time series is from the equity release council (ERC) Spring 2022 report. In the Spring 2023 ERC report, we extracted the data from the time series plot of annual average roll-up interest rates supplied by Moneyfacts Group Plc. We present this data in Panel B. The data in panel be is from August 2022 to April 2023.

Panel A: Equity Release Council, Spring 2022 market report	
Date	Avg roll-up rates
Jan-17	5.45%
Jan-18	5.14%
Jan-19	5.21%
Jan-20	4.48%
Jan-21	3.97%
Jan-22	4.16%
Panel B: Equity Release Council, Spring 2023 market report	
Date	Avg roll-up rates
Aug-22	4.81%
Sep-22	5.03%
Oct-22	5.62%
Oct-22	6.83%
Nov-22	6.80%
Dec-22	6.29%
Jan-23	6.15%
Feb-23	6.04%
Mar-23	5.94%
Apr-23	5.86%

Notes: This Table reports the time series of annual average roll-up interest rate for the UK ERM market space for the month of January, from 2016 to 2022 in Panel A. The time series is from the equity release council (ERC) Spring 2022 report. In the Spring 2023 ERC report, we extracted the data from the time series plot of annual average roll-up interest rates supplied by Moneyfacts Group Plc. We present this data in Panel B. The data in panel be is from August 2022 to April 2023.

Figure 6.12 illustrates the difference in market LTVs between a standard borrower and a borrower that benefits from medical improvements or operations that are known to improve longevity. The difference is almost constant between 55 and 85-year-olds but it switches in sign and increases in size after the age of 85. This is the age bracket where most loan termination events are expected to happen.

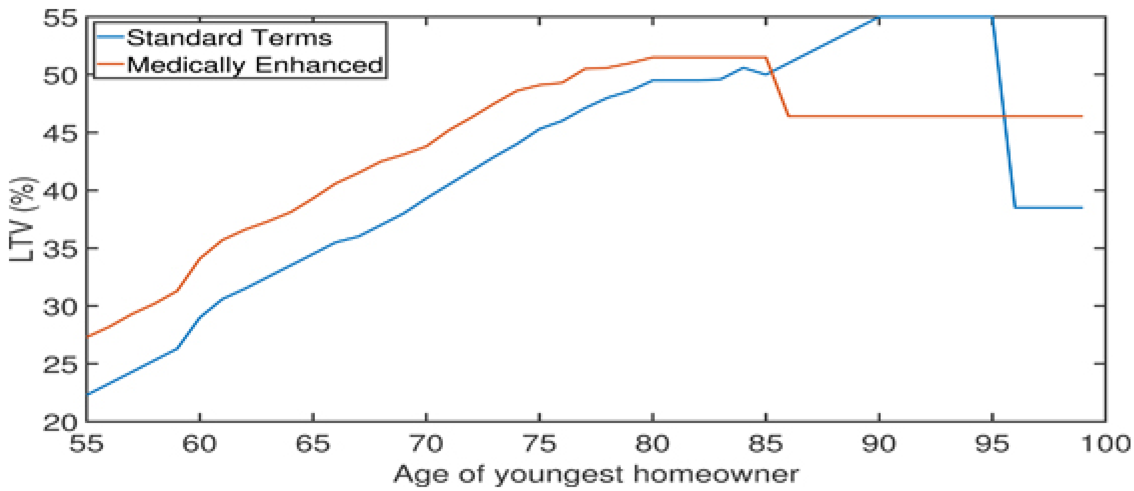
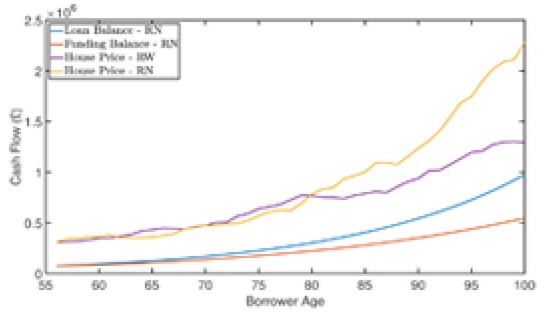
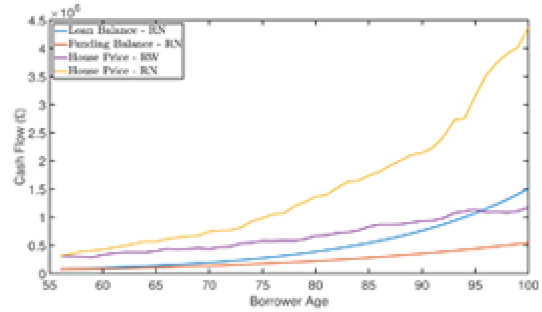


Figure 6.12 This figure presents a plot of the maximum loan-to-value (LTV) associated with a lifetime equity release mortgage loan on the market as of 17 November 2023. The age is based on that of the youngest homeowner. The data is from https://www.moneyrelease.co.uk/Equity-Release/Max-Release-Calculator/

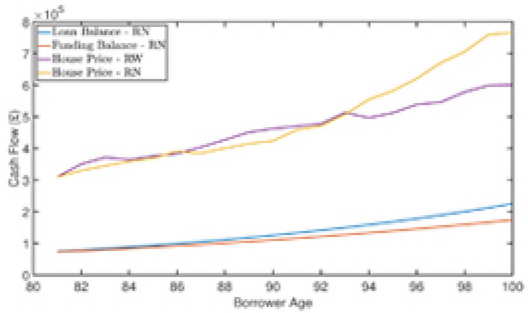
For real-world pricing practitioners use a risk-premium that is usually determined exogenously. The graphs in Figure 6.13 illustrate the relative evolution of various cash-flows defining an ERM for the GBM model we fitted to the Nationwide data. The house price pathways are described for the fifth Monte Carlo simulated path, under the real-world measure (where the drift and volatility parameters change monthly) and also under risk-neutral measure (where the volatility changes monthly) while the funding balance and loan balance evolution are model independent and change only with respect to the driving interest rate. In the scenarios illustrated in Figure 6.13 there is a risk for an NNEG to be in the money.



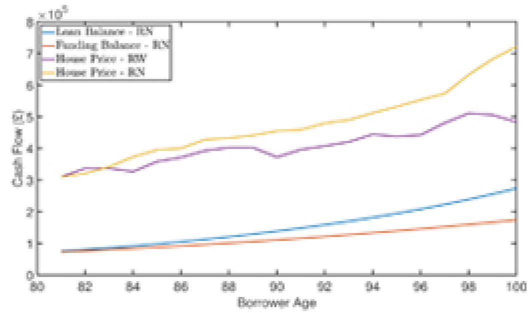
(a) Age 55 and $R = 5.86\%$



(b) Age 55 and $R = 6.83\%$



(c) Age 80 and $R = 5.86\%$



(d) Age 80 and $R = 6.83\%$

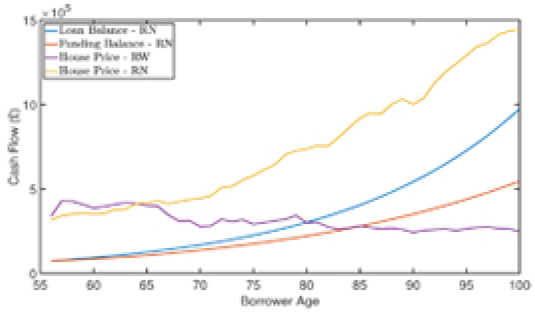
Figure 6.13 Cash-flow paths for funding balance, loan balance, and a single GBM Monte Carlo simulated pathway for monthly house prices risk-neutral and real-world, under baseline scenario where rental yield $g = 1\%$, house price volatility under GBM with maximum-likelihood simulation is $\sigma = 3.83\%$, and the corresponding drift parameter $\mu = 4.89\%$. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9. The roll-up interest is taken from the ERC Spring 2023 report. We set this equal to 5.86%, the value reported for April 2023 in sub-plot (a) and (c) and 6.83%, October 2022. The loan-to-value ratio is 22.50%

The graphs in Figure 6.13 illustrate comparatively the evolution of loan balance, funding balance and house prices from one simulated pathway under the GBM physical measure and also under a risk-neutral GBM measure, for borrowers aged 60 and 70 and also different roll up rates. In these singular simulations the risk-neutral house prices are always above the physical (also called historical) house prices. Since the volatility parameter for GBM would be the same under both physical and risk-neutral measure, the faster growth of real-world house prices is due to a larger drift, or in other words the risk-neutral rate that was applied for the risk-neutral simulation is less than the drift parameter μ applied for the real-world or physical GBM measure.

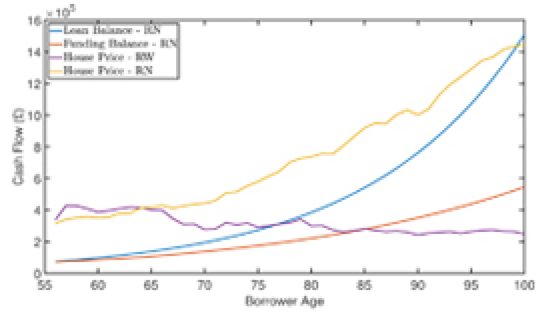
One should also note that a larger roll-up rate would imply a crossover point between the loan balance and the house price under the risk-neutral house prices pathway but there is no cross-over to the pathway of the real-world pathway. This points out to an important issue. If the regulator insists that risk management calculus for ERP to be done under a risk-neutral measure then, as this example illustrates, the investors or lenders/insurers could mark an occurrence of negative equity risk. By contrast, if all calculations were accepted to be done under the real-world measure, in this example we would not see a cross-over to the house price path simulated under the real-world measure.

Another remark here is that the difference between the funding rate and the loan balance increases exponentially with time. This is not quite an exact reflection on how things are done on the ground. While the roll-up rate is fixed and therefore the projected cash-flow is deterministic, the funding curve could be affected if funding is done in a revolving manner. For example, the lenders/insurers may get their funding on an yearly basis to issue new loans and pay back old loans. The funding gap may be small or negligible for many years but it may change suddenly based on new market conditions.

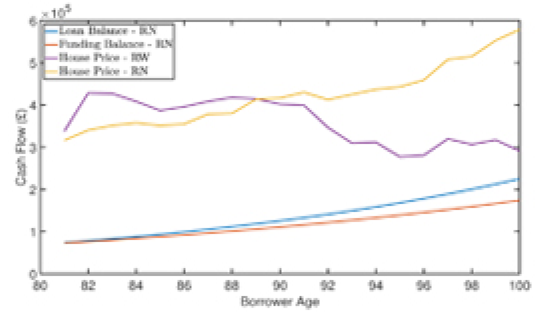
One important issue with the simulations in Figure 6.13 is the strong assumptions that house prices are following a geometric Brownian motion. We advocate strongly in our previous project presented to IFoA, see Tunaru and Quaye (2019), and this book that the GBM is not appropriate for modelling the dynamics of house prices. Furthermore, we have identified a family of models that fit the data much better. Hence, in Figure 6.14 we have redone the same exercise but with house prices following an ARMA-EGARCH process. The model parameters are estimated from monthly Nationwide house price time series spanning from January 1991 to December 2022. We split this historical sample into an in-sample data between January 1991 and December 2018 and an out-of-sample between January 2019 and December 2022. For explanatory purposes we simulate one Monte Carlo house price return pathway. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9. The roll-up interest is taken from the ERC Spring 2023 report and we take this as being equal to 5.86%, the value reported for April 2023 in sub-plot (a) and (c) and 6.83%, October 2022. The loan-to-value ratio is 22.50%, which is typical for loans in this market.



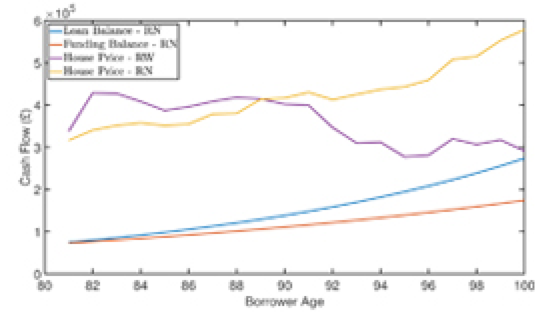
(a) Age 55 and $R = 5.86\%$



(b) Age 55 and $R = 6.83\%$



(c) Age 80 and $R = 5.86\%$

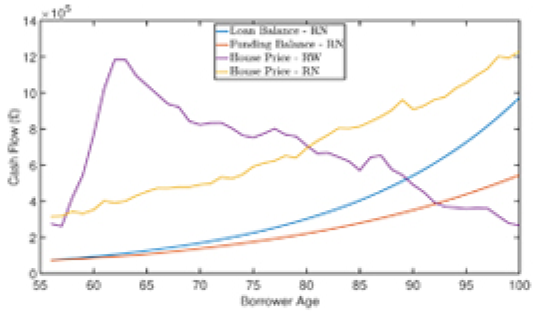


(d) Age 80 and $R = 6.83\%$

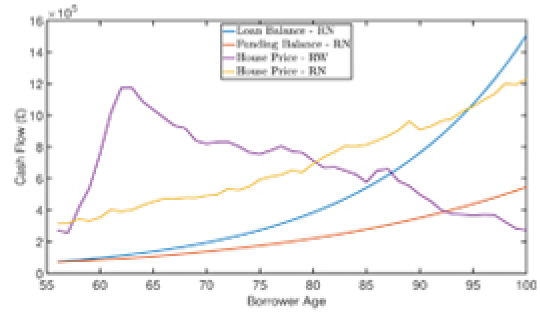
Figure 6.14 Cash-flow paths for funding balance, loan balance, and a single ARMA(4,4)-EGARCH(1,1) Monte Carlo simulated pathway for monthly house prices under the objective measure, with baseline scenario where rental yield $g = 1\%$. The ARMA(4,4)-EGARCH(1,1) parameters are estimated from monthly Nationwide house price time series spanning from January 1991 to December 2022. The in-sample data is from January 1991 to December 2018. We simulate one Monte Carlo house price return pathway in this exercise. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9. The roll-up interest is taken from the ERC Spring 2023 report. We set this equal to 5.86%, the value reported for April 2023 in sub-plot (a) and (c) and 6.83%, October 2022. The loan-to-value ratio is 22.50%

Importantly, the relationship between simulated house prices under the real-world measure and the risk-neutral measure is starting the same as with the GBM but very quickly it swaps order. In addition, we can observe the cross-over point of the real-world house prices much sooner and there is a secondary cross-over point with the funding curve.

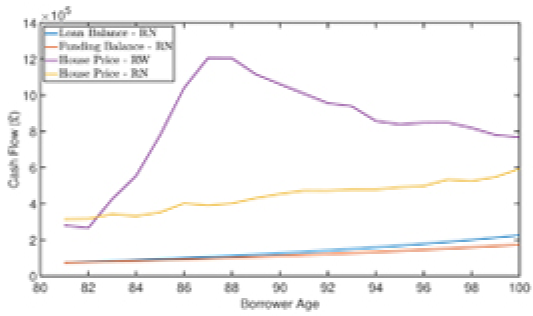
It would be interesting to see what would be the effect of using a model from the same ARMA-EGARCH family but with parameters estimated from a different data set. The example in Figure 6.15 show one pathway simulation based on the results of the ARMA-EGARCH model selected from the quarterly data. The evolution of the house price under the real-world measure is quite different from the example illustrated previously in Figure 6.14. This highlights that there could be substantial model risk present and the regulators, lenders, investors and so on could benefit from running multiple analyses to gauge the potential negative equity risk. It also emphasizes the important problem that may arise if real-world calculations are more relevant than the risk-neutral or market consistent calculations. The difference between risk-neutral house prices and real-world house prices in Figure 6.15 is getting very rapidly very large with time on the loan (marked on the figure as borrower's age), with the real world being a lot higher, then half way through it flips with risk-neutral prices becoming much larger than real-world house prices.



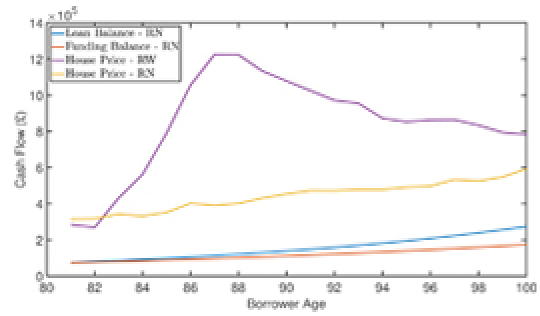
(a) Age 55 and $R = 5.86\%$



(b) Age 55 and $R = 6.83\%$



(c) Age 80 and $R = 5.83\%$



(d) Age 80 and $R = 6.83\%$

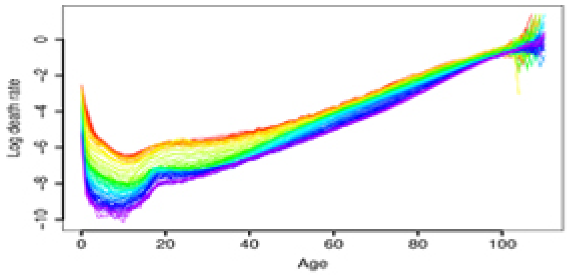
Figure 6.15 Cash-flow paths for funding balance, loan balance, and a single ARMA(4,3)-EGARCH(1,1) Monte Carlo simulated pathway for quarterly house prices under the objective measure, with baseline scenario where rental yield $g = 1\%$. The ARMA(4,3)-EGARCH(1,1) parameters are estimated from monthly Nationwide house price time series spanning from 1952Q4 to 2022Q4. The in-sample data is from 1952Q4 to 2018Q4. We simulate one Monte Carlo house price return pathway in this exercise. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9. The roll-up interest is taken from the ERC Spring 2023 report. We set this equal to 5.86%, the value reported for April 2023 in sub-plot (a) and (c) and 6.83%, October 2022. The loan-to-value ratio is 22.50%

6.4.2 Empirical Modelling for UK Mortality Data

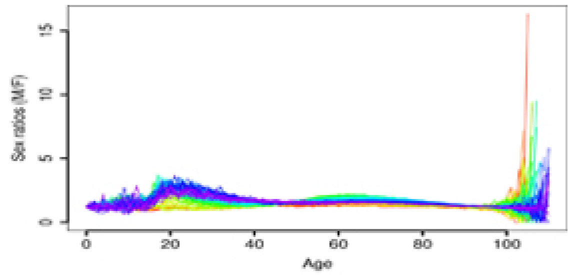
We collect a set of death rate time series data for the United Kingdom and do mortality projections and simulations for the NNEG valuation. The empirical exercise is based on the population data we collected from the human mortality database (HMD). The population mortality data is able to capture structural breaks and long-term changes that may have impact mortality of the UK population over the period of analysis. We also need to appreciate the fact that mortality projections based on general population mortality will bare some of the desirable features of the ERM product consumer. For example, users of ERM contracts may likely have an improved mortality compared to any ordinary unit of the population. Actuaries have ways special models to link population-based mortality projections to insured population projections. Pitacco (2009) presents some excellent discussions on how to implement such approaches. Specific to our stochastic mortality projection exercise, we select from the following age-period-cohort (APC) models: Lee and Carter (1992) (LC model), Cairns, Blake, and Dowd (2006) (CBD model), Renshaw and Haberman (2006) (RH model), Plat (2009) (PLAT model), and Cairns et al. (2009) (M7 model).

The empirical fitting and performance comparison are based data with ages 55 - 100 using calendar years 1950 to 2021. We have restricted analysis to ages above 55 in order to make projections that suit the age profile of a potential ERM product user. According to Pitacco (2009), the pace of mortality decline is even across all ages post 1950. In addition to the high quality of the HMD mortality rate data, this creates a desirable situation for our mortality projection exercise. We plot and compare the estimated model parameters for mortality projections for males, females, and the total population in the specified age bracket. In-sample fitting is from 1950 to 2021.

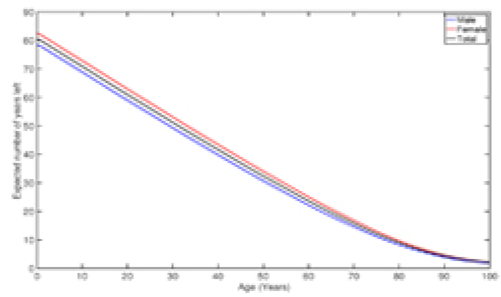
We present exploratory analysis of the goodness of fit for each model. Figure 6.17 presents residual plots by calendar year and age for each of the models. A reasonably good-fit should technically possess some randomness in the residual plots. On this basis, concentrations of residuals is not desirable.



(a) UK Death Rates 1922: 2021



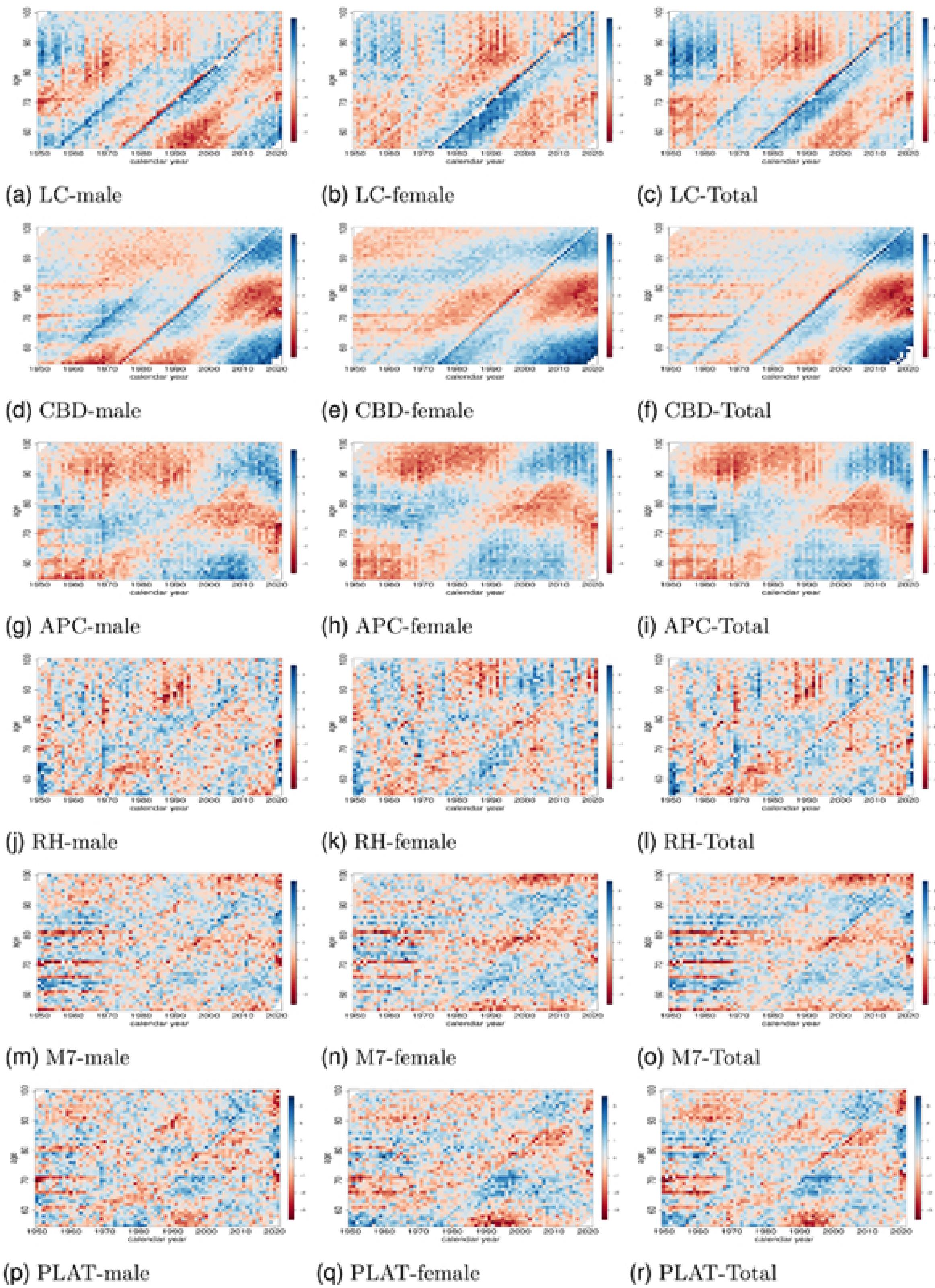
(b) Sex ratio (Male/Female) 1922: 2021



(c) Life Expectancy 2021

Figure 6.16 UK Total death rates, sex ratio, and life expectancy in 2021

Notes: Subplot (a) illustrates the combined male and female mortality rates for the United Kingdom use data from the human mortality database between 1922 and 2021. Each line in subplot a represents the cohort from a given year, with the recent years depicted in violet, the older values are shown in a red colour. The death rates are plotted on a log scale to accentuate visualisation for small changes that occur over the period of analysis. The age range is from 0 - 110 years. In (a), more negative values correspond to smaller death rates and vice versa. Besides the elderly, it is evident how mortality rate has decreased across all age groups. Subplot (b) reports the male-female mortality ratios for all age groups spanning the data from 1922 to 2021. Subplot (c) presents the 2021 life expectancy for male, female and the total, which combines male and female life expectancy.



(a) LC-male (b) LC-female (c) LC-Total
(d) CBD-male (e) CBD-female (f) CBD-Total
(g) APC-male (h) APC-female (i) APC-Total
(j) RH-male (k) RH-female (l) RH-Total
(m) M7-male (n) M7-female (o) M7-Total
(p) PLAT-male (q) PLAT-female (r) PLAT-Total

Figure 6.17 Goodness of fit: Deviance Residual Plots

Notes: This figure illustrates the deviance residual plots for each fitted model using female UK death rate timeseries. The models are fitted to general population aged 55 years to 100 years, using HMD time series data from 1950 to 2021 for the United Kingdom. For each model, we report the deviance residuals when fitted for males, females, and total population.

6.5 EMPIRICAL PRICING EXERCISE

According to the Equity Release Council, the average fixed roll-up interest rate on UK equity release mortgage contracts in August 2022 was 5.74%. By October 2022, the roll-up rate had skyrocketed to an average of 7.55%. Nonetheless, borrowers were able to get an average rate of 3.71% throughout the first and second quarters of 2022. It is important to note that roll-up rates are determined by the borrower's personal circumstances. Another possible explanation for these sharp fluctuations in the fixed roll-up rate is asset repricing actions initiated by loan issuers. The abrupt move from a low to high rate environment may have spurred the asset repricing activity. Individual and institutional investors were both accustomed to the protracted low interest rate environment that existed before to the outbreak of covid-19. The low interest rate environment was caused by the Bank of England's quantitative easing measures, which were implemented to boost economic growth.

We can also specify a Mixed Data Sampling GARCH (GARCH-MIDAS) process to model and forecast house price volatility. This setup decomposes the conditional house price volatility into two parts i.e. a short-run component and a long-run component where the short-run volatility is driven by a general GARCH(1,1) and the long-run component is driven by some known exogenous factor(s). The usual suspects that drive the long-run volatility component include realised variance of house prices or macroeconomic factors. The setup of the GARCH-MIDAS allows the variables to enter the model at different frequencies while the impact of systematic risk on house price volatility cannot be overemphasised.

TABLE 6.11 Goodness of fit

	LC	CBD	APC	RH	M7	PLAT
			Panel A: Male			
LL	-27860.39	-35823.54	-26900.47	-19910.22	-19614.34	-19514.0
df_{LL}	162	144	226	272	324	296
AIC	56044.79	71935.07	54252.93	40364.44	39876.68	39620.1
BIC	57033.26	72813.71	55631.91	42024.1	41853.63	41426.2
			Panel B: Female			
LL	-28092.68	-43594.71	-34238.39	-20611.03	-20426.93	-20427.3
df_{LL}	162	144	226	272	324	296
AIC	56509.37	87477.07	68928.77	41766.06	41501.86	41446.6
BIC	57497.84	88356.07	70307.75	43425.75	43478.81	43252.7
			Panel C: Total			
LL	-41415.03	-49330.13	-40511.83	-22848.61	-23182.24	-22774.6
df_{LL}	162	144	226	272	324	296
AIC	83154.05	98948.26	81475.66	46241.22	47012.47	46141.3
BIC	84142.52	99826.90	82854.64	47900.88	48989.41	47947.4

Notes: This table presents the log-likelihood (LL), AIC, BIC for the respective models. We also repo the degrees of freedom for the log-likelihood (df_{LL}).

The empirical exercise that follow from here constitute volatility model calibration in the real- or physical-world. The risk-neutral measure is required to formally calculate the NNEG cost and for that matter the value of the ERM contract.

6.5.1 Empirical Pricing Exercise of NNEGs

Here we compare the NNEG pricing between the GBM models and the ARMA-EGARCH models. Referring to the ERC Spring 2023 market report, the market average interest rate for January 2023, which is 6.150%, is taken as roll-up interest rate R in the baseline scenario. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9.. The standard Flexible Loan-to-Value (LTV) ratio are allocated to the corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-year increments. These valuations are 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. With the ERM contract, we employ a multiple decrement termination condition that includes voluntary termination, transition to long-term care, and death of borrower. We simulate 100,000 house price paths in each Monte Carlo exercise.

In the GBM model, the house price volatility σ is calculated by maximum likelihood estimation using quarterly house price time series from 1952Q4 to 2022Q4 obtained from the Nationwide house price database, it equals 4.88%. The in-sample data set for both ARMA(4,3)-EGARCH(1,1) and GBM parameter estimates spans from 1952Q4 to 2018Q4. Using the risk-neutral GBM and ARMA(4,3)-EGARCH(1,1) house price volatility estimation techniques, the ratio of the NNEG value to initial loan is reported in Table 6.12.

TABLE 6.12 Risk-neutral valuation of ERPs with quarterly house price tin series

Age	LTV	Initial Loan	NNEG			ERP Cost		
			Male	Female	Joint Life	Male	Female	Joi Lif
Baseline: Risk-neutral Valuation under GBM with $R = 6.150\%$								
60	17.00%	52,700	0.88%	1.28%	1.59%	268,310	277,750	334,2
65	22.50%	69,750	1.22%	1.75%	2.15%	317,110	326,880	391,9
70	28.50%	88,350	1.26%	1.79%	2.17%	360,420	369,870	441,5
75	32.40%	100,440	0.54%	0.78%	0.93%	370,460	378,420	448,9
80	36.50%	113,150	0.18%	0.26%	0.30%	378,900	384,700	451,3
85	41.50%	128,650	0.05%	0.07%	0.08%	391,020	394,840	449,8
90	41.50%	128,650	0.00%	0.00%	0.00%	351,220	352,240	370,8
Baseline: Risk-neutral Valuation under ARMA(4,3)-EGARCH(1,1) with $R = 6.150\%$								
60	17.00%	52,700	0.09%	0.14%	0.18%	268,730	278,350	334,9
65	22.50%	69,750	0.19%	0.29%	0.36%	317,830	327,890	393,1
70	28.50%	88,350	0.24%	0.37%	0.45%	361,310	371,120	443,0
75	32.40%	100,440	0.08%	0.12%	0.14%	370,930	379,080	449,7
80	36.50%	113,150	0.02%	0.02%	0.03%	379,090	384,970	451,7
85	41.50%	128,650	0.00%	0.00%	0.00%	391,080	394,920	449,9
90	41.50%	128,650	0.00%	0.00%	0.00%	351,220	352,250	370,8

Age	LTV	Initial Loan	NNEG			ERP Cost		
			Male	Female	Joint Life	Male	Female	Joir Lif
Baseline: Risk-neutral Valuation under GBM with $R = 6.150\%$								

Notes: Using the risk-neutral GBM and ARMA(4,3)-EGARCH(1,1) house price volatility estima techniques, the ratio of the NNEG value to initial loan is plotted in this figure. With the ERM contr we employ a multiple decrement termination condition that includes voluntary termination, transitio long-term care, and death of borrower. Referring to the ERC Spring 2023 market report, the ma average interest rate for January 2023, which is 6.150%, is taken as roll-up interest rate R in the base scenario. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9 the GBM model, the house price volatility σ is calculated by maximum likelihood estimation us quarterly house price time series from 1952Q4 to 2022Q4 obtained from the Nationwide house p database. The in-sample data set for both ARMA-EGARCH and GBM parameter estimates spans fr 1952Q4 to 2018Q4. The standard Flexible Loan-to-Value (LTV) ratio are allocated to the correspond borrower ages, beginning at age 60 and ending at age 90 in 5-year increments. These valuations 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simulate 100,000 house price path each Monte Carlo exercise.

The NNEg valuations presented in Figure 6.12 indicate that the NNEG calculations under the ARMA-EGARCH models are a lot less than the corresponding calculations under the GBM model. If we recall that the ARMA-EGARCH model also fitted the house prices data much better than the GBM model, this conclusion should at least invite for reflection. Using a simpler model (GBM in this case) may lead to inflating the NNEG values. The discrepancies are substantial for male, female and joint borrowers. This is also no surprise. The GBM model *by design* will increase the house price volatility with time whereas the ARMA-EGARCH model exhibits a regression-to-the mean effect for volatilities that a) is more relevant to the actual dynamics of house prices and b) allows the model to produce NNEG values that do not overpenalize through the NNEG lenses.

TABLE 6.13 Risk-neutral valuation of ERPs with monthly house price tim series

Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Join Lif
Panel A: Risk-neutral Valuation under GBM with $R = 6.150\%$								
60	17.00%	52,700	0.26%	0.39%	0.49%	268,640	278,220	334,7
65	22.50%	69,750	0.45%	0.67%	0.83%	317,650	327,630	392,8
70	28.50%	88,350	0.52%	0.77%	0.93%	361,070	370,770	442,6
75	32.40%	100,440	0.19%	0.29%	0.34%	370,810	378,910	449,5
80	36.50%	113,150	0.05%	0.07%	0.09%	379,050	384,910	451,6
85	41.50%	128,650	0.01%	0.01%	0.02%	391,070	394,910	449,9
90	41.50%	128,650	0.00%	0.00%	0.00%	351,220	352,250	370,8
Panel B: Risk-neutral Valuation under ARMA(4,4)-EGARCH(1,1) with $R = 6.150\%$								
60	17.00%	52,700	0.12%	0.19%	0.23%	268,710	278,330	334,9
65	22.50%	69,750	0.24%	0.37%	0.45%	317,790	327,840	393,1
70	28.50%	88,350	0.30%	0.45%	0.55%	361,260	371,050	442,9
75	32.40%	100,440	0.10%	0.15%	0.18%	370,910	379,050	449,7
80	36.50%	113,150	0.02%	0.03%	0.04%	379,080	384,960	451,6
85	41.50%	128,650	0.00%	0.00%	0.01%	391,080	394,920	449,9
90	41.50%	128,650	0.00%	0.00%	0.00%	351,220	352,250	370,8

Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Joir Lif
Panel A: Risk-neutral Valuation under GBM with $R = 6.150\%$								

Notes: Using the risk-neutral GBM and ARMA-EGARCH house price volatility estimation techniq the ratio of the NNEG value to initial loan is plotted in this figure. With the ERM contract, we empl multiple decrement termination condition that includes voluntary termination, transition to long-t care, and death of borrower. Referring to the ERC Spring 2023 market report, the market ave interest rate for January 2023, which is 6.150%, is taken as roll-up interest rate R in the base scenario. The risk-free interest rate is interpolated from the yield-to-maturity presented in Table 6.9 the GBM model, the house price volatility σ is calculated by maximum likelihood estimation u monthly house price time series from January 1991 to December 2022 obtained from the Nationw house price database. The in-sample data set for both ARMA-EGARCH and GBM parameter estim spans from January 1991 to December 2018. The standard Flexible Loan-to-Value (LTV) ratio allocated to the corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-y increments. These valuations are 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simu 100,000 house price paths in each Monte Carlo exercise.

We repeat the NNEG calculations but using the models with parameters estimated from monthly house price data. Recall that with monthly versus quarterly, we get more granular frequency but shorter available period for the data. The plot in Figure 6.13 depicts a similar story, the ARMA-EGARCH model NNEG calculations are below the corresponding NNEG calculations done under GBM. This is true for all ages, although for very elderly borrowers such as 85 and more, there is virtually no difference between the NNEG values under the two models and between male, female and joint borrowers. One should also note that under both sets of data, monthly and quarterly, the largest NNEG values are obtained for the 70 years old borrowers. However, this is only circumstantial, based on the combination of parameters inputs. Repeating the calculations with different inputs the results in Figure 6.14 show a very different picture. The highest NNEG values are for age 60 now and the models lose differentiation from the age of 80.

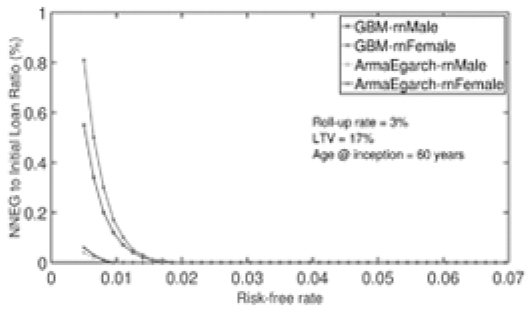
TABLE 6.14 NNEG to initial loan ratio using quarterly house price time se with baseline roll-up interest rate $R = 6.150\%$ increased by 100 basis point

Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Jo L
Panel A: Risk-neutral Valuation under GBM with $R = 7.150\%$								
60	17.00%	52700	9.75%	13.67%	16.94%	333,900	349,200	421
65	22.50%	69750	9.63%	13.38%	16.42%	378,130	393,150	472
70	28.50%	88350	7.73%	10.71%	12.96%	413,970	428,060	511
75	32.40%	100440	3.26%	4.57%	5.46%	413,410	425,510	505
80	36.50%	113150	1.09%	1.53%	1.80%	411,430	420,380	498
85	41.50%	128650	0.30%	0.41%	0.48%	414,450	420,380	478
90	41.50%	128650	0.01%	0.01%	0.01%	365,400	367,410	387
Panel B: Risk-neutral Valuation under ARMA(4,3)-EGARCH(1,1) with $R = 7.150\%$								
60	17.00%	52,700	4.09%	6.10%	7.57%	336,880	353,200	426
65	22.50%	69,750	4.67%	6.85%	8.42%	381,590	397,710	478
70	28.50%	883,50	3.95%	5.76%	6.99%	417,310	432,430	517
75	32.40%	100,440	1.35%	2.00%	2.39%	415,330	428,100	508
80	36.50%	113,150	0.31%	0.46%	0.55%	412,310	421,580	494
85	41.50%	128,650	0.05%	0.07%	0.08%	414,780	420,820	479
90	41.50%	128,650	0.00%	0.00%	0.00%	365,410	367,410	387

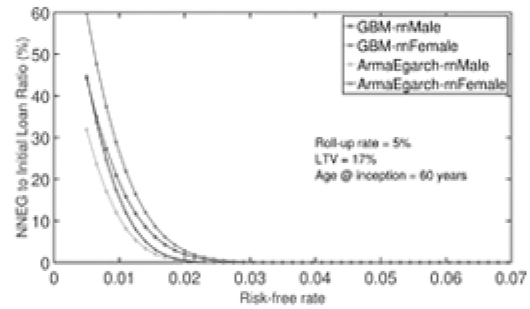
Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Jo L
Panel A: Risk-neutral Valuation under GBM with $R = 7.150\%$								

Notes: Using the risk-neutral GBM and ARMA-EGARCH house price volatility estimation techniqu ratio of the NNEG value to initial loan is plotted in this figure. With the ERM contract, we em multiple decrement termination condition that includes voluntary termination, transition to long-term and death of borrower. The baseline roll-up interest rate $R = 6.150\%$, is bumped up by 100 basis po $R = 7.150$ in this scenario. The risk-free interest rate is interpolated from the yield-to-maturity pre in Table 6.9. In the GBM model, the house price volatility σ is calculated by maximum likel estimation using quarterly house price time series from 1952Q4 to 2022Q4 obtained from the Natio house price database. The in-sample data set for both ARMA-EGARCH and GBM parameter esti spans from 1952Q4 to 2018Q4. The standard Flexible Loan-to-Value (LTV) ratio are allocated corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-year increments. valuations are 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simulate 100,000 house paths in each Monte Carlo exercise.

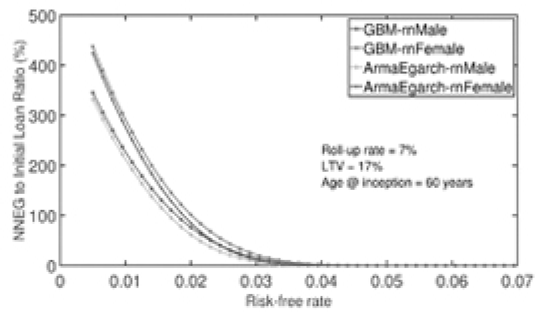
In Figures 6.18 we compare the sensitivity of NNEG to initial loan ratio for various risk-free (r) and roll-up (R) interest rates used in the Black-Sholes pricing model and the ARMA-EGARCH model, respectively. One can notice that for borrowers aged 60 the ratios are not much different between the two models at the extreme combination of interest rates when R is high and r is low. However, for both models it is very clear that the NNEG to loan value ratio is quite flat until the *difference* between the roll-up rate R and risk-free rate r is more substantial. The risk-free rate can be influenced only partially by the policy makers but the roll-up rates could be subjected to more stringent regulation. This highlights that a better regulatory tool for managing NNEG risk would be R. To be more precise, it is not R itself that should be subjected to some sort of capping but the difference $R - r$. It is very clear that this difference drives substantially the premium charged for issuing ERP loans. It is the “overcharging” that leads to NNEG problems more so than the volatility of the house prices.



(a) $R = 3\%$



(b) $R = 5\%$



(c) $R = 7\%$

Figure 6.18 Comparing NNEG cost to initial loan ratio at various risk-free interest rate

Notes: The plot depicts the range of NNEG to initial loan ratio in percentages, for various risk-free (r) and fixed roll-up (R) interest rates used in the Black-Sholes and ARMA(4,3)-EGARCH(1,1) pricing models. The loan-to-value (LTV) ratio at inception is set to 17% in all cases. It is the least value in the Flexible LTV grid. The scenario considers how the ratio evolves in relation to r and R in the context of male and female borrowers who are aged 60 years at the inception of the loan contract. The initial house price is £310000.

TABLE 6.15 NNEG to initial loan ratio using monthly house price time se with baseline roll-up interest rate $R = 6.150\%$ increased by 100 basis point

Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Jo Li
Panel A: Risk-neutral Valuation under GBM with $R = 7.15\%$								
60	17.00%	52,700	5.97%	8.67%	10.75%	333,900	349,200	421,
65	22.50%	69,750	6.38%	9.15%	11.23%	378,130	393,150	472,
70	28.50%	88,350	5.28%	7.52%	9.12%	413,970	428,060	511,
75	32.40%	100,440	2.00%	2.88%	3.45%	413,410	425,510	505,
80	36.50%	113,150	0.56%	0.80%	0.95%	411,430	420,380	493,
85	41.50%	128,650	0.11%	0.16%	0.19%	414,450	420,380	478,
90	41.50%	128,650	0.00%	0.00%	0.00%	365,400	367,410	387,
Panel B: Risk-neutral Valuation under ARMA(4,4)-EGARCH(1,1) with $R = 7.15\%$								
60	17.00%	52,700	4.51%	6.68%	8.29%	336,880	353,200	426,
65	22.50%	69,750	5.05%	7.38%	9.06%	381,590	397,710	478,
70	28.50%	88,350	4.24%	6.14%	7.45%	417,310	432,430	517,
75	32.40%	100,440	1.48%	2.18%	2.61%	415,330	428,100	508,
80	36.50%	113,150	0.36%	0.53%	0.62%	412,310	421,580	494,
85	41.50%	128,650	0.06%	0.08%	0.10%	414,780	420,820	479,
90	41.50%	128,650	0.00%	0.00%	0.00%	365,410	367,410	387,

Age	LTV	Initial Loan	NNEG to Initial Loan Ratio			ERM Cost		
			Male	Female	Joint Life	Male	Female	Jo Li
Panel A: Risk-neutral Valuation under GBM with $R = 7.15\%$								

Notes: Using the risk-neutral GBM and ARMA-EGARCH house price volatility estimation techni the ratio of the NNEG value to initial loan is plotted in this figure. With the ERM contract, we emp multiple decrement termination condition that includes voluntary termination, transition to long-term and death of borrower. The baseline roll-up interest rate $R = 6.150\%$, is bumped up by 100 basis p to $R = 7.150$ in this scenario. The risk-free interest rate is interpolated from the yield-to-ma presented in Table 6.9. In the GBM model, the house price volatility σ is calculated by maxi likelihood estimation using monthly house price time series from January 1991 to December obtained from the Nationwide house price database. The in-sample data set for both ARMA-EGA and GBM parameter estimates spans from January 1991 to December 2018. The standard Flexible I to-Value (LTV) ratio are allocated to the corresponding borrower ages, beginning at age 60 and endi age 90 in 5-year increments. These valuations are 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150 0.4150. We simulate 100,000 house price paths in each Monte Carlo exercise.

6.5.1.1 De-smoothing approach

One approach to deal with serial-correlation in house prices that is apparently being used by life actuaries working on annuities is to use a desmoothing procedure and get the modelling that way. While we do not fully agree with the standard desmoothing procedure that is normally applied to commercial real estate valuations because the indices there are appraisal based, a potentially good line of modelling in the context of real estate derivatives is described in van Bragt, Francke, Singor, and Pelsser (2015), see also an earlier report van Bragt, Francke, Kramer, and Pelsser (2009) or Tunaru and Quaye (2019). They consider the observed real estate price index as the convex combination of an "efficient market" price or true market price $y(t)$ and the previously observed market price $a(t-1)$

$$a(t) = \kappa y(t) + (1 - \kappa)a(t - 1)$$

with K a confidence parameter linking the two. This model is equivalent to an exponentially weighted moving average (EWMA) model that is well-known in financial risk management. To account properly for time value of money the model is adjusted using an expected annual return π

$$a(t) = \kappa y(t) + (1 - \kappa)(1 + \pi)a(t - 1)$$

van Bragt et al. (2015) assume that the underlying market returns follow a random walk process with drift. For a total return real estate index they prove that the price of a forward contract would then be equal to

$$F_t(T) = \frac{1}{df(t,T)}[y(t)(1 - \alpha_{\kappa,T}(t) + a(t)\alpha_{\kappa,T}(t)]$$

where $\alpha_{\kappa,T}(t) = (1 - \kappa)^{T-t}$.

Moreover, van Bragt et al. (2015) derive an approximative formula for the forward *and* a European put option contingent on a real estate index, using techniques developed for pricing Asian options and based on calculate the first moment M_1 and the second moment M_2 of $a(t)$, under the risk-neutral measure. Thus, the forward price formula is

$$F_t(T) = M_1; \;\; \sigma = \sqrt{\frac{1}{T-t}\ln\left(\frac{M_2}{M_1}\right)}$$

while the European put option formula for strike X is

$$p(t) = df(t,T)[\Phi(-d_2) - F_t(T)\Phi(-d_1)]$$

with $d_1 = \frac{\ln(F_t(T)/X)+0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$.

The model developed by van Bragt et al. (2015) can also be adapted to include seasonality effects and there are analytical formulae for pricing swaps on real-estate index as well.

6.6 THOMAS' APPROACH

6.6.1 Some Useful Background

Thomas (2021) proposed a different methodology focusing on the assumption that house prices cannot drop to zero so therefore there exist a natural barrier below which it is highly improbable that house prices can drop. The idea is that government will intervene if a large fall of house prices occurs. Here is the list of possible interventions that governments in the UK may do, as discussed in Thomas (2021):

- government could subsidize mortgage payments to homeowners who are in financial distress (e.g. like several new schemes introduced by the Ministry of Housing, Communities and Local Government in 2009: the Mortgage Rescue Scheme, the Repossession Prevention Fund, and the Homeowner Mortgage Support Scheme);
- government could facilitate better financing terms for purchases of houses (e.g. low interest rate environment by reducing the Bank of England base rate, cover risk for high loan to-value, high loan-to-

income, long terms, limited recourse, or interest-free equity loans as in Help to Buy);

- government could announce new tax breaks (e.g. stamp duty transaction tax cuts, deposit grants, reintroduction of mortgage interest relief for owner-occupiers);
- permit individuals early access to their pension funds to fund house purchase (as suggested by the Secretary of State for Housing and Communities in July 2019, and the Association of Consulting Actuaries in November 2019);
- it may provide guarantees of mortgage loans for lenders (e.g. the Help to Buy mortgage guarantee scheme ran from 2013 to 2017 and offered to guarantee up to £12bn of loans; under EU rules, these guarantees had to be charged for, but post Brexit this might not be necessary);
- government can introduce regulatory changes to help lenders to relax lending criteria, or to exercise forbearance on delinquent loans (e.g. the "Mortgage Preaction Protocol" promulgated by the Ministry of Justice in November 2008);
- the Bank of England/government may buy portfolios of mortgages from lenders, and then relax criteria or exercise forbearance as above;
- Bank of England/government could nationalise one or more mortgage lenders, and then relax criteria or exercise forbearance as above;
- the Government may use temporary guarantees on house prices (cf. the US Treasury's temporary guarantees on money market mutual funds announced on 19 September 2008);
- Bank of England/government could buy directly houses on the open market to sustain prices

There are other economical, political and purely practical reasons as of why the top authority of the state may "do something" to stop the rapid decline of house prices.

Another very important point made by Thomas is that "houses considered for equity release are usually freehold (or very long leasehold, which is nearly equivalent to freehold). A freehold house price notionally comprises two parts: the land value and the construction costs (indexed to current prices)."

The land value is quite important to recognise in this context because a major component of the house price is the associated land. If it is discovered that the land for an entire neighbourhood is actually contaminated with chemicals, or mines, or even geological problems akin to landslide or radon gas, it may take years for the local government to manage the situation and clean the land. This is a very important reason why in a series of works, Tunaru disagrees that *in all states of the economy* the value of the house now is strictly larger than the price now to own the house in the future.

Some estimates from the Office of National Statistics as of 2015 (see https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts) give 72% of the value of all dwellings in the UK is directly given by the corresponding land values. Knoll et al. (2017) also estimate that approximately 73% of the evolution of house prices in the UK between 1950 and 2012 was due to land prices, with the remaining 27% provided by the construction costs.

6.6.2 Risk-Neutral Valuation of NNEG Assuming a Reflecting Barrier

Thomas (2021) proposes to compute the risk-neutral indexrisk-neutral value of the NNEG under the assumption that there is a flat barrier at a threshold b such that the house price process never goes under. He then applies the formulae derived in a foreign exchange context by Hertrich (2015), Hertrich and Veestraeten (2013), Hertrich and Zimmermann (2017) to obtain an analytical formula for the European put option on house price representing the NNEG.

The value of the put option under the model including the reflecting barrier is calculated generically as

$$Put_B(K_T) = \int_0^\infty \max\left[K_T - H_T, 0\right] f(H_T) dH_T \tag{6.28}$$

$$Put_B(K_T) = \int_0^{K_T} (K_T - H_T) f(H_T) dH_T \tag{6.29}$$

$$Put_B(K_T) = \int_0^{b} (K_T - H_T) f(H_T) dH_T + \int_b^{K_T} (K_T - H_T) f(H_T) dH_T \tag{6.30}$$

$$Put_B(K_T) = \int_b^{K_T} (K_T - H_T) f(H_T) dH_T \tag{6.31}$$

where K_T is the exercise price equivalent to the accrued balance and $\{H_t\}_{t\geq 0}$ is the data drive process for house prices. Please note that this approach requires tacitly that $b < K_T$ and evidently also that $b < H_0$ so we also know that $b < min(K_T, H_0)$.

If $\{H_t\}_{t\geq 0}$ is a Brownian motion with a reflecting barrier at *b*, an analytical formula was stated by Thomas based on previous results in the literature. This is

$$\begin{aligned} Put_B(K_T) = K_T e^{-rT}\Phi(-d_1 + \sigma\sqrt{T}) - H_0 e^{-qT}\Phi(-d_1) \\ -be^{-rT}\Phi(-d_3 + \sigma\sqrt{T}) + H_0 e^{-qT}\Phi(-d_3) \\ +\frac{1}{\theta}\Big\{ be^{-rT}\Phi(-d_3 + \sigma\sqrt{T}) - H_0 e^{-qT}(\frac{b}{H_0})^{1+\theta}[\Phi(d_4) \\ - \Phi(d_2)] - K_T e^{-rT}(\frac{K_T}{b})^{\theta-1}\Phi(d_2 - \theta\sigma\sqrt{T})\Big\} \end{aligned} \tag{6.32}$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{H_0}{K_T}\right) + \left(r - q + 0.5\sigma^2\right)T\right] \tag{6.33}$$

$$d_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{b^2}{K_T H_0}\right) + \left(r - q + 0.5\sigma^2\right)T\right] \tag{6.34}$$

$$d_3 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{H_0}{b}\right) + (r - q + 0.5\sigma^2)T\right] \tag{6.35}$$

$$d_4 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{b}{H_0}\right) + (r - q + 0.5\sigma^2)T\right] \tag{6.36}$$

$$\theta = 2\frac{(r-q)}{\sigma^2} \tag{6.37}$$

Here H_0 is the current value of the house price, T is the time to maturity of the option, q is the deferment rate, σ is the volatility of the diffusion process that is combined with the reflective barrier and Φ is the cumulative normal distribution. in addition, r is the risk free rate that is applied for discounting for the risk-neutral valuation. Thomas makes the point that "the PRA prescribes r as *the published Solvency II basis risk-free rate for maturity T, adjusted for use on a continuously-compounded basis* (see SS 3/17, para 3.20).

The exact level of the barrier depends on the assumptions made by the regulator or the lender, or both. Thomas (2021) makes a compelling case for using a value of $b = 0.5$. This is not a conservative value in our opinion, the house prices in Germany in the aftermath of the collapse of Berlin wall did drop by 50%. As of 2006 the house prices in Japan dropped by 54% since the bubble burst at the end of 1989. Hence, $b \approx 0.5$ is a value that has been seen before and even being crossed. Ultimately, the value of *b* will be determined exogenously.

Depending on the local regulatory body, the Thomas' approach could be employed for reserving. In the UK, the PRA's Effective Value Test insists on the application of the Black '76 formula to value NNEG, and therefore implicitly they do not account for the possibility of policy intervention.

6.7 MACROECONOMIC FUNDAMENTALS AND ERP VALUATION

The housing prices are intrinsically related to real macroeconomic and financial variables. In more recent times they are also related to monetary and credit variables. Goodhart and Hofmann (2008) advocate that because cycles are correlated and housing prices are driven by fundamentals, there should be some house price comovement across regions. The intuition is that news on housing prices for some regions may influence investors to revise their expectations about prices in other regions. Furthermore, Goodhart and Hofmann (2008) argue that there could be even international fundamentals that may drive housing prices.

It would be very useful to be able to price contingent claims on house prices using information on macro-finance variables. Chang, Wang, and Yang (2012) consider the relationship between house prices and key macroeconomic variables when pricing mortgage insurance. Alai et al. (2014) developed a VAR model to cover the effects of house price risk, interest rate risk, rental yield risk and the gross domestic product for the valuation of reverse mortgages in the US.

It is possible to use a Mixed Data Sampling GARCH (GARCH-MIDAS) process for modelling and forecasting the house price index by allowing its conditional volatility to have a multiplicative decomposition into a short-run component driven by a general GARCH(1,1) model, and a long run-component which depends on past information on the Realised Variance (RV) and/or macroeconomic factors, see Badescu, Quaye, and Tunaru (2022). The impact of systematic risk on the ERM valuation cannot be overemphasised. Per the second quarter 2022 equity release market statistics, the UK equity release council (ERC) showed the slowdown of 2019/20 market takeup - influenced by the uncertain post-Brexit economic outlook and the outbreak of the Covid-10 pandemic.

More specifically, the framework uses the GARCH-MIDAS process for modelling the house price returns by linking the house price volatility observed on a monthly/quarterly basis to the RV and macroeconomic variables that are sampled at a different frequency (e.g. monthly/quarterly/annually). GARCH-MIDAS allows directly to investigate macro-volatility links while avoiding two-step procedure as seen in (Schwert, 1989). More specifically, one could adopt a similar multiplicative component framework for the house price returns process as pioneered by (Engle et al., 2013) for stock returns modelling. This approach provides two advantages, first, the short and long-run house price volatility components are separated, and second we employ a filtered RV and/or a direct approach imputing macroeconomic time series to capture the economic sources of house price volatility. The macroeconomic variables explored are restricted to the gross domestic product (GDP) and industrial production (IP), but the framework can accommodate for the simultaneous inclusion of other factors. Under a general GARCH-MIDAS setting under the physical measure, we construct a pricing framework for the valuation of the NNEGs by employing an exponential affine pricing kernel, which is linear in the return process and the macroeconomic variables. One could derive the risk-neutral dynamics when the house price returns are modelled with a multiplicative two-component volatility model driven by Gaussian innovations, while the macroeconomic variable is governed by an AR(1) process recorded at a different frequency than the underlying.

Embedding the macro-finance information in the ERP valuation is the next frontier in terms of the complexity of the modelling approach. This section follows Badescu et al. (2022) where a lot more technical explanations are provided.

The valuation of ERPs are most of the time done in a discrete-time framework. To this end suppose that there is a time index $\mathscr{T} = \{t \mid t = 0, \dots, T\}$, where T refers to a certain time-frame, quantifying the number of periods such as quarters/years. For each $t \in \mathscr{T}$, we introduce another time index $\mathscr{I}^{(t)} = \{i \mid i = 0, \dots, I_t\}$ with $I_0 = 0$, where I_t represents the number of subperiods months/quarters in period t. Henceforth, we shall denote the cartesian product of these sets by $\mathscr{S} = \mathscr{I}^{(t)} \times \mathscr{T} = \{(i,t) \mid i \in \mathscr{I}^{(t)} \text{ and } t \in \mathscr{T}\}$. We assume that $(I_{t-1}, t-1) = (0, t)$ for any $t = 1, \dots, T$. The time index $\mathscr{T}$ can be conceptualised as a subset of $\mathscr{S}$ if we employ the notational convention that $t := (I_t, t)$ for any $t \in \mathscr{T}$.

We further assume that the house prices follow a dynamics given by a stochastic process $H = \{H_{i,t}\}_{(i,t)\in\mathscr{S}}$. The corresponding log-return (henceforth return) process is denoted by $Y = \{Y_{i,t}\}_{(i,t)\in\mathscr{S}}$, and therefore $Y_{i,t} = \log\left(\frac{H_{i,t}}{H_{i-1,t}}\right)$ for any $i = 1, \dots, I_t$ and $t = 1, \dots, T$. Furthermore, under the physical measure $\mathbb{P}$ the return process is described by:

$$Y_{i,t} = \mu_{i,t} + \sqrt{v_{i,t}}\epsilon_{i,t}^{(Y)} \tag{6.38}$$

where e $\epsilon_{i,t}^{(Y)}$ is a sequence of i.i.d. innovations following a standard Gaussian distribution. In addition, for any $(i,t) \in \mathscr{S}$, we let $\mathscr{F}_{i,t}^{(Y)} = \sigma\left(\epsilon_{j,s}^{(Y)}, j \leq i, s \leq t\right)$ be the filtration associated with our framework. The conditional mean return is denoted by $\mu_{i,t}$, which is typically chosen as a function of the conditional variance $v_{i,t}$ of $Y_{i,t}$. The standard GARCH literature assumes that $v_{i,t}$ is $\mathscr{F}_{i,t}^{(Y)}$-predictable and the relationship to the past squared returns is non-linear. Badescu et al. (2022) consider a GARCH(1,1) specification.

Under the physical measure $\mathbb{P}$ the GARCH-MIDAS dynamics for modelling the house price return process can be described as (more details for the constructive build of this process are detailed in Badescu et al. (2022)):

$$Y_{i,t} = r_{i,t} - q_{i,t} + \lambda\sqrt{v_{i,t}} - \frac{1}{2}v_{i,t} + \sqrt{v_{i,t}}\epsilon_{i,t}^{(Y)}, \tag{6.39}$$

$$v_{i,t} = g_{i,t}\tau_t, \tag{6.40}$$

$$g_{i,t} = \left(1 - \alpha \mathrm{E}^{\mathbb{P}}\left[\zeta\left(\epsilon_{i-1,t}^{(Y)}; \boldsymbol{\xi}\right)\right] - \beta\right) + \alpha g_{i-1,t}\zeta\left(\epsilon_{i-1,t}^{(Y)}; \boldsymbol{\xi}\right) + \beta g_{i-1,t}, \tag{6.41}$$

$$\tau_t = \exp\left(m + \theta^{(Y)}\sum_{k=1}^{K}\psi_k(\omega_1^{(Y)}, \omega_2^{(Y)})RV_{t-k} + \theta^{(X)}\sum_{k=1}^{K}\psi_k(\omega_1^{(X)}, \omega_2^{(X

(6.42)

Here ξ denotes more loosely all other parameters that may be involved in the model specification of the innovations ζ, see footnote 7 in Badescu et al. (2022).

For practical purposes, a simplified variant of this model given by

$$\mu_{i,t} = r_{i,t} - q_{i,t} + \lambda\sqrt{v_{i,t}} - \frac{1}{2}v_{i,t}.$$

(6.43)

where $r_{i,t}$ and $q_{i,t}$ are the risk-free interest rate and the rental yield for period (i,t), respectively. It should be noted that the parameter λ modulates the house price market risk. This important parameter is estimated from historical returns prior to any other fitting of the model. It is important to see that the returns conditional variance $v_{i,t}$ in (6.40) has a multiplicative decomposition into a *short-run component*, denoted by $g_{i,t}$ and a *long-run (secular) component*, denoted here by τ_t. The former is driven by the monthly/quarterly liquidity concerns and possibly other short-lived shocks whilst the latter is driven by the expected future cash flows (in the form of rents) and other macroeconomic variables that hold information about housing market volatility. Following the standard approach in GARCH-MIDAS literature, Badescu et al. (2022) consider that τ_t is observed at a lower frequency (e.g. quarterly/yearly) than $g_{i,t}$. Also, τ_t depends on macroeconomic variables such as GDP growth rate, IP growth rate, and so on and on the realised volatility. The RV for period t is:

$$RV_t = \sum_{i=1}^{I_t} Y_{i,t}^2,$$

(6.44)

The vector of macroeconomic variable process denoted by $X = \{X_t\}_{t \in \mathscr{T}}$ is assumed in Badescu et al. (2022) to follow the AR(1) model

$$X_t = \phi_1 + \phi_2 X_{t-1} + \sigma_x \epsilon_t^{(X)}$$

(6.45)

where $\epsilon_t^{(X)}$ are i.i.d. N(0,1).

One important aspect of this line of modelling is that it allows to capture the potential correlation between the house price returns and the macroeconomic variables. To this end it is assumed that $\epsilon_{i,t}^{(Y)}$ and $\epsilon_t^{(X)}$ follow a bivariate standard Gaussian distribution with correlation coefficient ρ. Badescu et al. (2022) prove that

$$\mathrm{Var}^{\mathbb{P}}\left[Y_{i,t} \mid \mathscr{G}_{i-1,t}\right] \quad = g_{i,t}\tau_t = v_{i,t},$$

(6.46)

$$\mathrm{Cov}^{\mathbb{P}}\left(Y_{i,t}, X_t | \mathscr{G}_{i-1,t}\right) \quad = \sqrt{g_{i,t}\tau_t}\sigma_x\rho = \sqrt{v_{i,t}}\sigma_x\rho,$$

(6.47)

where ρ is the correlation coefficient of the driving noise processes of Y and X.

The risk-neutralisation procedure is technically more involved and beyond the scope of this monograph.

Under this complex modelling framework the NNEG valuations are adapted to the GARCH-MIDAS setting. Suppose that the loan balance process $K = \{K_{i,t}\}_{(i,t)\in\mathscr{S}}$, with initial value given by:

$$K_{0,1} = H_{0,1} \times \Gamma, \tag{6.48}$$

where $H_{0,1}$ is the initial house price evolves according to the equations

$$K_{i,t} = K_{i-1,t} \exp\left(R_{i,t}^{(l)}\right), \tag{6.49}$$

where $R_{i,t}^{(l)}$ is the roll-up interest rate on the loan over the period $(i-1,t)$ to (i,t). For simplicity Badescu et al. (2022) assume that roll-up and risk-free rates are constant for every period $R_{i,t}^{(l)} = R^{(l)}$ and $r_{i,t} = r$.

If the loan maturity is (I_M, T_M), with $1 \leq T_M \leq T$ and $1 \leq I_M \leq I_{T_M}$, then the terminal NNEG payoff is given by:

$$F := F\left(H_{I_M,T_M}\right) = \max\left[L_{I_M,T_M} - H_{I_M,T_M}, 0\right] \tag{6.50}$$

where L_{I_M,T_M} denotes the accumulated balance of the loan and H_{I_M,T_M} the house price, at time point (I_M, T_M). It is then straightforward to compute the current or preset risk-neutral value of NNEG at time $(0,1)$, for a contract with time to maturity $\tau_M = \sum_{k=1}^{T_M - 1} I_k + I_M$ periods. This is $\Pi_{0,1}(\tau_M) = E^{\mathbb{Q}}[\exp(-r\tau_M) \cdot F]$.

The value of the ERP loan at inception is given by:

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}\left[\min\left\{L_{I_M,T_M}, H_{I_M,T_M}\right\}\right] \tag{6.51}$$

and this can be decomposed as follows

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}\left[\left(L_{I_M,T_M} - \max\left(L_{I_M,T_M} - H_{I_M,T_M}, 0\right)\right)\right] \tag{6.52}$$

As discussed earlier in the book, the value of the ERP can be split into a risk-free bullet bond with face value equal to the accumulated ERP loan balance at time (I_M, T_M) and a short position in a put option contingent to the nominal collateral house price and with a strike equal to the accumulated loan balance

$$V_{0,1}(\tau_M) = \exp(-r\tau_M) \cdot E^{\mathbb{Q}}\left[L_{I_M,T_M}\right] - \Pi_{0,1}(\tau_M). \tag{6.53}$$

The value of the ERP is also equivalent to the value today of future house possession minus a call option on the value of the house with the strike price L_{I_M,T_M}:

$$A_{0,1}\left(\tau_M\right) = \exp\left(-r\tau_M\right) \cdot E^{\mathbb{Q}}\left[H_{I_M,T_M}\right] - \exp\left(-r\tau_M\right) \cdot E^{\mathbb{Q}}\left[\max\left(H_{I_M},\right.\right.$$

The causes of contract termination generally denoted by J are death, entry into LTC and exit by voluntary prepayment. For an individual of age (x), the probability at the current time of an NNEG for causes of decrement J can be computes by:

$$P_x^{(J)} = \sum_{k=0}^{\omega-x-1} {}_kp_x^{(\tau)} \cdot q_{x+k}^{(J)} \cdot \Pi_{0,1}\left(k + \frac{1}{2} + \delta\right). \tag{6.54}$$

where ω is the maximum attainable age, ${}_kp_x^{(\tau)}$ is the multiple decrement probability for x to survive age $x+k$ and $q_{x+k}^{(J)}$ is the probability that (x) fails due to cause J during ages $x+k$ and $x+k+1$.

Practitioners use Life Tables for the computation of these decrement probabilities while academics assume some specific well-known stochastic framework for mortality modelling.

All these characteristics imply that the ERP market is an incomplete market from a financial economics point of view Bjork (2009) and this makes the valuation of the NNEG based on risk-neutral approach quite challenging.

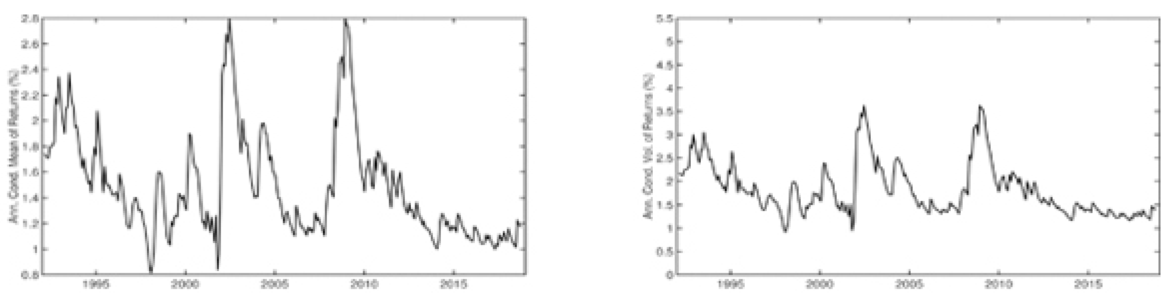
In Table 6.16 we present the statistical inference results for the GARCH-MIDAS model. These estimates are then used to produce the conditional mean returns and conditional volatilities presented in Figure 6.19. The conditional volatilities over the estimated period vary between 1% and 4%. There are spikes of volatility in the aftermath of the dotcom crisis and the global financial crisis. Moreover, post 2014 the conditional house price volatilities were very small for quite a long period.

TABLE 6.16 Maximum likelihood estimates for monthly log-returns of house price

Parameter	Coeff	StdErr	tStat	Prob
Panel A: GARCH - MIDAS - GDP				
λ	0.1614	0.0616	2.6194	0.0088
α	0.0900	0.0273	3.3003	0.0010
β	0.8623	0.0381	22.6564	0.0000
θ	0.0007	0.0000	130.0848	0.0000
w_1	6.4907	1.1044	5.8771	0.0000
w_2	10.4026	0.2337	44.5043	0.0000
m	0.0001	0.0000	111.9893	0.0000
Log-Likelihood	1024.77		AIC	-2035.54
AR(1) Model for GDP				
ϕ_1	0.0092	0.0037	2.5162	0.0119
ϕ_2	0.5161	0.0672	7.6788	0.0000
σ_x	0.0012	0.0000	31.7206	0.0000
Panel B: GARCH - MIDAS - IP				
λ	0.1717	0.0493	3.4805	0.0005
α	0.1929	0.0419	4.6034	0.0000
β	0.7095	0.0700	10.1411	0.0000
θ	0.0020	0.0000	552.6814	0.0000
w_1	3.1103	0.5643	5.5123	0.0000
w_2	3.7023	0.3479	10.6415	0.0000

Parameter	Coeff	StdErr	tStat	Prob
Panel A: GARCH - MIDAS - GDP				
m	0.0001	0.0000	135.6604	0.0000
Log-Likelihood	1022.04		AIC	-2030.09
AR(1) Model for Industrial Production (IP)				
ϕ_1	0.0025	0.0026	0.9922	0.3211
ϕ_2	0.6694	0.0792	8.4479	0.0000
σ_x	0.0007	0.0001	10.1829	0.0000

Notes: The table reports the maximum likelihood estimates together with their standard errors, t-statistics, and p-values for the GARCH-MIDAS models. Panel A presents results for the model constructed using GDP as explanatory variable (GARCH-MIDAS-GDP), and Panel B is constructed using industrial production (IP) as explanatory variable (GARCH-MIDAS-IP). The MIDAS weighting scheme is constructed with $K = 4$ past quarterly GDP and IP observations in the respective models. Both specifications are based on a 2-parameter Beta weighting scheme. The period January 1991 - December 1991 is used to initialise the MIDAS filter while the MLE is based on monthly house price log-returns over the period January 1992 to December 2018. The optimal values of the log-likelihood functions and AIC are reported for each model. The table also presents the maximum likelihood estimates (MLE) results for the AR(1) models in Equation 6.45. The estimation is based on quarterly macroeconomic variables over the period January 1992 - December 2018.



(a) Cond. Mean Return GARCH-MIDAS-GDP (b) Cond. Vol GARCH-MIDAS-GDP

Figure 6.19 GARCH-MIDAS-GDP Estimation

After fitting the GARCH-MIDAS model we employ it to computed the corresponding NNEG and loan values. The results are presented in Table

6.17. One can notice the very small NNNEG values across the board. Reflecting on the state of the economy one may wonder if the calculations are correct. We believe that the methodology based on the GARCH-MIDAS model is correct. However, the variables included may be expanded to consider the level of national indebtedness as well as other geopolitical risk indices. This could be a topic for further research for some academics and practitioners alike..

TABLE 6.17 Pricing the NNEG and ERP considering the role of macroeco fundamentals

Age	LTV	Initial loan	NNEG			ERM Cost	
			Male	Female	Joint Life	Male	Female
GARCH-MIDAS-GDP							
60	17.00%	52700	354.91	358.95	428.20	449089.91	504383.17
70	28.50%	88350	322.18	324.97	386.71	351120.75	394361.46
80	36.50%	113150	291.03	292.64	343.46	189882.89	210157.48
90	41.50%	128650	250.07	250.37	263.50	84344.20	88972.94
GARCH-MIDAS-IP							
60	17.00%	52700	354.88	358.93	428.18	449106.80	504395.78
70	28.50%	88350	322.11	324.91	386.65	351187.91	394413.23

Notes: The table presents report on the NNEG and ERP value using the risk-neutral GARCH-MIL GARCH-MIDAS-IP models for house price volatility estimation. We employ a multiple decreme condition that includes voluntary termination, transition to long-term care, and death of borrower. R ERC Spring 2023 market report, the market average interest rate for January 2023, which is 6.150 roll-up interest rate *R*. The risk-free interest rate is interpolated from the yield-to-maturity presente The standard Flexible Loan-to-Value (LTV) ratio are allocated to the corresponding borrower ages age 60 and ending at age 90 in 5-year increments. These valuations are 0.17, 0.2250, 0.2850, 0. 0.4150, and 0.4150. We simulate 100,000 house price paths in each Monte Carlo exercise.

Age	LTV	Initial loan	NNEG			ERM Cost	
			Male	Female	Joint Life	Male	Female
GARCH-MIDAS-GDP							
80	36.50%	113150	290.83	292.48	343.27	190110.56	210346.40
90	41.50%	128650	249.56	249.91	263.02	85004.15	89573.59

Notes: The table presents report on the NNEG and ERP value using the risk-neutral GARCH-MIL GARCH-MIDAS-IP models for house price volatility estimation. We employ a multiple decremei condition that includes voluntary termination, transition to long-term care, and death of borrower. R ERC Spring 2023 market report, the market average interest rate for January 2023, which is 6.150' roll-up interest rate *R*. The risk-free interest rate is interpolated from the yield-to-maturity presentec The standard Flexible Loan-to-Value (LTV) ratio are allocated to the corresponding borrower ages age 60 and ending at age 90 in 5-year increments. These valuations are 0.17, 0.2250, 0.2850, 0. 0.4150, and 0.4150. We simulate 100,000 house price paths in each Monte Carlo exercise.

6.8 SHALL WE GO REAL?

Practitioners in financial and insurance markets have recently questioned the risk-neutral valuation of financial instruments that cannot be hedged with other financial market instruments, see Section 4.3.1, page 64, in Kenyon and Stamm (2012).

This seems to be the case with the NNEG valuation. To start with, let us recall that, at the moment of writing this book, there is no financial (including insurance) instrument with a payoff matching the NNEG payoff. The value of the NNEG is in spirit more like collateral or margin calculation for a risk that may cause a tangible loss on the loan in a given state of the economy. While computationally practitioners and regulators are happy to consider equivalent the NNEG with a portfolio of European put options, one should note that there is no instrument behind the NNEG that could be traded even over-the-counter.

Moreover, in many countries there is no exchange traded derivatives contract with a payoff contingent on the house prices in that country. For US there is the S & P Case-Shiller index futures family that is openly traded and although volumes of trades in those futures are not always substantial, at least theoretically, one could use those contracts to organise a pricing framework based on risk-neutral valuation. However, in the UK, there are no such contracts and therefore, risk-neutral valuation may be called into questioning. For the case of UK, at this moment in time, it may well be the case that the NNEG valuation should be done under the real (also called physical or historical measure).

We envisage that with advances in machine learning into econometric techniques there will be more advanced techniques capable to produce better forecasting results that would enhance a more robust NNEG valuation under the physical or real-world pricing measure. There is a clear need for more research in this area from this point of view but we think that it would be just a matter of time until advanced econometric models involving machine learning would become more mainstream for NNEG calculus. We will revisit partially this issue in Chapter 8 when we will discuss various risk premia related to ERP.

6.9 SUMMARY

Various studies conclude that house price time-series exhibit serial correlation that is at odds with the GBM assumption, see Kogure et al. (2014) as an example. Li et al. (2010) considered the Nationwide House Price index and they remarked that, for this property index, a) there is a strong positive autocorrelation effect among the log-returns, b) the volatility of the log-returns varies with time, c) a leverage effect is present in the log-return series. All these three properties invalidate the use of the GBM for house prices.

There are many issues surrounding a more relevant calculation of NNEG, all subject to model risk though. One aspect that is frequently neglected in modern financial modelling is parameter estimation risk. We highly recommend a return to basics when applying any model proposed for NNEG valuation. Statistical estimation comes as a package with main requirements regarding checks about whether the model fits the data well or not. This is important regardless if the NNEG is calculated under market consistent or no-arbitrage risk-neutral principles or under a more actuarial historical frequentist approach.

There are many avenues for further improvement. Shall we use continuous time or a discrete time grid? The mechanics of the ERPs and the fact that changes in important risk drivers are observable only at monthly, quarterly or annually point out that the latter scheme is the natural one and a continuous time approach may introduce some biases in calculations. What is the best way to deal with the correlation between interest rates and house prices and should we account for changes in macro-finance variables? Rental yields estimation is an exercise in itself but is the house price just the present value of the series of future rental payments?

The modelling for ERP advances every year. Chen, Yang, and Huang (2021) use a continuous-time modelling framework to capture contagion effects across house prices in different regions captured under Merton jump-diffusion model specification. They also use a one-factor Heath-Jarrow-Morton model to capture the correlation between house prices and short interest rates. They conclude that the contagion effects could increase the NNEG values significantly.

The literature that can help advancing NNEG calculus grows rapidly every year. Below there are several references that we believe would help any researcher to consolidate their views after reading our chapter but also to find out new research questions that will lead to future papers in this area.

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CHAPTER 7

Risk Management of Equity Release Products

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7.1 INTRODUCTION

THE RISK MANAGEMENT of ERP products requires various valuation exercises by the sellers of these financial products prior to their trade but also subsequently after they are taken onto their balance sheet.

The previous chapter was dedicated to the NNEG calculus so in a sense it was more relevant to the UK market. This chapter examines other type of concepts and calculations that are relevant to other markets such as US, Korea, Japan or Australia.

Some of the calculus presented here is done in continuous time while some other is still carried out over a discrete time grid. Since both set-ups are encountered in the literature we made an effort to include both sides.

While this may be more challenging at a first sight, we believe it is also useful to have this challenge in order to comprehend better those concepts.

Ideally a risk manager would prefer to work with closed form solutions but sometimes insisting on a simplified approach may lead us towards solutions that are carrying some hidden pitfalls. To this end, in this chapter we would like to emphasise the important role played by model risk. We point out that some solutions that are easy to understand and even to implement may lead to inconsistent results in some scenarios.

One important aspect we would like to highlight is the danger presented by the so called “equilibrium models”. When the valuation of ERPs is done under market consistent principle, or in other words using risk-neutral pricing, one should be aware that the set of equilibrium models may be larger than the set of market consistent models. Secondly, but equally importantly, the equilibrium models are in general in contradiction with the reality of crisis such as house price collapse. Therefore, precisely when needed, equilibrium models will not offer any insight for exactly the period a model is needed the most.

From a market perspective, the perfect solution to risk management would be to securitise those assets. This would put a ring-fence the potential future problems and would allow a sensible transfer of risks to other market participants who can afford to absorb risks.

7.2 A FRAMEWORK FOR VALUATION OF REVERSE MORTGAGE IN CONTINUOUS-TIME

Chinloy and Megbolugbe (1994a) describe a continuous-time framework for disentangling the various options and risks embedded in a reverse mortgage. This framework is reviewed here in this section.

The original loan amount when the reverse mortgage is issued comes to $L = min(H, \lambda)$, where H is the value of the house and λ is the loan limit. The borrower can draw a maximum of $\lambda = \Gamma L$ where Γ is the LTV. The r^* is used for discounting; this rate is equal to either the risk-free rate r when the valuation ought to be done under a risk-neutral valuation principle or a risk-adjusted rate if valuation can be done under actuarial risk management principles using physical (also called objective or historical) measure. Under this framework, a strong assumption is that the house price grows at the rate h with continuous compounding, where h is constant.

If we assume that there are a maximum number of payments (as monthly payments) η then would be $\eta = 456$ for calculations of a loan from HECM programme.[1]. The present value of the borrower's liability is calculated with continuous compounding as

$$\Gamma L e^{(h-r^*)\eta} \tag{7.1}$$

On the other hand, the borrower will receive a sequence of payments, some of them being possibly zero to accommodate also the case of a lump sum payment, at times $t = 0, 1, 2, \ldots, \eta$. The payments may include indexed adjustments for inflation and lump-sum draws on a line of credit.

To facilitate the calculus, h and r^* can be transformed into monthly equivalents. In order not to complicate the notation, we shall retain without loss of generality the same notation, h and r^*, for the equivalent monthly rates.

Denoting by q_t the loan survival at time t, spanned by the various characteristics of the borrowers such as age, sex, correlation (for couples), mortality, morbidity tables and trends, the cash-flow at time t, from a lender perspective, is $q_t A(t)$, where $A(t)$ is the annuity value at time t. If the inflation index growth rate is i then (Chinloy and Megbolugbe, 1994a) argue that the value of the sequence of the ERP loan payments will be

$$\int_0^{\eta} q_t A(t) e^{-(r^*-i)t} dt. \tag{7.2}$$

By matching the liabilities and payments over the life of the product it leads to

$$\Gamma L e^{(h-r^*)\eta} = \int_0^{\eta} q_t A(t) e^{-(r^*-i)t} dt \tag{7.3}$$

---

[1]Under the HECM programme for example, the maximum limit is considered to be an age of 100 and the borrower is at least 62 years old.

Simplifying the assumptions to have $A(t) \equiv A$ and $q_t) \equiv q$ leads to

$$\Gamma L e^{(h-r^*)\eta} = qA \int_0^{\eta} e^{-(r^*-i)t} dt \tag{7.4}$$

This equation can be solved for the annuity payment A as

$$A = \frac{(r^* - i)}{[e^{-h\eta}(e^{r^*\eta} - e^{i\eta}]} \frac{\Gamma L}{q} \tag{7.5}$$

Now we can construct some examples.

Example 7.1 *Assume that for a reverse mortgage loan the following values occur: the property with the initial house price estimation at* $L = 500,000$ *dollars, an LTV* $\Gamma = 30\%$, *inflation rate* $i = 5\%$, *discount rate* $r = 4\%$, *house price growth rate* $h = 5\%$, *constant exit rate (combining mortality, morbidity, prepayment)* $q = 6\%$, *all rates per annum, and using* $\eta = 456$ *months as the lifetime of the reverse mortgage that is typically assumed under the HECM programme for a borrower taking the loan at 62 and living until 100 years old. Then, feeding these values into formula* (7.5) *the value of the annuity A is equal to 25,264.32 USD.*

Let's consider now that the house prices increase faster but that is associated also with an increase in inflation.

Example 7.2 *Assume that for a reverse mortgage loan the following values occur: the property with the initial house price estimation at* $L = 500,000$ *dollars, an LTV* $\Gamma = 30\%$, *inflation rate* $i = 6.5\%$, *discount rate* $r = 4\%$, *house price growth rate* $h = 6\%$, *constant exit rate (combining mortality, morbidity, prepayment)* $q = 6\%$, *all rates per annum, and using* $\eta = 456$ *months as the lifetime of the reverse mortgage that is typically assumed under the HECM programme for a borrower taking the loan at 62 and living until 100 years old. Then, feeding these values into formula* (7.5) *the value of the annuity A is equal to 6392.83 USD.*

The explanation for the lower annuity is that inflation is rising faster than the house prices and, under this pricing framework, the effect is quite dramatic on the cash-flow sums that can be extracted, leading to a reduction to almost a quarter.

The next example shows what happens in the HECM loan if the discount rate goes above the inflation rate and everything else stays the same.

Example 7.3 *Assume that for a reverse mortgage loan the following values occur: the property with the initial house price estimation at* $L = 500,000$ *dollars, an LTV* $\Gamma = 30\%$, *inflation rate* $i = 4\%$, *discount rate* $r = 6\%$, *house price growth rate* $h = 6\%$, *constant exit rate (combining mortality, morbidity, prepayment)* $q = 6\%$, *all rates per annum, and using* $\eta = 456$ *months as the lifetime of the reverse mortgage that is typically assumed under the HECM programme for a borrower taking the loan at 62 and living until 100 years old. Then, feeding these values into formula* (7.5) *the value of the annuity A is equal to 50,005.47 USD.*

The effect of having a stronger house price growth than inflation is substantial, the annuity value going almost double.

Here we recalculate the annuity rate by considering a time evolving exit rate. As in Chinloy and Megbolugbe (1994a) we consider that

$$q_t = b(1-b)^{t-1} \tag{7.6}$$

where b is a base exit rate, taken at $b = 0.1$. Thus, the annuity value is calculated from the equation

$$
\begin{aligned}
\Gamma L e^{(h-r^*)\eta} &= A \sum_{t=1}^{\eta} b(1-b)^{t-1} e^{-(r^*-i)t} \\
&= A \frac{b}{1-b} \sum_{t=1}^{\eta} (1-b)^{t} e^{-(r^*-i)t} \\
&= A \frac{b}{1-b} \frac{\omega^{\eta+1}-\omega}{\omega-1}
\end{aligned}
$$

where $\omega = (1-b)e^{-(r^*-i)}$. Hence

$$
A = \frac{\Gamma L e^{(h-r^*)\eta}}{b/(1-b)} \frac{\omega-1}{\omega^{\eta+1}-\omega} \tag{7.7}
$$

The above framework provided analytical formulae for the ERP valuations. Many other risk management quantities can be generated easily on the back of the above assumptions. Here we can employ the framework to discuss how the some of the ERP main risks can be transferred to another insurer or reinsurer.

The crossover option at η is determined by the strike price $A\frac{e^{r^*\eta}-1}{r^*}$. The present value of the strike crossover barrier is

$$
X(t) = e^{-r^*t} A \frac{e^{r^*\eta}-1}{r^*} \tag{7.8}
$$

Then if $M(H,t;\theta)$ denotes the value of the reverse mortgage, with θ denoting model parameters vector, the payoff of the crossover option at time t is

$$P(t) = \max\left[X(t) - M(H,t;\theta), 0\right] \tag{7.9}$$

The assignment option can be defined as the mechanism of transferring the NNEG risk to an insurer. This option is relative to a bond $B(r,t;\theta)$ representing the market value of the annuity contract. The fixed dollar annuity has a future value ΓL. The lender of the ERP product has an option to sell the loan with maturity η to an insurer, for ΓL. At the term date the value of the assignment option is given by

$$S(r,\eta;\theta) = \max\left[\Gamma L - B(r,\eta;\theta), 0\right] \tag{7.10}$$

If the accumulating balance reaches ΓL at some time t prior to the maturity of the loan η then it becomes optimal for the lender to exercise the assignment option. This is a method to pass over crossover risk from the ERP lender to the insurer (reinsurer). What makes the assignment option valuable is the adjustable rates used for growing the outstanding balance. This reinsurance scheme operated in the U.S. with HUD as the insurer for HECM lenders.

The above framework has the disadvantage that it assumes a fixed rate of growth for the house prices. If this is positive then house prices in all subsequent calculations will grow with time. If the rate is negative it will always decrease with time.

7.2.1 Break-even Cost Calculations

The break-even level of monthly payments that the borrower must pay in the ERP contract structured with an insurance premium, as is the case in US for example, is determined by equating the present value of the mortgage insurance premium (PVMIP) to the present value of expected loss (PVEL) associated with this loan. For clarity of explanation we will consider some notations below. Thus:

- ${}_tp_x$ is the probability that a borrower at age x will survive at $x+t$ (the probability that life aged x today is alive t years from today)
- ${}_{t|}q_x$ is the probability that a loan will survive to the age $x+t$ but not to the age $x+t+1$. (This is deferred mortality, where the borrower dies more than t years after the inception of the loan contract.)
- P_0 is the upfront mortgage insurance premium
- $T(x)$ is the remainder of the payment period until loan termination when the loan starts at the age x of the borrower,
- μ is the rate of mortgage insurance premium,
- $OLB(t)$ is the outstanding loan balance accrued up to time t,
- pm is the monthly payments to time $T(x)$
- H_t is the expected house value at time t,
- h is the growth rate of housing price,
- R is the roll-up actual interest rate of the loan during its period

- r^* the discount rate.

There is a well-known link between the mortality survival probabilities ${}_tp_x$ and the termination probabilities ${}_{t|}q_x$ given by

$$ {}_{t|}q_x = {}_tp_x - {}_{t+1}p_x = {}_tp_x \times q_{x+t} $$

(7.11)

Then the valuations of the two sides of the contract are

$$ PVMIP = P_0 + \sum_{t=1}^{t=T(x)} \frac{(K_t + pm) \times \mu \times {}_tp_x}{(1+r^*)^{t-1}} $$

(7.12)

$$ PVEL = \sum_{t=1}^{t=T(x)} \left\{ \frac{\max\left[(K_t - H_t), 0\right] \times {}_{t|}q_x}{(1+r^*)^t} \right\} $$

(7.13)

The monthly premium pm_t is computed by requiring that $PVMIP = PVEL$ and applying a search method. It is important to realise that both sides depend on the monthly premium because the dynamics of the loan balance is given by the equation

$$ K_t = (K_{t-1} + pm)(1+\mu)(1+R) $$

(7.14)

If the house price dynamics is given by the following equation

$$H_t = H_0 \times (1+h)^t \tag{7.15}$$

then exact calculations can be performed. However, this tacitly assume that house prices evolve in a deterministic manner, which is quite an unrealistic assumption.[2]

Note that there is one rate *R* that applies to compute the accrued balance and another rate *r* to calculate the present value discounting. This is a general principle. As a particular example, for the Korean programme, commercial lenders were charging as the rolling rate the certificate of deposit (CD) rate plus 1.1%. This shows that the insurer faces interest rate risk on two very different channels.

The payments could be fixed, similar to an annuity, or floating, linked to a floating rate or a macroeconomic variable. With payments cash-flows structured like an annuity fixed scheme, based on matching (7.12) and (7.13) leads to the calculation of the pm_t in closed-form, with the formula

$$pm_t = \frac{PVEL - P_0 - \mu \sum_{t=1}^{t=T(x)} \frac{K_{t\,t}p_x}{(1+r^*)^{t-1}}}{\mu \sum_{t=1}^{t=T(x)} \frac{{}_t p_x}{(1+r^*)^{t-1}}} \tag{7.16}$$

7.2.2 Total Annual Loan Cost Rate

Some authors and practitioners introduced for reverse mortgages a type of interest rate as a yardstick to compare various reverse mortgage products on a like-for-like basis. This is called the *total annual loan cost rate* (TALCR) and it is an average annual rate at given period such as month or year $t = n$ capturing all costs associated with the respective ERP. For simplicity we shall assume that the borrowers will reach the age of 100 and therefore calculations can be done mathematically without involving survival probabilities. The TALCR can be used by lenders, regulators or NGOs to compare products among competitors.

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[2]The Korean government sponsored program, see Section 3.4.2, applies a fixed rolling interest rate, the borrowers receiving payments similar to an annuity until the loan is terminated. For calculations we need the expected interest rate μ that is involved in determining the outstanding loan balance in each period and the present values of each stream of cash-flows, PVMIP and PVEL.

For a reverse mortgage the TALCR can be calculated for each payment type. For constantly monthly payment the pricing equation for the TALCR is

$$Min(K_n, H_n) = pm \sum_{t=0}^{t=n} (1 + TALCR)^{n-t} \tag{7.17}$$

which can be rewritten simply as

$$\frac{(1 + TALCR)^{n+1} - 1}{TALCR} = \frac{Min(K_n, H_n)}{pm} \tag{7.18}$$

For graduate monthly payment design, the pricing equation is

$$Min(K_n, H_n) = pm_0 \sum_{t=0}^{t=n} [(1+i)(1+TALCR)]^{n-t} \tag{7.19}$$

which can be rearranged as

$$\frac{[(1+i)(1+TALCR)]^{n+1} - 1}{(1+i)(1+TALCR) - 1} = \frac{Min(K_n, H_n)}{pm_0}$$

Both equations (7.17) and (7.19) must be solved by searching methods given their high nonlinearity. They are in essence polynomial equations.

Denoting by φ the $(1+TALCR)$ quantity in (7.17) or the $(1+i)(1+TALCR)$ in (7.19), the equation can be reformulated as follows

$$1 + \varphi + \varphi^2 + \ldots \varphi^n = \kappa \tag{7.20}$$

with κ denoting the constant term on the right side of the equation (7.17) or (7.19). This is a polynomial equation and in general it is known to have multiple solutions, which would cause problems in the sense that one would not know a priori if a solution exists, or when it is known that it does exist we are not sure if the solution is unique.

Considering the function

$$h(\varphi) = \frac{1-\varphi^{n+1}}{1-\varphi} - \kappa$$

(7.21)

and taking into account that both φ and κ are greater than one, it follows easily to see that $h(0) = 1 - \kappa < 0$ and $\lim_{\varphi\to\infty} h(\varphi) = +\infty$. Furthermore

$$h'(\varphi) = 2\varphi + 3\varphi^2 + \ldots + (n+1)\varphi^n$$

(7.22)

which is strictly positive. Therefore, the function h is changing signs so there is at least one root solution and that solution is unique because h is strictly increasing.

7.2.3 Break Even Calculation for Lump-sum Scheme under GBM Assumption

One of the most useful concepts in ERP calculus is the break-even point. In order to perform this calculus the risk manager or analyst must make an assumption about the future dynamics of the house prices. As we have seen before, assuming that house prices evolve in a deterministic manner is far from idea.

The next step in refining the ERP calculus for a break even point is to assume a stochastic process evolution for house prices. Computationally speaking, the most convenient assumption is that of a geometric Brownian motion because in most situations calculus can be simplified based on prior known results. Pu et al. (2014) showcase break-even calculations under the assumptions that the termination time of the contract is fixed at T and the house prices dynamics is described by a geometric Brownian motion.

The analytical calculations are detailed here for a lump-sum type of contract and also for an annuity-like scheme, correcting some typos in the original paper (see also Tunaru, 2017; Quaye, 2021) and discussing some of the implication of our derivations under this simplistic stochastic assumption.

Let L_T be the fixed lump-sum advanced to the homeowner at time t_0 for a loan with maturity T. Usually this is calculated as $L_T = \Gamma H_0$. Without loss of generality we can assume that $t_0 \equiv 0$. The house price that is collateral in this loan is supposed to follow the dynamics of the process

$$dH_t = \mu H_t dt + \sigma H_t dW_t \tag{7.23}$$

There are two interest rates to consider: R is the rate charged on the loan and r is the discount rate, considered here as being represented by the risk-free rate. The balance outstanding at the exit T is calculated using R and this is $L_T e^{RT}$, while any discounted cash-flow is computed using r. For both rates we assume continuous compounding.

At T the lender will receive

$$H_T - \max\left[H_T - L_T e^{RT}, 0\right].$$

The break-even lump-sum of the loan L_T^* is calculated as the solution of the equation

$$L_T = e^{-rT}E(H_T) - e^{-rT}E\left(\max\left[H_T - L_Te^{RT}, 0\right]\right) \tag{7.24}$$

in the unknown L_T.

If the house price follows (7.23) then

$$H_T = H_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T} \tag{7.25}$$

and standard calculations for log-normal distribution gives

$$E(H_T) = H_0 e^{\mu T} \tag{7.26}$$

Following similar calculations as those known for the Black-Scholes formula gives[3]

[3]Here we are *not* in a Black-Scholes economy, there is no replication or no-arbitrage argument invoked.

$$E\left(\max\left[H_T - L_Te^{RT}, 0\right]\right) = H_0e^{\mu T}N(d_1(L_T)) - L_Te^{RT}N(d_2(L_T)) \tag{7.27}$$

where

$$d_1(L_T) = \frac{\ln\left(\frac{H_0}{L_T e^{RT}}\right) + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \;\; d_2(L_T) = d_1(L_T) - \sigma\sqrt{T}$$

The break-even lump-sum equation can be rewritten

$$L_T = H_0 e^{(\mu-r)T} - H_0 e^{(\mu-r)T} N(d_1(L_T)) + e^{(R-r)T} L_T N(d_2(L_T))$$

$$L_T = H_0 e^{(\mu-r)T} \left(1 - N(d_1(L_T))\right) + e^{(R-r)T} L_T N(d_2(L_T)) \tag{7.28}$$

$$L_T \left(1 - e^{(R-r)T} L_T N(d_2(L_T))\right) = H_0 e^{(\mu-r)T} \left(1 - N(d_1(L_T))\right) \tag{7.29}$$

The last equation requires to find out the value of lump-sum that makes the two sides equal, after knowing the current value of the house H_0, the expected rate of growth under GBM assumption μ and the corresponding volatility σ, the roll-up rate R and the discount rate r.

7.2.4 Annuity like scheme

In this subsection we consider that the fixed annuity coupon A is paid every period such as every month for example.[4] As in (Pu et al., 2014) we assume a fixed exit time T and that the uncertainty in the house price is driven by a geometric Brownian motion. The months are spanning a payment time grid

$\{t_1, t_2, \ldots, t_M\}$ with $t_M = T$. In this case $\Delta \equiv t_{i+1} - t_i = \frac{1}{12}$ year and we assume that $T = M\Delta$.

The balance that will accumulate at exit T can be calculated as

$$K_T = \sum_{i=1}^{M} Ae^{R(T-t_i)} \tag{7.30}$$

Working on the expression on the right side we get

$$\sum_{i=1}^{M} Ae^{R\Delta(M-i)} = Ae^{R\Delta M} \sum_{i=1}^{M} e^{-iR\Delta}. \tag{7.31}$$

Based on standard calculus we get

$$\sum_{i=1}^{M} e^{-iR\Delta} = \frac{e^{-R\Delta M} - 1}{1 - e^{R\Delta}} \tag{7.32}$$

Thus

$$K_T = A\frac{1 - e^{R\Delta M}}{1 - e^{R\Delta}} \tag{7.33}$$

[4]It can be any frequency like quarterly or annually.

At the termination of the contract the lender will receive the payoff

$$X_T \quad = \min\left[H_T, K_T\right] \tag{7.34}$$

$$= OLB(T) - \max\left(K_T - H_T, 0\right) \tag{7.35}$$

$$= H_T - \max\left(H_T - K_T, 0\right) \tag{7.36}$$

As it was observed previously, from (7.36) the payoff of this ERP at maturity is equal to a portfolio of a long position in the house and a short position in a European call option with a time dependent strike K_T.

The NPV of the lender's cash flow is

$$NPV \quad = E(e^{-rT}X_T) - E(\sum_{i=1}^{M} Ae^{-rt_i}) \tag{7.37}$$

$$= E(e^{-rT}X_T) - A\frac{e^{-r\Delta M} - 1}{1 - e^{r\Delta}} \tag{7.38}$$

Therefore, working with $K_T - \max\left(K_T - H_T, 0\right)$, the lender will have a positive NPV if and only if

$$e^{-rT} E\left(K_T - \max\left(K_T - H_T, 0\right)\right) - A \frac{e^{-r\Delta M} - 1}{1 - e^{r\Delta}} > 0$$

Standard algebra leads us to

$$A\left[\frac{1 - e^{R\Delta M} - 1}{1 - e^{R\Delta}} - \frac{1 - e^{r\Delta M} - 1}{1 - e^{r\Delta}}\right] > E\left(\max\left(K_T - H_T, 0\right)\right) \tag{7.39}$$

The lender would be interested to know the break-even annuity payment that they should afford, *under their modelling assumptions*. The break-even annuity payment rate A^* is computed as the solution of the equation

$$A\left[\frac{1 - e^{R\Delta M} - 1}{1 - e^{R\Delta}} - \frac{1 - e^{r\Delta M} - 1}{1 - e^{r\Delta}}\right] = E\left(\max\left(K_T - H_T, 0\right)\right) \tag{7.40}$$

Applying standard formulae from Black-Scholes calculus leads to the analytical expressions:

$$E\left(\max\left(K_T - H_T, 0\right)\right) = K_T N(-d_2(A)) - H_0 e^{\mu T} N(-d_1(A)) \tag{7.41}$$

where $d_1(A) = \frac{\ln\left(\frac{H_0}{K_T}\right)+(\mu+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$, $d_2(A) = d_1(A) - \sigma\sqrt{T}$.

7.3 RISK SENSITIVITY ANALYSIS

Table 7.1 reports the sensitivity analysis for NNEG calculations for a joint couple loan in the UK at the end of 2019 first quarter. Panel A shows the results under GBM model preferred by the regulator while Panel B shows the corresponding results, for the same scenarios, under the ARMA-EGARCH model identified as a suitable model for forecasting well house prices in the UK. Similar results are reported in the Appendix, for the single female borrower and single male borrower.

TABLE 7.1 Non-negative equity guarantee sensitivity analysis for joint cou
baseline flexible LTV in the UK

Age	60	65	70	75	80
LTV	17.0%	22.5%	28.5%	32.4%	36.5%
Initial Loan	65,100	82,150	102,300	114,700	130,20
Panel A: Joint Life - GBM					
Baseline parameters: g=1%,σ=4.88% *R*=6.150%, *r*=3.422%					
Baseline outcomes:	1.59	2.15	2.17	0.93	0.31
Rental yield					
g = 0.00% (↓ 1.00%)	-0.768%	-0.711%	-0.661%	-0.655%	-0.653%
g = 2.00% (↑ 1.00%)	16.849%	10.961%	8.129%	8.644%	9.534%

Notes: This Table reports the percentage deviation of the NNEG-to-initial-loan ratio from baseline va
the risk-neutral pricing under the Black-Scholes framework and Panel B report results from the
ARMA-EGARCH model using the quarterly house price time series data. The standard Flexibl
allocated to the corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-year
0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simulate 100,000 house price paths i
baseline parameter values used in the sensitivity analysis are *g*=1%,σ=4.88% *R*=6.150%, *r*=3.422%.

Age	60	65	70	75	80
House price volatility					
$\sigma = 3.25\%(\downarrow 50\%)$	-0.876%	-0.811%	-0.765%	-0.825%	-0.893
$\sigma = 7.32\%(\uparrow 50\%)$	4.098%	2.989%	2.503%	3.196%	4.528
$\sigma = 9.76\%(\uparrow 100\%)$	12.075%	8.178%	6.615%	9.007%	14.021
$\sigma = 14.64\%(\uparrow 300\%)$	37.484%	23.611%	18.520%	27.363%	47.663
Risk-free rate					
r = 2.422% ($\downarrow$ 1.00%)	119.69%	90.88%	60.62%	26.43%	9.29%
r = 4.422% ($\uparrow$ 1.00%)	-0.938%	-0.906%	-0.871%	-0.861%	-0.853
Roll-up rate					
R = 5.150% ($\downarrow$ 1.00%)	-0.998%	-0.995%	-0.989%	-0.987%	-0.984
R = 7.150% ($\uparrow$ 1.00%)	9.677%	6.626%	4.977%	4.883%	4.912
Panel B: Joint Life - ARMA-EGARCH					
Baseline parameters: g=1%,σ=4.88% R=6.150%, r=3.422%					
Baseline outcomes:	0.18	0.93	1.20	0.06	0.00
Rental yield					
g = 0.00% ($\downarrow$ 1.00%)	-0.998%	-0.994%	-0.987%	-0.984%	-0.979
g = 2.00% ($\uparrow$ 1.00%)	35.730%	18.872%	12.507%	14.396%	17.289
House price volatility					
$\sigma(\downarrow 50\%)$	-0.943%	-0.877%	-0.821%	-0.895%	-0.955
$\sigma(\uparrow 50\%)$	7.204%	4.365%	3.296%	4.737%	8.157

Notes: This Table reports the percentage deviation of the NNEG-to-initial-loan ratio from baseline va the risk-neutral pricing under the Black-Scholes framework and Panel B report results from the ARMA-EGARCH model using the quarterly house price time series data. The standard Flexibl allocated to the corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-year 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simulate 100,000 house price paths i baseline parameter values used in the sensitivity analysis are g=1%,σ=4.88% R=6.150%, r=3.422%.

Age	60	65	70	75	80
$\sigma(\uparrow 100\%)$	26.433%	13.769%	9.682%	15.333%	31.022'
Risk-free rate					
r = 2.422% (↓ 1.00%)	48.196%	24.641%	15.752%	17.260%	19.678'
r = 4.422% (↑ 1.00%)	-0.998%	-0.994%	-0.988%	-0.985%	-0.980'
Roll-up rate					
R = 5.150% (↓ 1.00%)	-0.998%	-10.994%	-0.987%	-0.984%	-0.978'
R = 7.150% (↑ 1.00%)	43.425%	22.532%	14.529%	15.847%	17.931'

Notes: This Table reports the percentage deviation of the NNEG-to-initial-loan ratio from baseline va the risk-neutral pricing under the Black-Scholes framework and Panel B report results from the ARMA-EGARCH model using the quarterly house price time series data. The standard Flexibl allocated to the corresponding borrower ages, beginning at age 60 and ending at age 90 in 5-year 0.17, 0.2250, 0.2850, 0.3240, 0.3650, 0.4150, and 0.4150. We simulate 100,000 house price paths i baseline parameter values used in the sensitivity analysis are g=1%,σ=4.88% R=6.150%, r=3.422%.

The baseline scenario comparison across the two models indicates an over-conservative NNEG valuation for the GBM model. The main reason for that is that the variance of house price returns increases linearly with time.

The NNEG has an almost insignificant weight for near-term maturities given that mortality probabilities are increasing to more significant levels towards long-term maturities. Furthermore, in the first few years, the relative low LTV ratio protects the accumulated value of the loan and so there is very little risk that the loan will not be repaid if it comes due. The NNEG risk manifests after the accumulated balance has had sufficient time to increase and, at the same time, the house prices had time to experience a crash or market correction. On the other hand, for the ARMA-EGARCH model the variance of house price returns does not increase linearly with time.

A decrease in the rental yield parameter g will decrease substantially the NNEG values while an increase in g will determine very large values for NNEG. A similar effect occurs for the changes in the volatility, smaller volatility leads to smaller NNEG and larger volatility means larger NNEG values. The risk-free rate r should be assessed vis-a-vis g, since the market NNEG valuation depends on the drift $r - g$ combination. Hence, the effect of changing r is the opposite of g. When r decreases the NNEG increases, and when r increases the NNEG value decreases. The roll-up rate R has the expected effect, decreasing the NNEG value when R decreases and increasing the NNEG value when R is increasing. Overall, g and σ are the parameters inducing the highest sensitivity to the NNEG value. Their estimation is therefore a very important exercise.

7.4 THE DATA GENERATING PROCESS AND MODEL RISK

We define the accumulator of one unit of local currency from time t to time T as $A(t, T)$. Therefore, if the initial value of the loan is K_0 and the roll-up mortgage applies a fixed continuously compounded rate R on the loan then

$$K_t = K_0 A(0, t) = K_0 e^{Rt} \tag{7.42}$$

The average roll-up rate for the top ten equity release lenders in the UK in May 2007 was 6.39%. This has decreased post subprime crisis to values between 4.15% and 5.25% as of October 2018 (Tunaru and Quaye, 2019; Quaye, 2021).

One of the most important factors determining the value of the NNEG is the ERM termination time, regardless of the NNEG option being in the money or not. Since mortality and moving into long-term care are stochastic quantities, the NNEG put option has a random maturity making the valuation of NNEG difficult. Moving into care is influenced by either morbidity or mobility problems, called long-term care risk. A semi-Markov model capable to capture well the termination times for reverse mortgages is developed in Ji et al. (2012) and applied by Alai et al. (2014) to a single female borrower with only two possible states, death and long-term care.

Suppose that K_t and $H(t)$ are the values of the loan balance and the collateral house at time t, respectively. At termination time τ the lender will receive

$$\min\left(K_\tau, H(\tau)\right) = K_\tau - \max\left(K_\tau - H(\tau), 0\right) \tag{7.43}$$

Hence, the borrower is long a put option contingent on the house price value, with random maturity and strike price equal to K_τ and short a negative amortising bond. Likewise, the lender is long the same bond and short the option. This is usually called in the literature a European put. However, we argue that this option has some non-standard characteristics that makes it difficult to associate with a European put option. The maturity τ is a stochastic quantity driven by the filtrations of information related to the mortality and morbidity risks, as well as to the voluntary prepayment risk. Secondly, the strike price K_τ could be also a stochastic quantity, even in the case of fixed interest rates. Therefore, the derivative payoff appearing in (7.43) can be seen more as an asset exchange type of option, i.e. a bullet bond versus the value of a collateral house, with the exchange taking place at a random time τ.

7.4.1 General Considerations

It is paramount to estimate as accurately as possible the probability of the contract termination, given that underestimating the termination time in a reverse mortgage will lead to heavier loan balance accumulation and consequently to higher risk of the NNEG option being in the money. For home reversion type products, the same underestimation may cause a longer lease life and a larger payment towards the borrower. Let $DF(t)$ be the discount factor for maturity t calculated now. The most general formulation of the risk-neutral NNEG put option value is given by

$$V_{NNEG} = \mathbb{E}^{\mathbb{Q}} \left\{ DF(\tau+\delta)[K_{\tau+\delta} - H(\tau+\delta)]^{+} \right\} \tag{7.44}$$

where δ reflects the delay time until the property is actually sold, with the usual assumption that termination time is always in the middle of the period and $\mathbb{Q}$ is an equivalent martingale pricing measure.

The risk factors that are driving the NNEG are embedded in a) the stopping time τ, whose filtration is $\mathscr{F}_\tau = \mathscr{F}_{mortality} \bigotimes \mathscr{F}_{morbidity} \bigotimes \mathscr{F}_{prepayment}$, b) the interest rate environment manifesting through discount factors $\{DF(t)\}_{t\geq 0}$ and the accrual factors $\{A(0,t)\}_{t\geq 0}$ as $K_t = K_0 A(0,t)$, and c) the house price $\{H(t)\}_{t\geq 0}$ dynamics. Without loss of generality, we may assume that τ is independent of interest rate risk and house price risk. However, house price levels may be correlated with the interest rates.

One common approach to bypass the problem caused by the random stopping time associated with the termination of an ERM contract is to take a conditional approach, working on a grid $\{t_k\}_{k\geq 1}$, usually monthly or annually. Then, the risk-neutral valuation of NNEG would be

$$V_{NNEG} = \mathbb{E}^{\mathbb{Q}}\left\{DF(\tau+\delta)[K_{\tau+\delta} - H(\tau+\delta)]^{+}\right\} \tag{7.45}$$

$$= \sum_{k\geq 1} \mathbb{Q}(\tau \in (t_{k-1}, t_k])\mathbb{E}^{\mathbb{Q}}\left\{DF(t_k+\delta)[K_{t_k+\delta} - H(t_k+\delta)]^{+}\right\} \tag{7.46}$$

$$= \sum_{k\geq 1} \mathbb{Q}(\tau \in (t_{k-1}, t_k])\mathbb{E}^{\mathbb{Q}}(DF(t_k+\delta))\mathbb{E}^{\mathbb{Q}}[K_{t_k+\delta} - H(t_k+\delta)]^{+} \tag{7.47}$$

where the last identity follows from the assumed independence between interest rates and house prices, which is generally assumed in the NNEG literature (Li et al., 2010).

We denote by $Q_S(t)$ the survival probability $\mathbb{Q}(\tau \geq t)$, in the sense that the loan is still active so at least one borrower is alive, not in the long-term care and the loan has not been prepaid, i.e. staying in the mortgaged property, on a last survivor basis. We can model this survival probability as

$$Q_S(t) = exp\left(-\int_0^t h(v)dv\right)$$

(7.48)

where h is a hazard function that can be any integrable function. If τ has the probability density ψ then

$$Q_S(t) \quad = 1 - \int_0^t \psi(v)dv$$

(7.49)

$$Q'_S(t) \quad = -\psi(t)$$

(7.50)

$$\psi(t) \quad = h(t)Q_S(t)$$

(7.51)

where the last equation follows from using that $Q_S(0) = 1$. Without loss of generality, we can assume that τ and $\{A(0,t)\}_{t\geq 0}$ are independent under both $\mathbb{P}$ and $\mathbb{Q}$. One advantage of this modelling approach is that, if the risk-free discount factor takes the form $\mathbb{E}^{\mathbb{Q}}(DF(t)) = e^{-r(t)t}$, then a NNEG-risky discount factor can be expressed as follows

$$\widetilde{DF}(t) = DF(t)Q_S(t) = \exp\left(-r(t)t - \int_0^t h(\tau)d\tau\right)$$

(7.52)

Therefore, we obtain

$$V_{NNEG} = \sum_{k \geq 1} \widetilde{DF}(t) \mathbb{E}^{\mathbb{Q}}[K_{t_k+\delta} - H(t_k + \delta)]^+$$

(7.53)

The hazard function h can be decomposed based on the three types of risk, mortality (h_1), morbidity (h_2) and voluntary prepayment (h_3),

$$h(t) = h_1(t) + h_2(t) + h_3(t)$$

(7.54)

For numerical purposes, we take ω as the highest attainable age by a borrower, usually 100 or 120 in most studies.[5] Consider a time grid $t_0 < t_1 < \ldots < t_n < \ldots$ with $\Delta_k = (t_k - t_{k-1})$ and assume that all terminations occur in the middle of the yearly time period. In addition, we denote by $\widetilde{x}$ the longest survival time of a couple, usually denoted by the $\overline{xy}$, where the wife has age x and y is the age of the husband. We note that the younger partner does not necessarily die later. Hence, if $\tau \in (t_{k-1}, t_k]$, where k is a positive integer, then $\mathbb{Q}(\tau \in (t_{k-1}, t_k]) = {}_{t_k}p_{\widetilde{x}}$ is the probability that the last survivor borrower from couple will still be in the property in t_k years time. Furthermore, we can calculate the value of the NNEG as

[5]In this way we fix *conditionally* the time for the discount factor, balance accrual and house price valuation time. However, a problem still remains, since there is no theory to indicate what is highest attainable value for τ. However, it is standard practice to consider that borrowers may survive to 100 years old or 120 years old.

$$V_{NNEG} = \int_0^{\omega-\widetilde{x}+1} DF(t)[K_{t+\delta} - H(t+\delta)]^+ \psi(t)dt \tag{7.55}$$

which can be approximated with

$$V_{NNEG} = \sum_{k=1}^{\omega-\widetilde{x}+1} [{}_{t_{k-1}}p_{\widetilde{x}} - {}_{t_k}p_{\widetilde{x}}]\mathbb{E}^Q \left\{DF(t_k+\delta)[\mathfrak{B}(t_k+\delta) - H(t_k+\delta)]\right. \tag{7.56}$$

7.4.2 NNEG Pricing by Actuarial Matching

In the US, the NNEG premium is often calculated as a combination of an upfront premium π_0 and a running premium π_a

$$V_{premium} = \pi_0 H(0) + \sum_{k=1}^{\omega-\widetilde{x}} \pi_a \Delta_k Q_S(t_k)\mathbb{E}^Q\left\{DF(t_k)K_{t_k}\right\} \tag{7.57}$$

such that $V_{premium} = V_{NNEG}$. A more precise approach should take into consideration the pro-rata premium payment that the insurer (seller of NNEG) should receive for the fractional period of time during the termination year and for which NNEG cover is still provided. This leads to the premium

$$\begin{aligned} V_{premium} &= \pi_0 H(0) + \sum_{k=1}^{\omega-\widetilde{x}} \pi_a \Delta_k Q_S(t_k) \mathbb{E}^Q \left\{ DF(t_k) K_{t_k} \right\} \\ &\quad + \int_0^{\omega-\widetilde{x}} \pi_a (\lfloor t \rfloor - t + 1) DF(t) \psi(t) dt \end{aligned} \tag{7.58}$$

where $\lfloor t \rfloor = \max\{t_k | t_k \leq t\}$.

The actuarial matching approach will determine the periodic[6] premium π_a by matching (7.56) with (7.57) (or 7.58 if termination premium is included contractually).

One should notice that this procedure is feasible only after fixing π_0.

The running premium reflecting the NNEG cover can be calculated as

$$\pi_a = \frac{\sum_{k=1}^{\omega-\widetilde{x}+1} [{}_{t_{k-1}}p_{\widetilde{x}} - {}_{t_k}p_{\widetilde{x}}] \mathbb{E}^Q \left(DF(t_k+\delta)[K_{t_k+\delta} - H(t_k+\delta)]^+ \right) - \pi}{\sum_{k=1}^{\omega-\widetilde{x}} {}_{t_k}p_{\widetilde{x}} \mathbb{E}^Q \left(DF(t_k) K_{t_k} \Delta_k \right)} \tag{7.59}$$

Since this running premium is a positive quantity, there is an upper boundary for the upfront cost given by

$$\pi_0 < \frac{1}{H(0)} \sum_{k=1}^{\omega-\widetilde{x}+1} [{}_{t_{k-1}}p_{\widetilde{x}} - {}_{t_k}p_{\widetilde{x}}] \mathbb{E}^Q \left(DF(t_k+\delta)[K_{t_k+\delta} - H(t_k+\delta)]^+ \right) \tag{7.60}$$

The difficult part in all the calculations presented above is, for any given time $t > 0$, the computation of $\mathbb{E}^Q\left(DF(t)[K_t - H(t)]^+\right)$, representing the NNEG market valuation and this is the focus in the remainder of the paper.

[6]This is an annualised rate.

7.4.3 Equilibrium versus No-arbitrage Models for Pricing Property Options

Real-estate derivatives represent a viable solution for managing real-estate price risk in developed economies (Case, Shiller, and Weiss, 1993; Shiller, 2008; Tunaru, 2017). One reason for the currently underdeveloped real-estate derivatives markets is the lack of reliable models that can be used for pricing and hedging strategies. Models on pricing property derivatives published in finance and economics literature can be classified into equilibrium models, see Cao and Wei (2010) and no-arbitrage models such as those proposed by (Bjork and Clapham, 2002; Fabozzi et al., 2012).[7]

The modelling approach to house price derivatives described in Cao and Wei (2010) has not been discussed in the reverse mortgages literature in relation to the NNEG valuation. Their modelling approach produces analytical solutions that are appealing for computational reasons. We show that an equilibrium model for pricing property derivatives proposed by Cao and Wei (2010) does not always satisfy the put-call parity for European options, which is a model-free condition in financial markets. This equilibrium model has the advantage to lead to analytical formulae for European options on a real-estate index. However, we argue that the formulae implied by the model should be used with care since they do not satisfy the put-call parity and therefore allow for arbitrage. One instance when this model becomes useful is when temporary absence of market prices for the real-estate derivatives occurs. Then a marked-to-model valuation would be the only valuation alternative and arbitrage would be impossible due to illiquidity of options on house prices.

7.4.3.1 Cao-Wei Model

Starting with Lucas' model, Cao and Wei (2010) considered a continuous-time economy with a financial market and a housing market. They assumed that any economic agent can trade in a single risky stock, pure discount bonds and a finite number of other contingent claims on either the risky stock, the pure discount bond or on the price of the house. They also assumed the existence of an aggregated dividend stream in the economy $\{\delta_t\}_{t\geq 0}$ and that the risk-free bond and the contingent claims are all in zero net supply.

The equilibrium model assumes that at time zero the market representative agent has one share of the market portfolio X and one house H, while the agent has a fixed working life up to time T and a post-retirement life span up to $T^* > T$. The agent receives a constant salary y per unit of time.

In addition, the dividend stream $\{\delta_t\}_{t\geq 0}$ and the housing value $\{H(t)\}_{t\geq 0}$ are considered exogenous Markov processes. For a financial asset with a price X_t, generating dividends q_t, the cum-dividend vector of dividends is given by $D_t \equiv \int_0^t q_u du$. The agent will finance consumption using the salary income and a continuous trading strategy $\{\theta_t\}_{t\geq 0}$, where

$\theta_t = (\theta_t^s, \theta_t^B, \theta_t^{x'})$ denotes the market portfolio, the discount bond and other contingent claims, respectively.

The aim of the Cao-Wei model is to optimise the present value of the agent's expected utility from the pre-retirement consumption and the post-retirement wealth, under the assumption that the agent will not sell the house until time T. The optimisation problem can be formulated as follows

[7] A recent review of modelling approaches in this area can be found in (Tunaru, 2017).

$$\max_{(c_t,\theta_t)} \mathbb{E}\left[\int_0^T e^{-\phi t} U(c_t)dt + e^{-\phi T} U(W_T)\right]$$

subject to

$$\begin{cases} H(t) + \theta_t X_t + \int_0^t (c_u - y_u)du = \theta_0 X_0 + H(0) + \int_0^t \theta_u dD_u + \int_0^t \theta_u d\text{…} \\ W_T = H_T + \theta_T X_T \end{cases}$$

where U denotes the local utility function of consumption, ϕ is the rate of time preference, c_t is the consumption rate measured in dollars per unit of time and W_T is the final wealth at time T. Cao and Wei (2010) show that the solution to this problem is

$$X_t = \frac{e^{-\phi(T-t)}}{U_c(\delta_t)} \mathbb{E}_t \left(\int_t^T U_c(\delta_u) dD_u\right) \tag{7.61}$$

for any $t \in (0, T)$, where $\mathbb{E}_t$ is the conditional expectation at time *t*, and U_c is the partial derivative as indicated by the subscript.

The equilibrium European call and put option prices for maturity *T* and strike price *K* are determined by Cao and Wei as

$$call_t(H_t, K, T) = e^{-\phi(T-t)}\mathbb{E}_t\left(\frac{U_c(\delta_T)}{U_c(\delta_t)}\max\left(H_T - K, 0\right)\right) \tag{7.62}$$

$$put_t(H_t, K, T) = e^{-\phi(T-t)}\mathbb{E}_t\left(\frac{U_c(\delta_T)}{U_c(\delta_t)}\max\left(K - H_T, 0\right)\right) \tag{7.63}$$

The solutions are too general for real applications and in order to get closed-form solutions, three more assumptions were made. First, the utility function is assumed to be the CRRA function $U(c_t) = \frac{c_t^{1-\gamma}}{(1-\gamma)}$. Secondly, the aggregate dividend process follows the geometric mean-reverting process

$$d\delta_t = (\mu_\delta - a_\delta \ln \delta_t)\delta_t dt + \sigma_\delta \delta_t dZ_t^\delta \tag{7.64}$$

Thirdly, the housing index process follows a geometric Brownian motion

$$dH_t = \mu_H H_t dt + \sigma_H H_t dZ_t^H \tag{7.65}$$

The noise Wiener process Z^H and Z^δ have correlation coefficient ρ.

With these extra assumptions, (Cao and Wei, 2010) derived closed form solutions for the equilibrium price of a zero coupon bond

$$B_t(T) = e^{-\phi(T-t)}\mathbb{E}_t\left(\frac{U_c(\delta_T)}{U_c(\delta_t)}\right)$$

Furthermore, they derive closed-formulae for the yield-to-maturity $R(t,T)$ and the instantaneous risk-free rate (the short rate) $r_t = R(t,t)$:

$$\begin{aligned} R(t,T) &= \phi - \frac{1}{2}\gamma^2\sigma_\delta^2\frac{1-e^{-2a_\delta(T-t)}}{2a_\delta(T-t)} \\ &+ \gamma\left(\mu_\delta - \frac{1}{2}\sigma_\delta^2 - a_\delta \ln \delta_t\right)\frac{1-e^{-a_\delta(T-t)}}{a_\delta(T-t)} \end{aligned} \tag{7.66}$$

$$r_t = \phi - \frac{1}{2}\gamma^2\sigma_\delta^2 + \gamma\left(\mu_\delta - \frac{1}{2}\sigma_\delta^2 - a_\delta \ln \delta_t\right) \tag{7.67}$$

One major obstacle in using this model is that the aggregate dividend process and the risk-aversion parameter of the representative agent are not observable. (Cao and Wei, 2010) advocate that r_t is observable and given the linear relationship between $\ln\delta_t$ and the endogenised interest rate r_t, they infer the parameters for the aggregate dividend process from the observed interest rates.

Denoting for simplicity

$$dr_t = a_r(b_r - r_t)dt + \sigma_r dZ_t^r$$

(7.68)

by identification with the corresponding quantities in (7.66) they obtain

$$a_r = a_\delta, \;\; b_r = \phi - \frac{1}{2}\left(\frac{\sigma_r}{a_r}\right)^2, \;\; \sigma_r = \gamma a_\delta \sigma_\delta, \;\; \rho_{rH} = -\rho$$

Under the modelling assumptions above, the final distribution for European contingent claims is known in closed form. (Cao and Wei, 2010) apply this to derive the equilibrium prices of the forward contract, and European call and put options as follows

$$F_t(H_t, T) = H_t \exp\left(\mu_H(T-t) + \rho_{rH}\sigma_H \frac{\sigma_r}{a_r^2}(1 - e^{-a_r(T-t)})\right)$$

(7.69)

$$call_t(H_t, K, T) = C_{BS}(F_t(H_t, T), T-t, K; R(t,T), R(t,T), \sigma_H)$$

(7.70)

$$put_t(H_t, K, T) = P_{BS}(F_t(H_t, T), T-t, K; R(t,T), R(t,T), \sigma_H)$$

(7.71)

where $C_{BS}(S_t, \tau, K; r, q, \sigma) = S_t e^{-qT} N(d_1) - K e^{-r\tau} N(d_2)$ is the usual Black-Scholes formula, likewise for P_{BS}.

The issue we raise is that this market is inherently incomplete and therefore the pricing measure for contingent claims on the housing price H_t is not unique. Furthermore, the economy may not always be in an equilibrium state, as the recent subprime crisis has demonstrated. For practical purposes, the equations describing the model needs to be risk-neutralised. Since the equilibrium price of a zero-coupon bond is given by the equation

$$dB_t = [r_t + \gamma^2 \sigma_\delta^2 (1 - e^{-a_\delta (T-t)})] B_t dt + \gamma \sigma_\delta (1 - e^{-a_\delta (T-t)}) b_T dZ_t^\delta$$

the risk-neutral version of this equation[8] is

$$dB_t = r_t b_t dt + \gamma \sigma_\delta (1 - e^{-a_\delta (T-t)}) b_T dZ_t^{\delta,q}$$

[8]That is the equation in continuous time under a risk-neutral measure Q associated with the transformation $Z_t^{\delta,Q} = \gamma \sigma_\delta t + Z_t^\delta$
Then the risk-neutral process for the risk-free rate is

$$dr_t = a_r \left[b_r + \left(\frac{\sigma_r}{a_r} \right)^2 - r_t \right] dt + \sigma_r dZ_t^{r,Q} \tag{7.72}$$

Cao and Wei (2010) observed that the only difference between the equilibrium process and the risk-neutral process in their model relates to the long-run mean. Moreover, they proved that the yield-to-maturity in the risk-neutral world is the same as its equilibrium version.

The risk-neutral process for the house index is considered to be

$$dH_t^Q = r_t H_t^Q + \sigma_H dZ_t^{H,Q}$$

(7.73)

which is in line with the assumption that the housing index is considered to be a tradable asset. In this risk-neutral world, Cao and Wei (2010) derive the housing index derivatives prices to be given by

$$\begin{aligned} F_t^Q(H_t, T) \quad & = H_t e^{R(t,T)(T-t)} \\ call_t^Q(H_t, K, T) \quad & = C_{BS}(H_t, T-t, K; R(t,T),, \sigma_H^*) \\ put_t^Q(H_t, K, T) \quad & = P_{BS}(H_t, T-t, K; R(t,T),, \sigma_H^*) \end{aligned}$$

$$\sigma_H^* = \sqrt{\sigma_H^2 + 2\rho_{rH}\sigma_H \frac{\sigma_r}{a_r}\left(1 - \frac{1-e^{-a_r(T-t)}}{a_r(T-t)}\right) + \frac{\sigma_r^2}{a_r^2}\left(1 - 2\frac{1-e^{-a_r(}}{a_r(T-}\right.}$$

7.4.3.2 A Numerical Counterexample

Here we discuss how to price a forward contract and an ATM European call and put options on the housing index following (Cao and Wei, 2010). The calculation is done as of December 2007 for one, two and three year maturity contracts. The composite housing price index for December 2007 was reported as 200.77 and the short rate at the time was calculated as

$r = 0.2478\%$. The parameters estimated by maximum likelihood in Cao and Wei (2010) using the data between January 1998 and December 2007 are described in Table 7.2.

TABLE 7.2 The estimated parameters in Table IV from Cao and Wei (2010), for the composite housing index H_t and the interest rate r_t processes.

parameter	μ_H	σ_H	a_r	b_r	σ_r	ρ_{rH}
ML Estimate	0.0559	0.0252	0.468	0.0428	0.002	0.084
Standard Error	0.544	0.113	0.249	0.010	0.000	0.066

Notes: The monthly composite housing index is retrieved from www2.standardpoors.com, whereas the three-month T-bill rates are downloaded from the Federal Reserve Board's website. All parameters are estimated using the maximum likelihood method over the period Jan 1987-Dec 2007. The numbers on third row are standard errors.

The equilibrium option and forward prices in Table 7.3 are different, as expected, from the corresponding risk-neutral prices. However, the put-call parity for the equilibrium prices fails for all three maturities, with the difference between the call side and the put side being equal to 2.8841, 5.0672, and 7.2884 for one, two and three years. Hence, the equilibrium prices are a lot smaller than the risk-neutral prices implying that equilibrium based NNEG valuations would be in our example lower than the risk-neutral or market consistent valuations, which is equivalent to saying that the ERM values under the equilibrium model would be much higher than the ERM values under the risk-neutral approach.

TABLE 7.3 Risk-Neutral and equilibrium forward and European at-the-money options prices using the formulae in Cao and Wei (2010).

T	Forward Risk-Neutral	Forward Equilibrium	Call Risk-Neutral	Call Equilibrium	Put Risk-Neutral	Put Equilibrium
1	209.2127	212.307	8.2109	11.0949	0.1088	0.0235
2	218.9753	224.5067	16.6945	21.7649	0.0028	0.0017
3	229.0908	237.4072	24.8197	32.1081	1.23E-04	2.34E-06

Notes: All options are at the money.

First, we remark that the put-call parity is verified for the risk-neutral prices while the equilibrium prices do not obey the put-call parity. Moreover, the put-call parity discrepancy increases with maturity. The fact that the equilibrium prices for the European call and put do not verify the put call parity is not only a numerical peculiarity for the parameter estimates utilised in the example. The formulae (7.70) and (7.71) imply that

$$call_t(H_t, K, T) - put_t(H_t, K, T) = e^{-R(t,T)(T-t)}[F_t(H_t, T) - K] \tag{7.74}$$

Second, in order for the put-call parity to be verified for the equilibrium call and put prices given by the formulae in Cao and Wei (2010), the following identity must hold (recall that $K = H_t$)

$$e^{-R(t,T)(T-t)}[F_t(H_t, T) - K] = H_t - Ke^{-R(t,T)(T-t)}$$

leading to

$$F_t(H_t, T) = H_t e^{R(t,T)(T-t)} \tag{7.75}$$

Combining this formula with formula (7.69) implies that

$$R(t, T) = \mu_H(T-t) + \rho_{rH}\sigma_H \frac{\sigma_r}{a_r^2}(1 - e^{-a_r(T-t)})$$

which is in contradiction with the yield-to-maturity formula (7.66). Hence, the equilibrium prices for European call and put in (Cao and Wei, 2010) do not verify the put-call parity. This shows that a market consistent valuation of NNEG cannot be achieved using this equilibrium model.

7.4.4 Problems and Solutions for NNEG Valuation

In this section we review critically the main models applied to extract the NNEG valuation and we highlight the main difficulties that these models pose. In addition, we derive model-free a lower bound for the NNEG value of an ERM.

7.4.4.1 Black-Scholes formula

Under a set of simplifying assumptions the NNEG can be computed with a Black-Scholes type formula. The first assumption is this case that risk-free discounting is a constant rate r and the balance compounding is at a fixed rate R. Hence, for any $t > 0$, it follows that $DF(t) = e^{-rt}$ and $A(0, t) = e^{Rt}$. Secondly, the house price process is assumed to have a dynamics given by a geometric Brownian motion. Hence, under a risk-neutral pricing measure

$$dH(t) = (r - g)H(t)dt + \sigma H(t)dW(t)$$

where g is a rental yield similar to the dividend yield for equity stock, that regulators argue that it should be taken into account, see (Prudential Regulation Authority, 2018) for the UK market.

For a given maturity T, we can calculate at time 0 the NNEG put option component value for year T as

$$\mathbb{E}^Q\left(DF(T)[K_T - H(T)]^+\right) = K_0 e^{(R-r)T} N(-d_2) - H(0)e^{-gT} N(-d_1) \tag{7.76}$$

where

$$d_1 = \frac{\ln\left(\frac{H(0)}{K_0}\right) + \left(r - g - R + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

While the Black-Scholes formula provides a very convenient computational tool, it also has several shortfalls. The first one relates to the geometric Brownian motion as a data generating process. This would be inadequate for house prices because those exhibit serial correlation. Secondly, the variance of house price returns under the GBM is increasing with time, implying that the longer the horizon the larger the variance of house price returns. Nevertheless, there is empirical evidence that house price returns are negatively correlated at long-term horizon, see Case and Shiller (1989), Chinloy, Cho, and Megbolugbe (1997), Chen, Chang, Lin, and Shyu (2010), Li et al. (2010) and Tunaru (2017), implying a mean-reversion effect that would dampen the volatility long-term. Last but not least, house prices lack granularity and fungibility as an asset and furthermore, they cannot be sold short.

7.4.4.2 *No-arbitrage Continuous Time Models*

The assumption of a geometric Brownian motion (GBM) as the data generating process for the house price process has a long tradition in the ERM literature, see Szymanoski (1994); Chinloy and Megbolugbe (1994a); Kau and Keenan (1995); Wang et al. (2008); Huang et al. (2011); Ji et al. (2012). The GBM is clearly not suitable for house prices, being unable to reproduce future price paths that have similar stylised features as the market house price series. In particular, the GBM returns are serially independent, the conditional volatility is constant and the future volatility grows infinitely with the square root of maturity.

In an attempt to circumvent this mismatch, the next steps in the evolution of continuous-time models that can be applied to house price contingent claims are the jump-diffusion models Chen et al. (2010); Lee et al. (2012) and the mean-reverting models discussed by Fabozzi et al. (2012) and Tunaru (2017). One inconvenience of these models is that the low frequency data characteristic of housing markets may impair the parameter estimation. Wang et al. (2014) aimed for a model capable to generate housing price jumps, hence they selected exponential Lévy processes for house prices. These models are known not to be able to produce the mean-reverting returns paths observed in house prices.

7.4.4.3 *Discrete-time Models*

A strand of NNEG literature considers macroeconomic factors as the major factors determining house prices. Chang et al. (2012) applied a multivariate linear regression model aiming to link house prices with fundamental macroeconomic factors. An important development in this direction is the approach proposed by Sherris and Sun (2010) and refined by Alai et al. (2014) whereby a VAR model jointly covers interest rates, house prices, rental yields and GDP and then pricing the NNEG risk follows a stochastic discount approach.

One of the most promising approaches for computing the value of NNEG is based on the AR(I)MA-GARCH family of models. This line of modelling has been developed by Chen et al. (2010), Li et al. (2010), Lee et al. (2012), Kogure et al. (2014) and it has been discussed and applied in Tunaru and Quaye (2019). In particular the ARMA-EGARCH model seems to perform very well in capturing the stylised features of house price indices. A combination of the macro and the GARCH volatility was proposed by Kim and Li (2017) who applied a VAR-DCC/GARCH model on the Korean market.

The main difficulty in applying these models in a NNEG context is to identify a suitable risk-neutral pricing measure process that is well calibrated over time.

7.4.4.4 *Statistical and Calibration Issues*

In general, parameter estimates of various models employed are presented in various papers without showing goodness-of-fit results for the fitted models. Moreover, comparative results are presented in the literature for only few selected scenarios. Overall, the conclusion regarding NNEG risk is that there is substantial model risk involved. Lee et al. (2012) found that an ARMA-GARCH model may produce higher[9] NNEG valuations than a jump-diffusion model.

If the timing of termination event is assumed to be known or conditionally constant (as in the actuarial matching approach) then pricing the NNEG put option depends only on the interest rate risk and housing price risk. In addition to the type of models described above one may consider the idea to use an equilibrium pricing approach as in Cao and Wei (2010). In times of crisis it may be difficult to use econometric or financial engineering models due to lack of available data for calibration. Equilibrium models have mainly theoretical underpinnings and they may be very useful in these situations. The class of equilibrium models, however, is larger than the class of no-arbitrage models, and it may allow arbitrage as demonstrated in the next section.

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[9]These results were based on selecting an ARMA(0,1)-GARCH(1,1) model, which is a bit surprising since this model implies that there is no serial correlation in house price returns.

One important problem emerging in this area of NNEG valuation is selecting the risk-neutral pricing measure. There are various competing alternatives. Li and Ng (2010) developed the canonical valuation method that is mainly based on minimising the Kullback-Leibler information criterion, see also Kogure et al. (2014) for a similar approach and this method has been followed by Kim and Li (2017) for pricing non-recourse protection in reverse mortgages. The main risk-neutralisation methodology emerging in the area of NNEG valuation is focused on the conditional Esscher transform going back to the seminal papers by Gerber and Shiu (1994) and Buhlmann et al. (1996) and applied more recently to ERMs by Li et al. (2010). This technique of selecting an equivalent martingale measure has been investigated, together with the extended Girsanov principle Elliot and Madan (1998), by Badescu and Kulperger (2008) who also proposed a nonparametric density estimator that seems to work well for pricing stock options but that has not been exploited extensively in the NNEG house price put option area.

7.4.4.5 Arbitrage-Free Bounds if Forwards on House Prices Existed

Ideally, the valuation of the NNEG embedded option should employ a market consistent approach based on risk-neutrality pricing. Recently, Dowd et al. (2019) and Buckner and Dowd (2018) argue that the NNEG valuation should be based on the widely used Black (1976) model. Unfortunately, in the case of UK there are no futures contracts traded on house prices, which invalidates this approach.[10] Their approach is still very useful as it highlights the importance of introducing a futures contract, so hedging the house price risk is more efficient and consequently many of the current difficult problems related to the market consistent valuation of the NNEG would disappear.

Given the current state of development of derivatives in housing markets, the best we can do at this moment in time is to rely on some model free no-arbitrage bounds that may help with market-wide risk-management. This section is motivated by the results derived in Syz and Vanini (2011), which we adjust slightly to our context. Let H_t be the value of a property asset or portfolio or index on a total return basis at time t, and consider that there is a house price forward contract[11] with maturity T traded with a forward price $F_t(T)$.

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[10] In Black (1976) the entire modelling is done in futures prices space, and on that basis forward prices are derived.

[11]There is evidence of forward contracts traded over-the-counter in the UK on the Halifax house price index.

Suppose that r is the risk-free interest rate and the transaction costs are k_b or k_s for buying or selling a property respectively, defined for a one way transaction in percentage terms of the corresponding house prices. For a given investment horizon $\tau = T - t$, an investor may purchase a property portfolio with a total value H_t, funding this operation through borrowing the equivalent amount, including associated friction costs, $H_t(1 + k_b)$. Simultaneously, the investor enters a short forward position contingent on the same property portfolio for the amount of $F_t(T)(1 - k_s)$.

At the end of the investment horizon, the investor could sell the property portfolio for $H_T(1 - k_s)$, whilst also unwinding the short forward contract $F_t(T)(1 - k_s) - F_T(T)(1 - k_s)$ and paying back the loan with interest, $H_t(1 + k_b)e^{r\tau}$. Requiring no-arbitrage leads to the condition

$$F_t(T)(1 - k_s) - H_t(1 + k_b)e^{r\tau} \leq 0. \tag{7.77}$$

or equivalently

$$F_t(T) \leq \frac{(1 + k_b)}{(1 - k_s)} H(t) e^{r\tau}. \tag{7.78}$$

Denoting the “convenience yield” ρ_τ that is locking the equation

$$F_t(T) = H_t e^{(r+\rho_\tau)\tau}$$

(7.79)

we find by reverse engineering the upper arbitrage-free bound of the convenience yield

$$\rho_\tau^U = \frac{1}{\tau}\ln\frac{(1+k_b)}{(1-k_s)}$$

(7.80)

that can be mapped into an upper bound $F_t^U \equiv H_t e^{(r+\rho_\tau^U)\tau}$. Syz and Vanini (2011) showed how one can calibrate k_b, k_s to over-the-counter house price derivatives on the Halifax house price index. The values of these parameters can be also taken exogenously from market information. In the U.K. the stamp duty rate was 2% in 2018 for properties (or portions) between £125,000-250,000, 5% for properties (or portion) between £250,001-925,000, 10% for properties (or portion) between £925,001-1.5mil. and 12% above that. As an example[12] suppose that the NNEG calculation is carried out with respect to a house collateral valued at £275,000. The stamp duty calculation is done in stages: 0% on the first £125,000, 2% on the next £125,000 = £2,500, and 5% on the final £25,000 = £1,250, leading to a total of £3,750 that implies $k_b = 1.3636\%$. For the year 2018, the average real estate agents' commission in the UK was between 1% and 3% (including VAT), so an average value of $k_s = 2\%$ is representative.

Under the assumption of a constant risk-free rate *r*, the forward prices and futures prices are equal, with the latter the risk-neutral expectation of the future underlying asset value H_T at the horizon *T*, Bjork (2009). Hence, the upper boundary discussed above is a model-free no-arbitrage upper boundary for the future house price.

[12]The example is based on approximate calculations since legal costs, moving house costs, and other hidden costs are not represented.

Since

$$\mathbb{E}_t^Q[H_T] \leq F_t^U \tag{7.81}$$

one can see that, for any strike price *K*, we can derive the following inequalities:

$$\begin{aligned} \mathbb{E}_t^Q[K - H_T] &\geq K - F_t^U \\ \mathbb{E}_t^Q[(K - H_T)^+] &\geq K - F_t^U \\ \mathbb{E}_t^Q[(K - H_T)^+] &\geq (K - F_t^U)^+ \\ e^{-rT}\mathbb{E}_t^Q[(K - H_T)^+] &\geq e^{-rT}(K - F_t^U)^+ \end{aligned}$$

Recalling the general formula (7.56), we can apply the newly derived lower bound for all NNEG put options components with[13] $K = K_{t_k}$ and thus we obtain a lower bound for the ERM NNEG value

$$V_{NNEG} \geq \sum_{k=1}^{\omega-\widetilde{x}+1} \left[{}_{t_{k-1}}p_{\widetilde{x}} - {}_{t_k}p_{\widetilde{x}}\right] \left[\mathfrak{B}(t_k)e^{-rt_k} - \tfrac{(1+k_b)}{(1-k_s)} H_0\right]^+ \tag{7.82}$$

This result can be used to differentiate between the various NNEG valuation models, so any model that produces values outside this range can be eliminated.

7.5 SUMMARY

In this paper, we show that the GBM model recommended by the regulator in the UK produces much higher values of the NNEG when compared with a best fit ARMA-EGARCH model selected on the basis of forecasting house prices well. Utilising an inappropriate model in the context of reverse mortgage loan market may in the end stifle this market by imposing very high capital reserve requirements in insurers. This is very important since there is no diversification benefit for an insurer issuing RM loans, each loan being valued separately for NNEG calculations purposes. Furthermore, inflating the volatility parameter will automatically imply a high variance of house prices at long maturities for the GBM model, therefore impacting directly RMs loan characteristics for the younger borrowers who would benefit the most from the this new asset class.

The study also finds evidence to suggest that rental yield parameter is not the key driver of underlying house prices in UK. If the majority of house prices do not pay rents, and this is verified in the Office of National Statistics, it would be wrong to assume that all houses have prices driven by rents. The proportion may be different from country to country but there is no known country where all houses generate a rental income. An overestimation of the rental yield induces downward trending house prices in the long run that ultimately will inflate the NNEG values.

While the RMs may offer a viable solution to long-term care and pension boosting to the elderly generation in most developed economies, there is a general lack of development of this market world-wide. One possible explanation is that the interaction between the consumers, the insurers and the regulator needs to be improved and the capital should work more efficiently.

[13]We consider that $\delta = 0$ for simplicity of exposition, without great reduction of generality.

In this chapter, we derive an upper boundary for the upfront risk premia charged on the reverse mortgage products in the US. In addition, we discuss the pitfalls of calculating market consistent valuations of the NNEG for the ERMs in the UK and we identify a model-free no-arbitrage lower boundary for the NNEG value.

We show that an equilibrium model proposed for pricing real-estate derivatives is incompatible with the put-call parity that should hold model-free. While not all equilibrium models are arbitrage-free, equilibrium models can still be useful when there is no liquid market to enforce a no-arbitrage approach. Since put-call parity is derived model-free, our example points out to a very important question for derivatives markets: should equilibrium prices for vanilla derivatives satisfy the put-call parity? This condition has not been yet investigated in relation to the derivation of equilibrium models.

The NNEG valuation combines life-insurance risk with non-life insurance risk. Under the current Solvency II set of regulations, it is not possible to take advantage of diversification in a large ERM loan portfolio, as the NNEG value is computed and applied on a loan-by-loan basis. Going forward, it would be useful to expand the framework described in Asimit, Badescu, Haberman, and Kim (2016) to the NNEG space, such that the liability transfers are optimised with respect to the Technical Provisions and Minimum Capital Requirement at the insurer group level.

Cho, Hanewald, and Sherris (2013) proposed a VAR model to simulate economic scenarios and determine the stochastic discount factors that can be applied directly for pricing the NNEG. They provide evidence that lump-sum reverse mortgages are generally more profitable and less risk-based capital intensive than ERP loans using income streams. Another comparative study of product design looking at risk versus profitability is Lee, Kung, and Liu (2018). They advocate that Solvency II capital requirement may lead to a reduction in the loan-to-value ratio and in order for the market to survive some government back-up may be required. Furthermore, they also confirm the lenders prefer the lump-sum products. On the other hand, Cocco and Lopes (2014) proved that adding to the ERP loan an insurance against a forced home sale is Pareto improving and can boost the demand for the ERP loans due to reduced risk.

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CHAPTER **8**

Estimating the Risk Premium

DOI: 10.1201/9781003340270-8

8.1 INTRODUCTION

THE PRICE OF RISK, or risk premium, is an important component when it comes to pricing, funding, and reserving for ERM contracts. We identify risk premium associated with house price growth, the NNEG valuation and with the equity release loan itself and we discuss briefly other ancillary types of risk premium that may come into focus when dealing with ERPs, such as product novelty or liquidity, driving product valuations. The impact of these different facets of risk premiums in ERM pricing and risk management may differ depending borrower- or product-specific features. Other issues like market concerns about accuracy of variables that trigger the NNEG clause, and difficulties insurers face when calibrating ERMs in their asset-liability management decisions can drive risk premiums.

Estimating risk premium in the context of ERPs is not a straightforward task given the multifaceted character to many risks involved. In addition, risk premium is often computed by fixing some “other” parameters to some values that may be relevant but in doing so the model risk or more precisely parameter estimation risk is largely ignored.

First, we focus on the calculation of the house price risk premium. This part should be of interest to more researchers, given that housing portfolios represent a substantial portion of total wealth in modern economies. In Section 8.3, we discuss the calculation of NNEG risk premium, pointing out that there is a substantial difference between calculating NNEG under risk-neutral or market consistent measure and calculations done under the physical measure. Then, in Section 8.4, we follow the analysis with the ERP risk premium, which essentially combines the previous two. A more modern type of risk premium is novelty premium or venturing premium which gives some insight into the business opportunity premium. This is a less known concept and it is only briefly discussed in Section 8.5. Last section provides a summary of the main ideas and it also contains some recommended further readings for those particularly interested in this topic.

8.2 HOUSE PRICE RISK PREMIUM

It is well known that residential real-estate represents a large proportion of wealth in most developed economies. Surprisingly, there is not a lot of empirical work on the house price risk premium, one possible explanation being the lack of data on returns on investments in the housing sector. Some notable exceptions are Ibbotson and Siegel (1984), Case and Shiller (1989) and Goetzmann and Ibbotson (1990) for the U.S. real estate market. The main conclusion of these studies is that the returns for this asset class are possibly superior or similar to other asset classes. Interestingly, diversified portfolios of housing investments could produce average returns with much higher Sharpe ratios due to lower volatilities than equity portfolios. There is evidence that since the 1980s, the U.S. real estate portfolios gave a large risk premium (defined as the excess over the risk-free rate or government yield rate) of about 6% annually, but only being exposed to a fraction of the volatility of the stock market, (Shiller, 2003).

Jorda, Schularick, and Taylor (2019) analysed a new database covering the U.S. and 15 other advanced economies, between 1870 to 2015, for total returns on house prices and common stock. The database is described in full in Jorda, Knoll, Kuvshinov, Schularick, and Taylor (2019) and it includes data for the following countries Australia, Belgium, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. They show that house price returns are comparable to those of equities, but the house price returns have lower covariance with consumption growth than equities. The conclusions are verified a range of horizons. This means that the implied risk aversion parameters for housing wealth are much larger than those for equities alone, by a factor of 2 or more.

The house price risk premium in the ERP context concerns the loan issuer's compensation charged for the collateral-effect inherited from house price volatility. Behaviour of stakeholder in ERM markets would more than likely feature a strong house price risk premium. Different facets of risk management processes could be developed to gauge the effect of property volatility and the corresponding risk premium. The loan issuers may include graduated payments indexed to house price growth rates. A direct approach to account for house price risk due to the positive relationship between house price volatility and ERM/NNEG price. All things equal, younger[1] borrowers will be impacted more by the house price risk premium. The calculation of house price risk premium would also be directly affected by the house-price model embedded in the valuation process. Using an aggregate house price index would result in risk-premiums which are significantly different from the models directly linked to the underlying property for example repeat-sales pricing model. Irrespective on the method adopted, the focus should be on gauging impact of sustained high levels of house price volatility. The ability of the data-generating process to produce realistic property crash will serve well when calculating risk premium associated with house price growth risk.

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[1] Younger borrowers include those within lower age range profile i.e. 55 - 60 years who may potentially have long maturity contracts.

Risk premium for house prices can be conceptualised in at least two different ways. First, one can think of the house price risk premium as the difference between the expectation of house price for some future maturity T under the risk-neutral measure $\mathbb{Q}$ and the expectation of the house price for the same maturity under the real-world or physical measure $\mathbb{P}$. In other words, a measure of risk premium is

$$HPRP_1(T) = E^{\mathbb{Q}}(H_T) - E^{\mathbb{P}}(H_T). \tag{8.1}$$

A second way that house price risk premium is defined in the literature is analogy with equity risk premium. This would be equal with the difference between rate of return produced by investing into a house (or in our case house price index) and the rate of return that the same investor can get by parking the same investment into a savings account. In the literature the latter rate is taken often as the risk-free rate but in our context is not facile to identify risk-free rates for very long maturities. In addition, post global financial crisis, other rates could be involved that have a funding rate character, such as government treasury rate, OIS rate, repo rate and so on. It should also be noted that this risk premium is defined in the returns space. Another pitfall that is rarely discussed for this second definition is the inclusion and measurement of rental yield. Even for house price indexes, rental yield is not directly observable and, as discussed in previous chapters, is not an easy exercise. Ignoring the rental yield, the house price premium is then computed as

$$HPRP_2(T) = \frac{H_T - H_0}{H_0} - r_{[0,T]} \times T. \tag{8.2}$$

If rental yield $g = \frac{rental\ income}{H_0}$ is taken into account then the previous formula ought to be adjusted to

$$HPRP_3(T) = \frac{H_T - H_0}{H_0} + g \times T - r_{[0,T]} \times T. \tag{8.3}$$

Note that the rental yield and the risk-free rate are for the entire [0,*T]* period. In order to make the formula per year we could just transform it in an equivalent way as

$$HPRP_3(T) = \frac{1}{T}\frac{H_T - H_0}{H_0} + g - r_{[0,T]}.$$

(8.4)

The *realised* risk premium can be computed once H_T is finally known. From the point of view of time zero, the above formula can be used to determine the *expected* risk premium. The former is an *ex post* calculation whilst the latter is an *ex ante* calculation. Applying the properties of the expectation operator leads to

$$E(HPRP_3(T)) = \frac{1}{T}\frac{E(H_T) - H_0}{H_0} + g - r_{[0,T]}.$$

(8.5)

Thus, once we know $E(H_T)$ the excess premium could be easily calculated. For the ARMA-EGARCH model, because of the lack of analytical formulae for future house price H_T, the only possibility to calculate the expected excess premium $E(HPRP_3)$ is to use Monte Carlo simulations to determine $E(H_T)$.

For the GBM model there is a convenient shortcut bypassing simulations. Since it is known that

$$H_T = H_0 \exp\left((\mu - g - \frac{1}{2}\sigma^2)T + \sigma W_T\right)$$

(8.6)

it follows that

$$E(H_T) = \exp\{(\mu - g)T\}$$

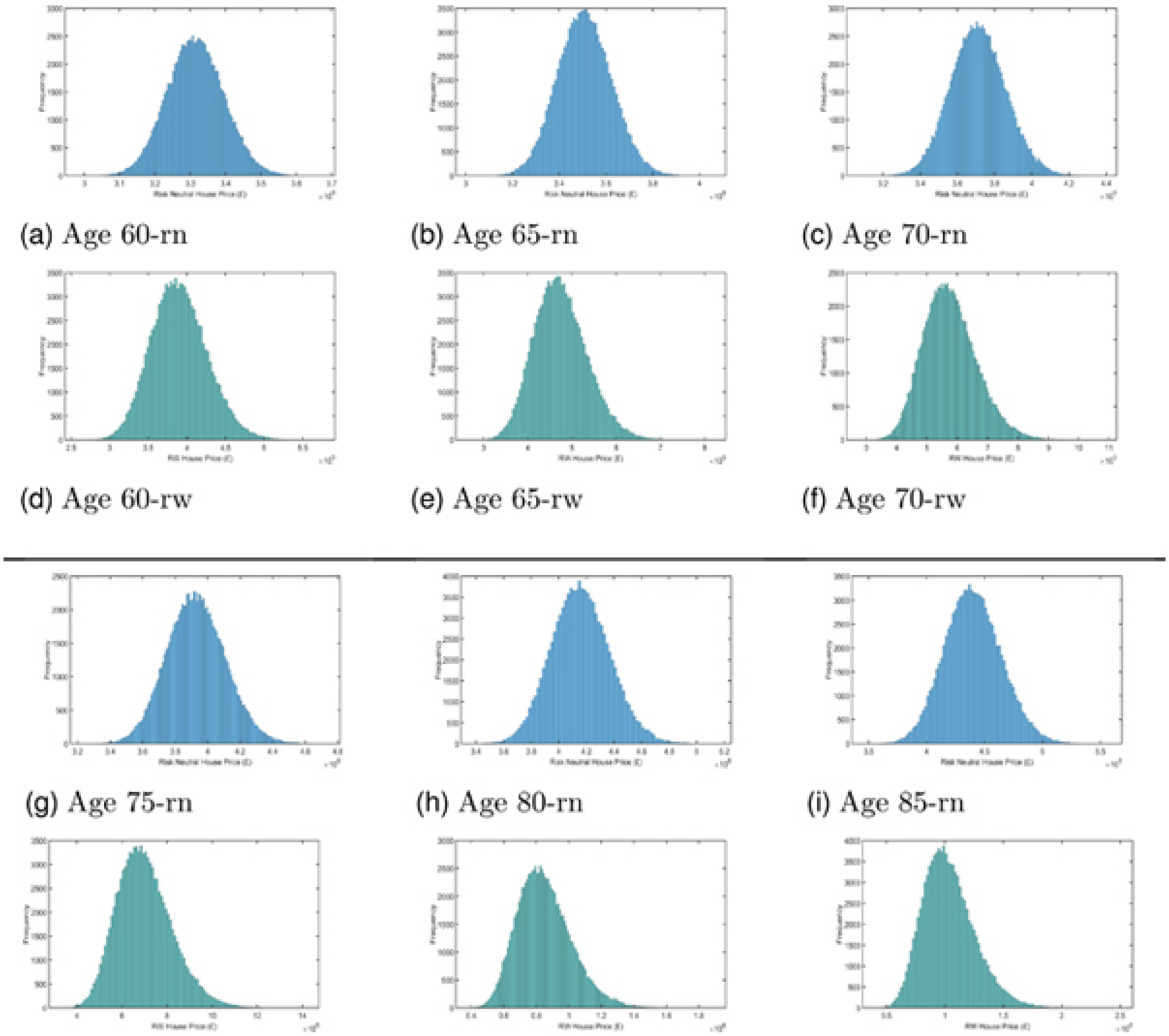
(8.7)

and then, for percentage returns,

$$E\left(\frac{H_T - H_0}{H_0}\right) = \frac{1}{H_0}E(H_T) - 1 = \frac{1}{H_0}\exp((\mu - g)T) - 1.$$

(8.8)

The first house price risk premium seems to be more relevant to the ERP calculus, so we shall start with this one first. Although we do not support the usage of GBM as a model for house prices, in Figure 8.1 we illustrate the differences between the house prices obtained by simulation under the risk-neutral measure and the real-world measure, respectively, for the sake of discussion. There are clear differences for all borrowers' ages, suggesting that the insurers using a real-world valuation mechanism may "benefit" from higher house price values expected in the future while the insurers employing risk-neutral methods have a more conservative view on house prices, under the same market parameters. The situation may be reversed if the volatility is increased and/or the risk-neutral drift exceeds the real-world drift. If the house prices go on a downward spiralling trend, with the same volatility, the risk-neutral drift stays the same while the real-world simulate pathways point downwards.



(a) Age 60-rn (b) Age 65-rn (c) Age 70-rn

(d) Age 60-rw (e) Age 65-rw (f) Age 70-rw

(g) Age 75-rn (h) Age 80-rn (i) Age 85-rn

(j) Age 75-rw (k) Age 80-rw (l) Age 85-rw

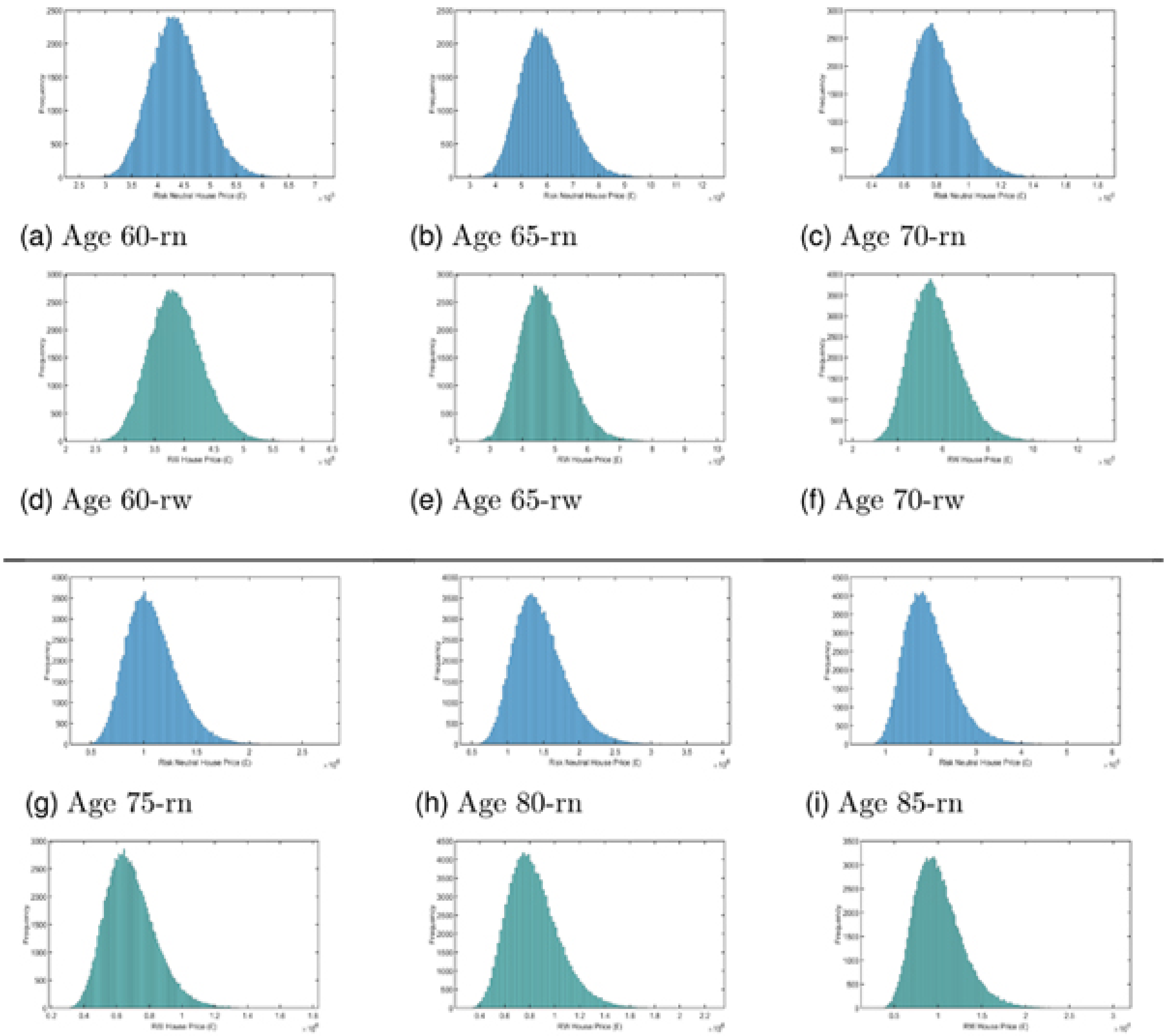
Age	60	65	70	75	80	85	90
Risk Premium (GBP)	18,112	33,923	50,892	69,174	88,939	110,250	133,069

Figure 8.1 Distribution of projected monthly house prices for UK under real-world and risk-neutral GBM

Notes: GBM-rw is the real-world GBM model and GBM-rn is the risk-neutral GBM, under baseline scenario $r = 4.98\%$, $g = 1\%$. We use the maximum likelihood estimates of μ and σ in Table 6.2 for the simulation exercise. There are 100,000 house price simulations.

For the given market conditions in UK described in the baseline scenario, there is some overlap between the risk-neutralised distribution of house prices and the real-world distribution of house prices for short horizon and then the two distributions separate, indicating that the house price risk premium increases with time.

We also repeat the simulation exercise for quarterly frequencies of house prices- recall that the model parameters would be different given that the historical data sample for house prices is longer. The results are illustrated in Figure 8.2. The positive risk premium is still present for all age categories.



(a) Age 60-rn (b) Age 65-rn (c) Age 70-rn

(d) Age 60-rw (e) Age 65-rw (f) Age 70-rw

(g) Age 75-rn (h) Age 80-rn (i) Age 85-rn

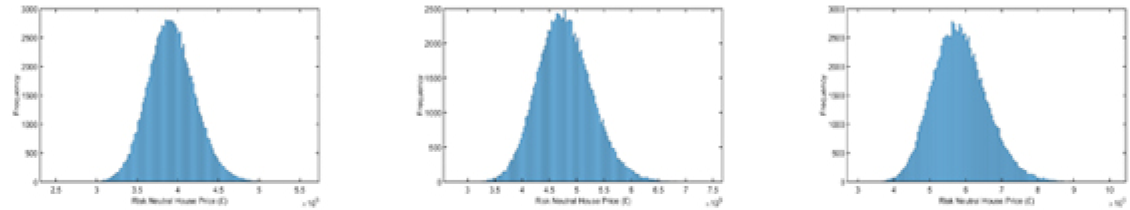
(j) Age 75-rw (k) Age 80-rw (l) Age 85-rw

Age	60	65	70	75	80	85	90
Risk Premium (GBP)	48,483	115,840	217,004	365,816	581,675	890,331	1,328,237

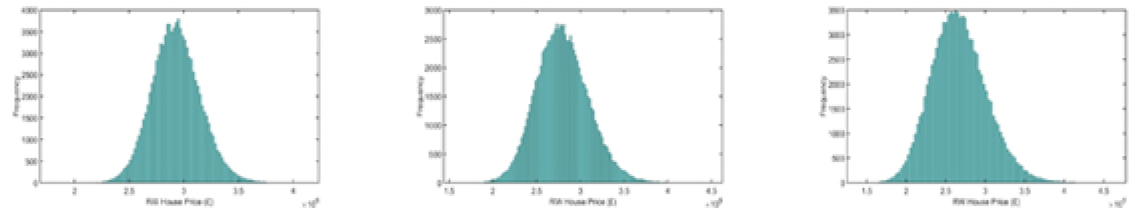
Figure 8.2 Distribution of projected quarterly house prices for UK under real-world and risk-neutral GBM

Notes: GBM-rw is the real-world GBM model and GBM-rn is the risk-neutral GBM, under baseline scenario $r = 4.98\%, g = 1\%$. We use the maximum likelihood estimates of μ and σ in Table 6.2 for the simulation exercise. There are 100,000 house price simulations.

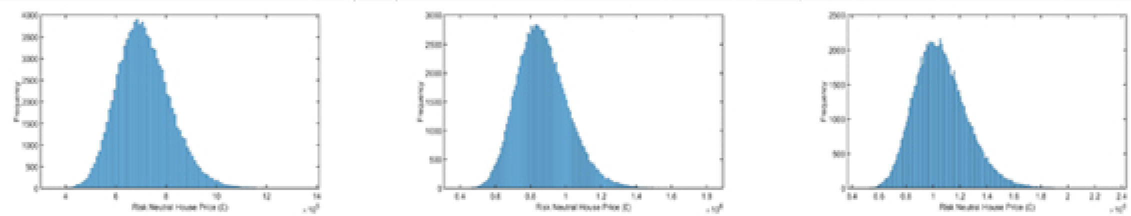
One way to consider the risk premium on ERM is to look at the difference in distributions for house prices under the real-world measure and under the risk-neutral measure, for the ARMA(4,3)-EGARCH(1,1) model, in Figure 8.3 (see also Figure A.8 in the Appendix) we illustrate these differences calculated at future time horizons defined by borrower's future age 60, 65, 70, 75 and 80, 85 respectively from the point of view of a 55-year-old. In other words house prices are calculated at future time horizons 5, 10, 15, 20, 25 and 30 years ahead. We also report in the attached table on the same figure the risk-premium on the *house prices* defined as the difference between the mean value under real-world distribution and the mean value under risk-neutral distribution. We can see that for the ARMA(4,3)-EGARCH(1,1) model, the risk premium as described by the Nationwide monthly series is increasing with the maturity horizon.



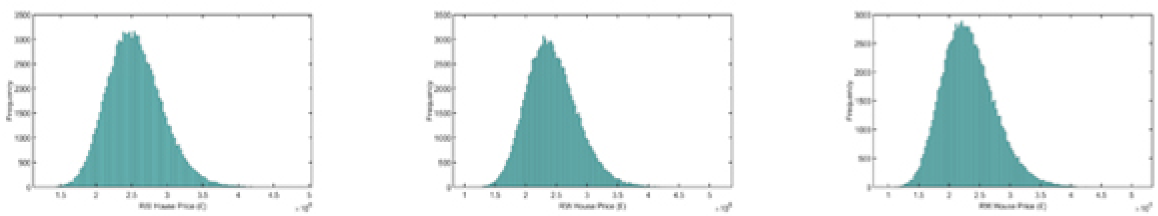
(a) Age 60-rn (b) Age 65-rn (c) Age 70-rn



(d) Age 60-rw (e) Age 65-rw (f) Age 70-rw



(g) Age 75-rn (h) Age 80-rn (i) Age 85-rn

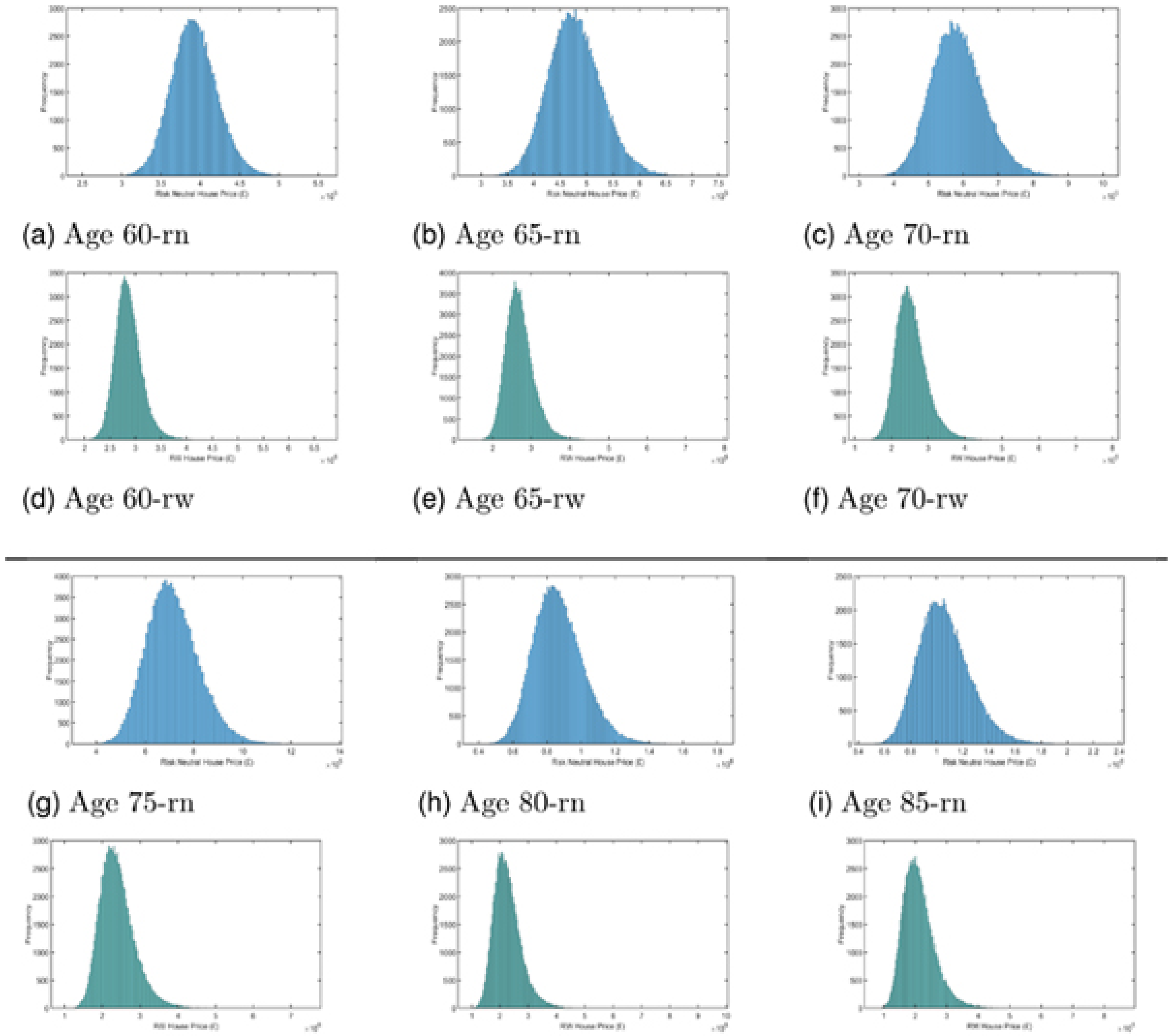


(j) Age 75-rw (k) Age 80-rw (l) Age 85-rw

Age	60	65	70	75	80	85	90
Risk Premium (GBP)	77,549	156,094	253,027	373,095	521,018	703,203	927,401

Figure 8.3 Distribution of projected monthly house prices for UK under real-world and risk-neutral ARMA-EGARCH

Notes: The real-world house price simulations denoted (-rw) have a green face-colour whereas the simulations from the risk-neutral world denoted (-rn) are blue-faced. We simulate 100,000 house price paths in this exercise.



(a) Age 60-rn (b) Age 65-rn (c) Age 70-rn

(d) Age 60-rw (e) Age 65-rw (f) Age 70-rw

(g) Age 75-rn (h) Age 80-rn (i) Age 85-rn

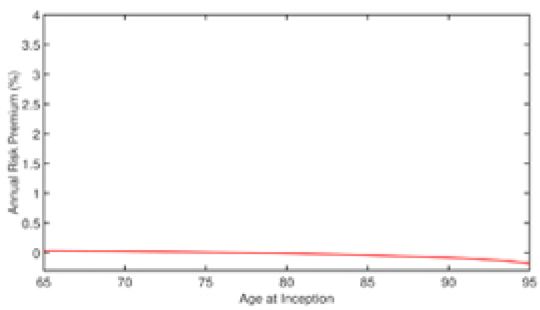
(j) Age 75-rw (k) Age 80-rw (l) Age 85-rw

Age	60	65	70	75	80	85	90
Risk Premium (GBP)	101,373	193,429	304,004	438,076	599,473	795,942	1,033,828

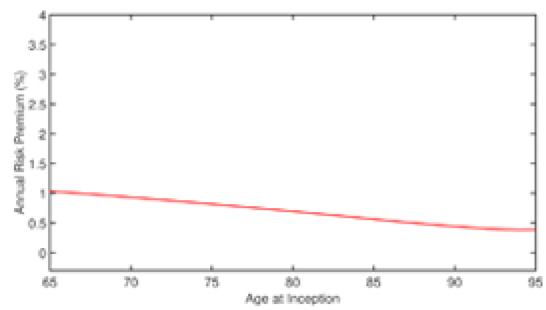
Figure 8.4 Distribution of projected quarterly house prices for UK under real-world and risk-neutral ARMA-EGARCH

Notes: The real-world house price simulations denoted (-rw) have a green face-colour whereas the simulations from the risk-neutral world (-rn) are blue-faced. We simulate 100,000 house price paths in this exercise.

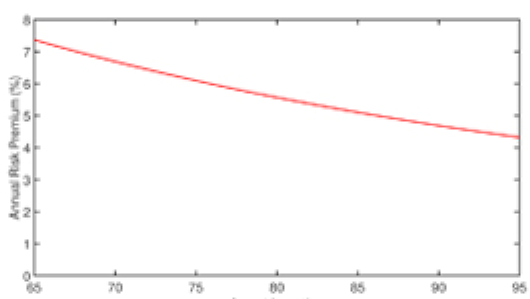
The second type of house price risk premium is illustrated in Figure 8.5 based on the monthly house price historical series and in Figure 8.7 for the quarterly series, for both the ARMA-EGARCH and GBM models. The graphs are constructed with different rental yield estimates $g = 1\%$ and $g = 3.3\%$. There is a striking difference between the profile of the house price premium $HPRP_3$ under the ARMA-EGARCH and the corresponding premium calculated under GBM. For the models calibrated with monthly series, and hence a shorter series, both models produce and $HPRP_3$ that is positive. One should note that the decreasing pattern with borrower's age is equivalent to an increasing pattern with time to maturity for the same borrowers. For the monthly data associated with Figure 8.5 we observe that the excess premium is much lower for the ARMA-EGARCH by comparison with the GBM.



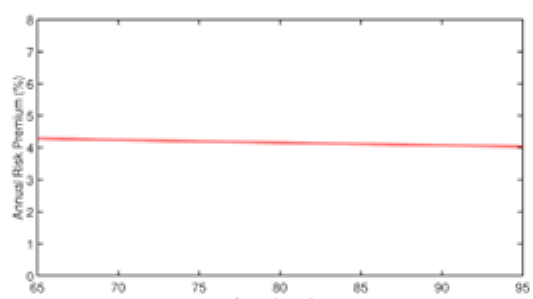
(a) Arma-Egarch ($g = 1\%$)



(b) Arma-Egarch ($g = 3.3\%$)



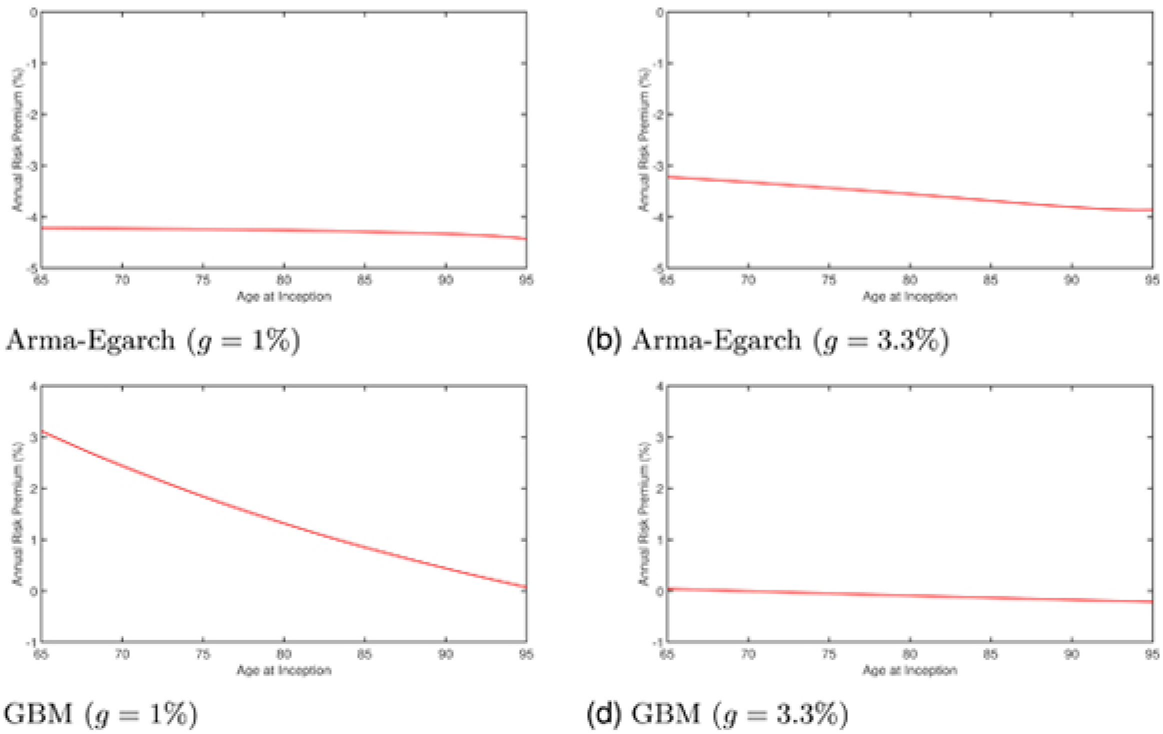
(c) GBM ($g = 1\%$)



(d) GBM ($g = 3.3\%$)

Figure 8.5 Expected excess house price return over the risk-free rate premium term structure (using parameter values in Scenario 1)

Notes: This figure depicts calculations based on Equation (8.3). The excess of the house price return ($Y_t = \frac{H_T - H_0}{H_0}$) over the risk-free rate of interest (r), taking also into account the rental yield g. In this illustration, we use the parameters values $r = 0.5\%$, $g = 1\%$, and $g = 3.3\%$ as provided in scenario 1 in Table 1.1. This excess house price return is calculated for different borrower ages at the inception of the contract. We simulate 100,000 monthly house price returns with their corresponding volatilities using the ARMA-EGARCH model on one hand and the GBM on the other, using the monthly Nationwide UK house price series in both cases.



(a) Arma-Egarch ($g = 1\%$)

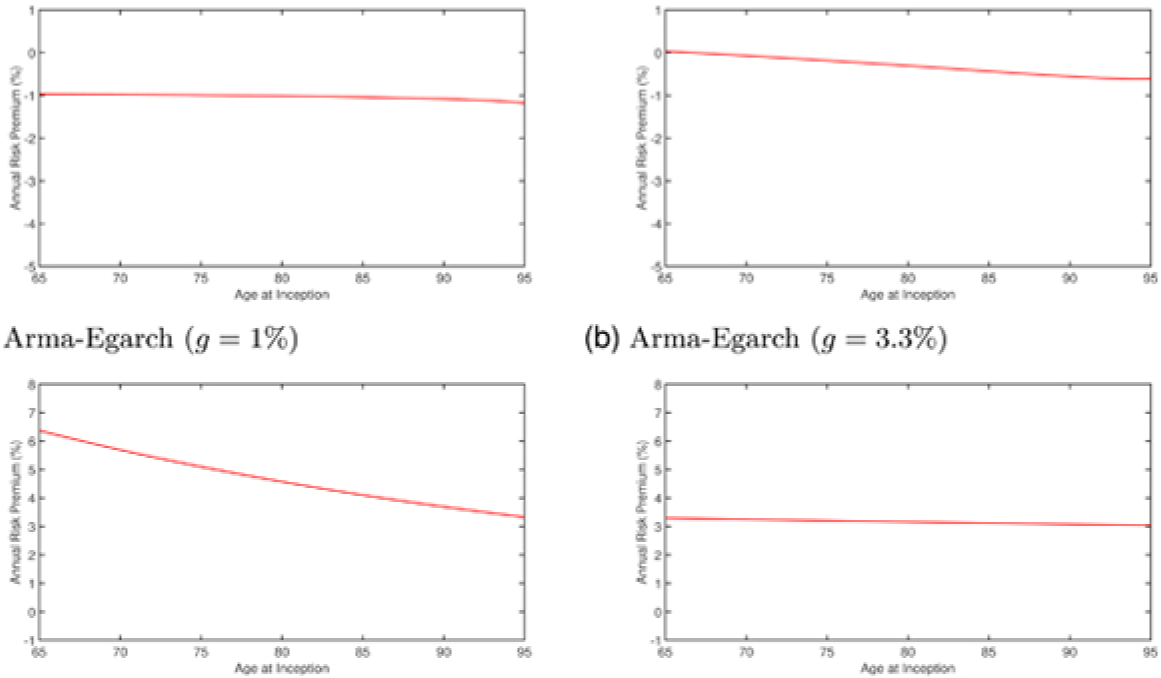
(b) Arma-Egarch ($g = 3.3\%$)

(c) GBM ($g = 1\%$)

(d) GBM ($g = 3.3\%$)

Figure 8.6 Expected excess house price return over the risk-free rate premium term structure (using parameter values in Scenario 2)

Notes: This figure depicts calculations based on Equation (8.3). The excess of the house price return $(Y_t = \frac{H_T - H_0}{H_0})$ over the risk-free rate of interest (r), taking also into account the rental yield g. In this illustration, we use the parameters values $r = 0.5\%$, $g = 1\%$, and $g = 3.3\%$ as provided in scenario 2 in Table 1.1. This excess house price return is calculated for different borrower ages at the inception of the contract. We simulate 100,000 monthly house price returns with their corresponding volatilities using the ARMA-EGARCH model on one hand and the GBM on the other, using the monthly Nationwide UK house price series in both cases.



(a) Arma-Egarch ($g = 1\%$)

(b) Arma-Egarch ($g = 3.3\%$)

(c) GBM ($g = 1\%$)

(d) GBM ($g = 3.3\%$)

Figure 8.7 Expected excess house price return over the risk-free rate premium term structure (using parameter values in Scenario 3)

Notes: This figure depicts calculations based on Equation (8.3). The excess of the house price return ($Y_t = \frac{H_T - H_0}{H_0}$) over the risk-free rate of interest (r), taking also into account the rental yield g. In this illustration, we use the parameters values $r = 0.5\%$, $g = 1\%$, and $g = 3.3\%$ as provided in scenario 3 in Table 1.1. This excess house price return is calculated for different borrower ages at the inception of the contract. We simulate 100,000 monthly house price returns with their corresponding volatilities using the ARMA-EGARCH model on one hand and the GBM on the other, using the monthly Nationwide UK house price series in both cases.

The differences in the shapes and levels of excess risk premia term structure for ARMA-EGARCH and GBM are even more striking in Figure 8.7 depicting the similar calculations for quarterly house price data. The levels are lower for ARMA-EGARCH and also differ as a shape. For the ARMA-EGARCH and rental yield $g = 1\%$ the excess premium is negative for all borrowers, whilst for the calculations performed with $g = 3.3\%$ there are positive and negative values for different borrowers' ages.

However, under GBM and employing the same quarterly data the excess premium stays positive for all borrower's ages.

8.3 NNEG RISK PREMIUM

The NNEG cash intrinsic values defined as the maximum between the difference between the accumulated loan and the value of the house price at the respective time will reverse the order of distributions. A similar analysis is carried out for the GBM model; in Figure 8.1 we illustrate these differences calculated at future time horizons defined by borrower's age 60, 65, 70, 75, and 80, 85, respectively, from the point of view of a 55-year-old. The two distributions being closer this time round the risk premia implied are lower than in the case of ARMA-EGARCH model. Under the GBM model the difference between the risk-neutral and real-world values simulated reflects only the parameter estimation uncertainty of inference between monthly and quarterly data. Recall that the quarterly series is historically longer than the monthly series. In addition, if the same estimates are obtained for drift and volatility for the GBM model then the same simulations would be obtained under monthly and quarterly steps. This follows from the theoretical properties of the geometric Brownian motion.

The risk-premium for the GBM as calibrated on the Nationwide monthly series is increasing with the maturity horizon.

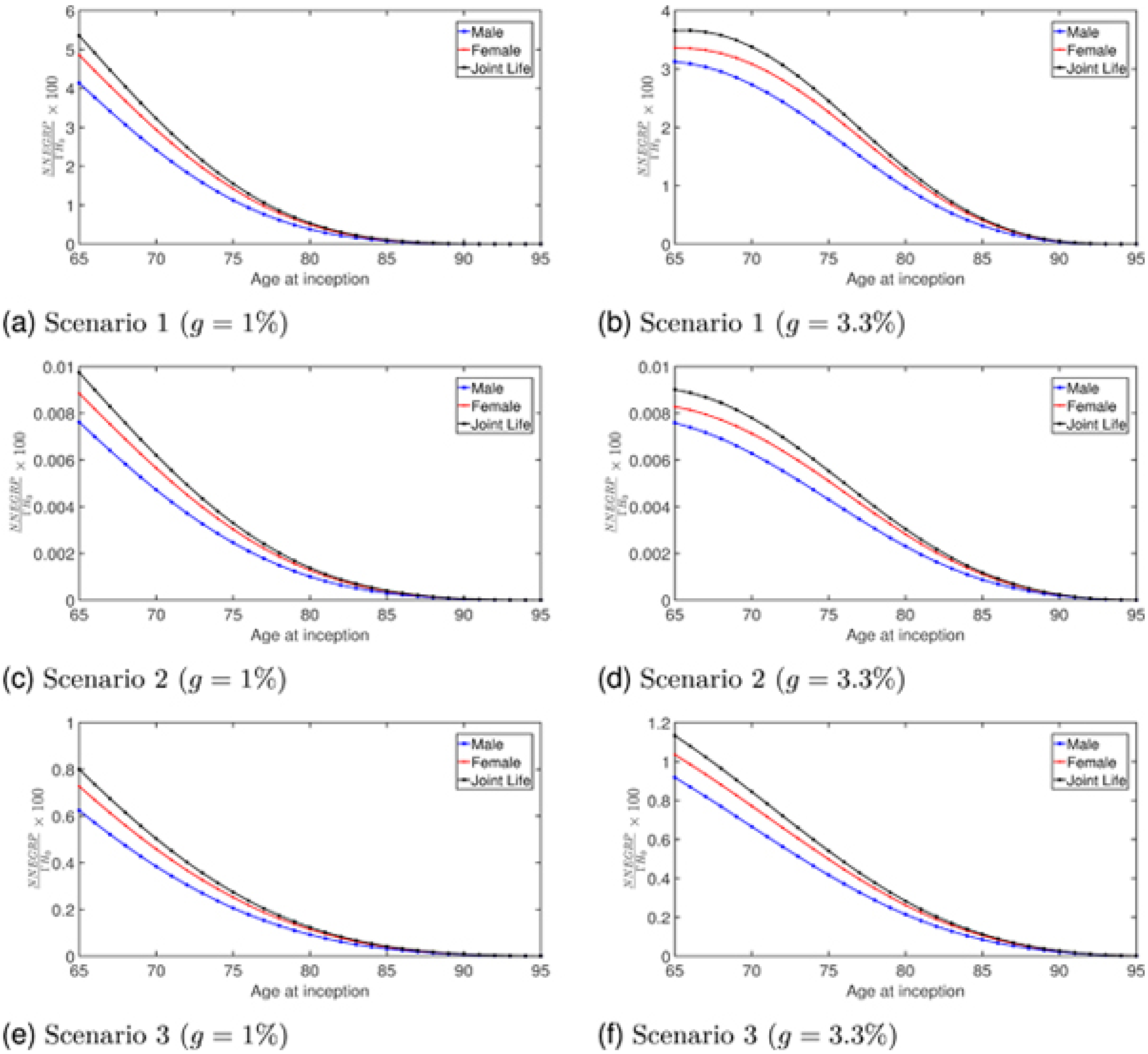
The NNEG values for the ARMA-EGARCH-rn will be much higher than the NNEG values for the ARMA-EGARCH-rw corresponding model. The same is also true for the calibrated GBM values over the same historical Nationwide monthly time-series, with the difference a lot larger. The risk-neutral house price distributions exhibit higher variance under the GBM model by comparison with the ARMA-EGARCH family. This confirms once again that the GBM model can induce a false sense of security. This is even more evident when quarterly data is used (and model parameters are different from the monthly data.

The NNEG risk premium induced by the house price valuations can also be calculated as the difference between the risk-neutral and physical NNEG price:

$$NNEGRP_{0,1} = \Pi^{\mathbb{Q}}_{0,1}\left(\tau_M\right) - \Pi^{\mathbb{P}}_{0,1}\left(\tau_M\right) \tag{8.9}$$

where $NNEGRP_{0,1}$ is the house price risk premium induced by the NNEG clause and $\Pi^{\mathbb{Q}}_{0,1}\left(\tau_M\right)$ is the NNEG price at inception i.e. at time (0,1) for a contract with maturity τ_M.

The graphs in Figure 8.8 illustrate the NNEG premia calculated with the Black-Scholes model for various borrowers' ages and type. In this case, the premia is always larger for couples and for females and it is decreasing over time, sometimes faster in a convex manner for small values of rental yield and sometimes slower for younger borrowers and then faster for older borrowers, for higher values of the rental yield g. In addition, the NNEG risk premia calculated with the Black-Scholes model is always positive.



(a) Scenario 1 ($g = 1\%$)

(b) Scenario 1 ($g = 3.3\%$)

(c) Scenario 2 ($g = 1\%$)

(d) Scenario 2 ($g = 3.3\%$)

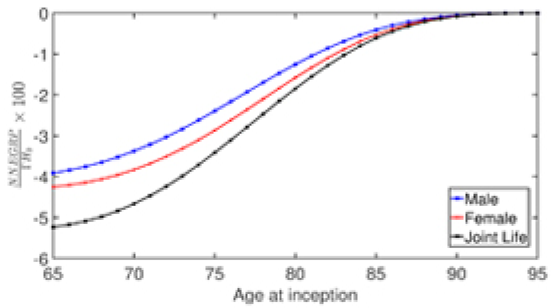
(e) Scenario 3 ($g = 1\%$)

(f) Scenario 3 ($g = 3.3\%$)

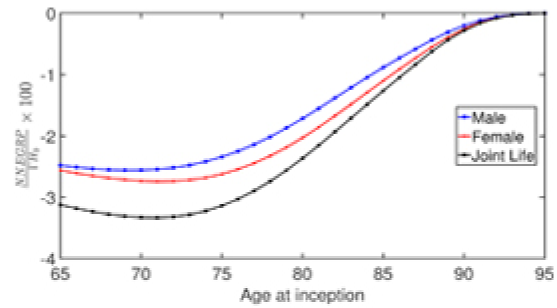
Figure 8.8 NNEG risk premium using the Black-Scholes model

Notes: This figure depicts calculations based on Equation (8.9). The NNEG risk premium (NNEGRP) is expressed as a percentage of the initial loan (ΓH_0) and calculated for various borrower-age at inception when the LTV is fixed at 30%. The required parameter values used in the calculations are taken from the scenarios described in Table 1.1.

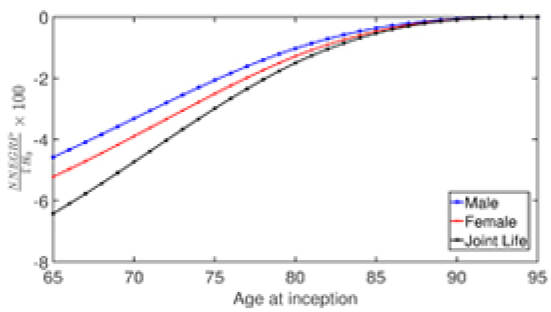
Remarkably, the same calculations carried out under the ARMA-EGARCH model, produces results that are qualitatively opposite. The NNEG risk premia depicted in the graphs in Figure 8.9 show that the premia is lower for couples and females compared to males, that the premia increases with the age of the borrower and that the increase is fast, most of the times convex and sometimes concave, depending on the scenario. Furthermore, our calculations show that for the scenario considered the NNEG premia computed with the ARMA-EGARCH model are negative.



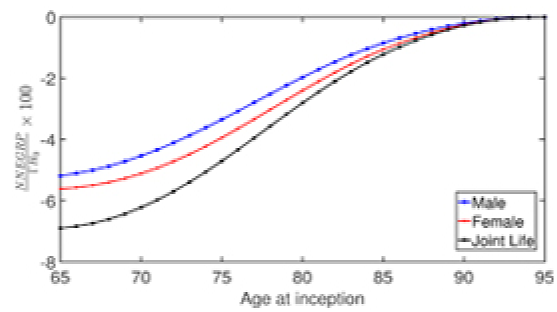
(a) Scenario 1 ($g = 1\%$)



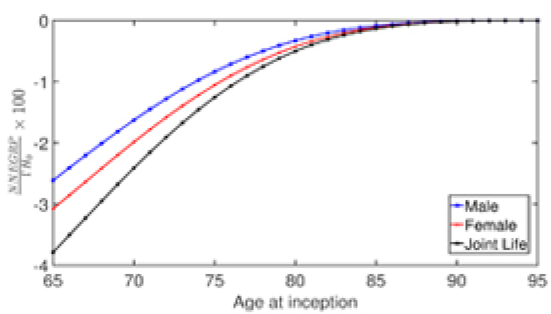
(b) Scenario 1 ($g = 3.3\%$)



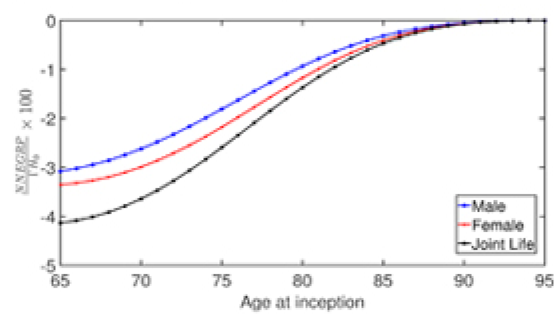
(c) Scenario 2 ($g = 1\%$)



(d) Scenario 2 ($g = 3.3\%$)



(e) Scenario 3 ($g = 1\%$)



(f) Scenario 3 ($g = 3.3\%$)

Figure 8.9 NNEG risk premium (using ARMA-EGARCH model)

Notes: This figure depicts calculations based on Equation (8.9). The NNEG risk premium (NNEGRP) is expressed as a percentage of the initial loan (ΓH_0) and calculated for various borrower-age at inception when the LTV is fixed at 30%. The required parameter values used in the calculations are taken from the scenarios described in Table 1.1.

The positivity or not of the NNEG premia should not be taken as a rule. Depending on the Black-Scholes model, and respectively ARMA-EGARCH model, parameters used one may get negative NNEG premia with Black-Scholes model and positive risk premia with the ARMA-EGARCH model. Thus, it becomes very important to recognise the importance of a) the parameter estimation risk and b) model selection risk.

8.4 ERP RISK PREMIUM

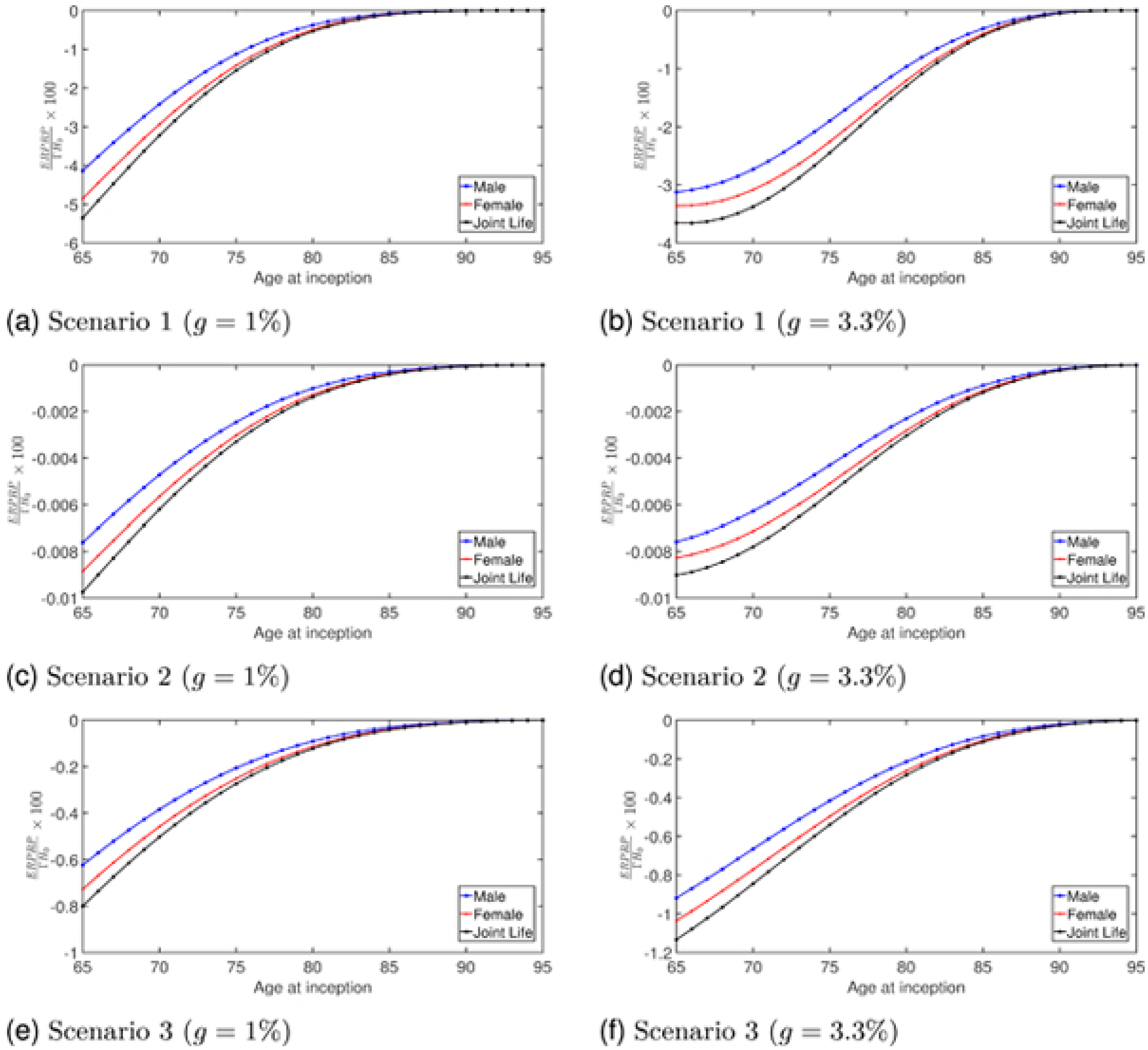
The ERP loan risk premium is defined as the difference between the risk-neutral and physical ERP price:

$$ERPRP_{0,1} = V_{0,1}^{\mathbb{Q}}\left(\tau_M\right) - V_{0,1}^{\mathbb{P}}\left(\tau_M\right) \tag{8.10}$$

where $ERPRP_{0,1}$ is the time zero hour price risk premium. $\mathbb{Q}$ is the risk-neutral measure, and $\mathbb{P}$ is the physical measure. $V_{0,1}^{\mathbb{Q}}\left(\tau_M\right)$ is the value of the ERM loan at inception under the risk-neutral measure introduced in Equation 6.53. The value of the ERM loan under the physical measure is denoted $V_{0,1}^{\mathbb{P}}\left(\tau_M\right)$.

From the way we have defined $V_{0,1}\left(\tau_M\right)$ in Equation (6.53), it is easy to see that the ERP loan risk premium can be expressed as a function of the NNEG risk premium. More specifically, this expression is the difference between $\Pi_{0,1}^{\mathbb{P}}\left(\tau_M\right)$ and $\Pi_{0,1}^{\mathbb{Q}}\left(\tau_M\right)$.

The results obtained for NNEG risk will transfer to the ERP risk premia calculations. These can be observed on the graphs in Figures 8.10 and 8.11.



(a) Scenario 1 ($g = 1\%$)

(b) Scenario 1 ($g = 3.3\%$)

(c) Scenario 2 ($g = 1\%$)

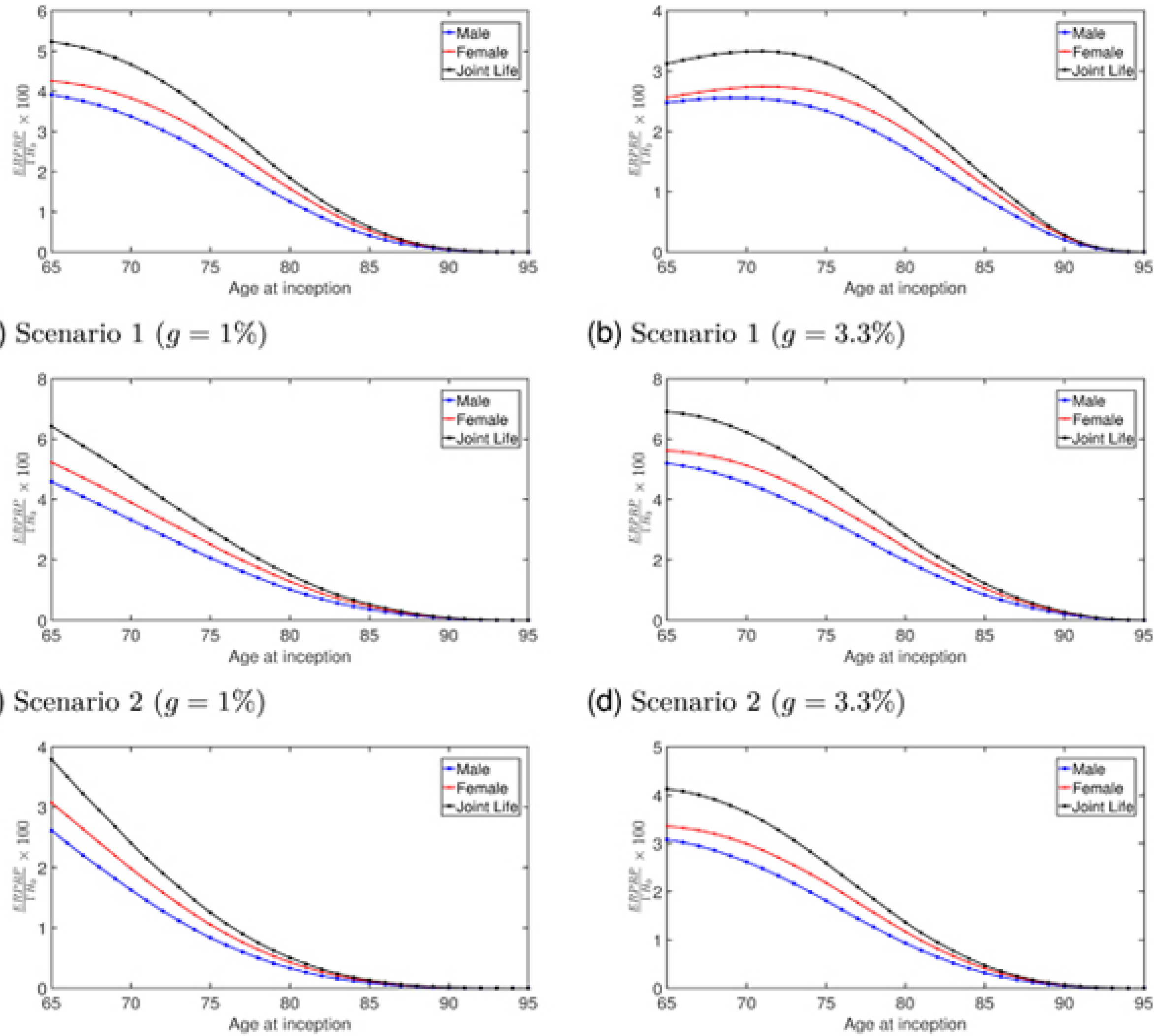
(d) Scenario 2 ($g = 3.3\%$)

(e) Scenario 3 ($g = 1\%$)

(f) Scenario 3 ($g = 3.3\%$)

Figure 8.10 ERP risk premium using Black-Scholes model

Notes: This figure depicts calculations based on Equation (8.9). The ERP risk premium (ERPRP) is expressed as a percentage of the initial loan (ΓH_0) and calculated for various borrower-age at inception when the LTV is fixed at 30%. The required parameter values used in the calculations are taken from the scenarios described in Table 1.1.



(a) Scenario 1 ($g = 1\%$)

(b) Scenario 1 ($g = 3.3\%$)

(c) Scenario 2 ($g = 1\%$)

(d) Scenario 2 ($g = 3.3\%$)

(e) Scenario 3 ($g = 1\%$)

(f) Scenario 3 ($g = 3.3\%$)

Figure 8.11 ERP risk premium using the ARMA-EGARCH model

Notes: This figure depicts calculations based on Equation (8.9). The ERP risk premium (ERPRP) is expressed as a percentage of the initial loan (ΓH_0) and calculated for various borrower-age at inception when the LTV is fixed at 30%. The required parameter values used in the calculations are taken from the scenarios described in Table 1.1.

There are some other excess type risk premia that may be considered. For example the excess ERP premium can be defined as the excess over the risk-free rate of the return an investor would make from investing into the ERM market.

For a lump sum advanced to the borrower at time zero this premium is formally defined as follows

$$ERMRP(T) = \frac{1}{T}\frac{\min\left(H_T, K_T\right) - \Gamma H_0}{\Gamma H_0} - r_{[0,T]} \tag{8.11}$$

where $r_{[0,T]}$ is the risk-free rate, annualised, for maturity *T*. This risk premium does not capture the NNEG valuation impact on capital considerations for the investor. The risk premium in 8.11 is useful for computing an *ex post* realised premium along the realised path of house prices.

The *ex ante* expected ERMRP depends intrinsically on the $E(\min\left(H_T, K_T\right))$ which is derived further as follows.

$$\begin{aligned}
\mathbb{E}^{\mathbb{P}}(\min\left(H_T, K_T\right)) &= \mathbb{E}^{\mathbb{P}}\left(K_T - \max\left(K_t - H_T, 0\right)\right) \\
&= K_T - \mathbb{E}^{\mathbb{P}}\left(\max\left(K_T - H_T, 0\right)\right) \\
&= K_T - \mathbb{E}^{\mathbb{P}}\left(\max\left(K_T - H_T, 0\right)\right) \\
&= K_T - PutPayoff(H_T, K_T, T)
\end{aligned}$$

where $PutPayoff(H_T, K_T, T)$ is the expected value of the put option on the house price index under the real-world measure $\mathbb{P}$. Then the expected ERMRP is calculated as

$$\mathbb{E}^{\mathbb{P}}(ERMRP(T)) = \frac{1}{T}\frac{\mathbb{E}^{\mathbb{P}}(\min\left(H_T, K_T\right)) - \Gamma H_0}{\Gamma H_0} - r_{[0,T]}$$

(8.12)

After replacement we get

$$\mathbb{E}^{\mathbb{P}}(ERMRP(T)) = \frac{1}{T}\frac{K_T - PutPayoff(H_T, K_T, T) - \Gamma H_0}{\Gamma H_0} - r_{[0,T}$$

(8.13)

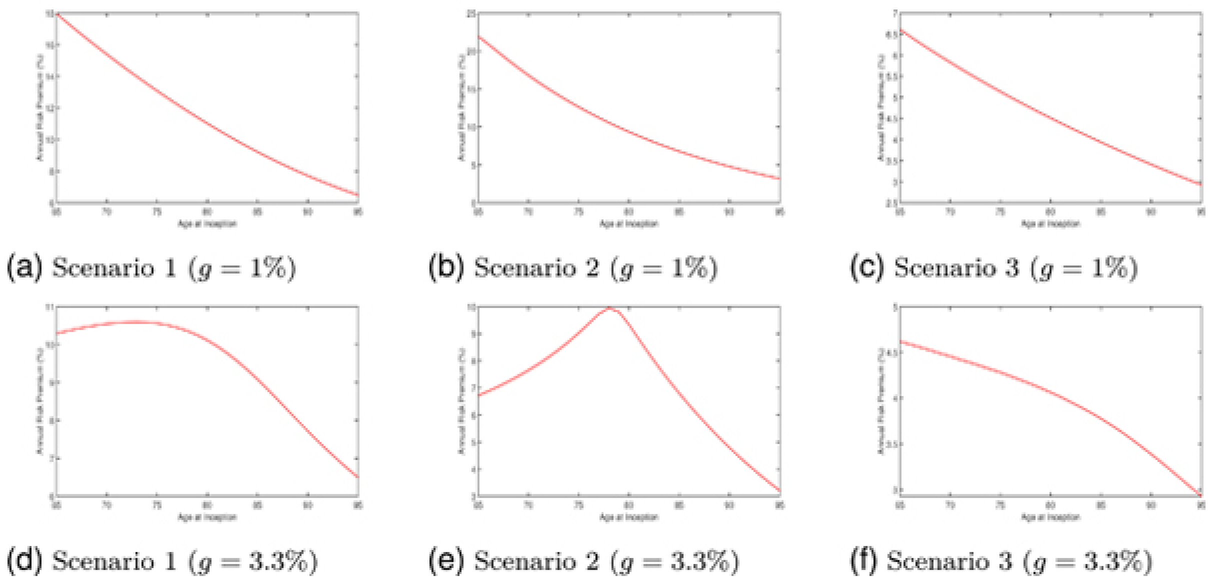
which can be further rewritten as

$$\mathbb{E}^{\mathbb{P}}(ERMRP(T)) = \frac{1}{T}\left[\frac{K_T - \Gamma H_0}{\Gamma H_0} - \frac{PutPayoff(H_T, K_T, T)}{\Gamma H_0}\right] - r$$

(8.14)

The last expression can be interpreted as the excess over the risk-free rate of the annualised difference between the expected return on the loan if the balance is paid in full and the total return of the NNEG payoff relative to the lump payment provided to the borrower.

In Figure 8.12 we illustrate the ERMRP term structure for the three main scenarios under the GBM model. In general the ERMRP decreases with the age of the borrower. When the rental yield is higher the shape of the ERMRP is increasing first and then decreasing fast. For “younger” borrowers the ERMRP is positive and substantial, varying across our scenario combinations between 4.6% and 22.5%.

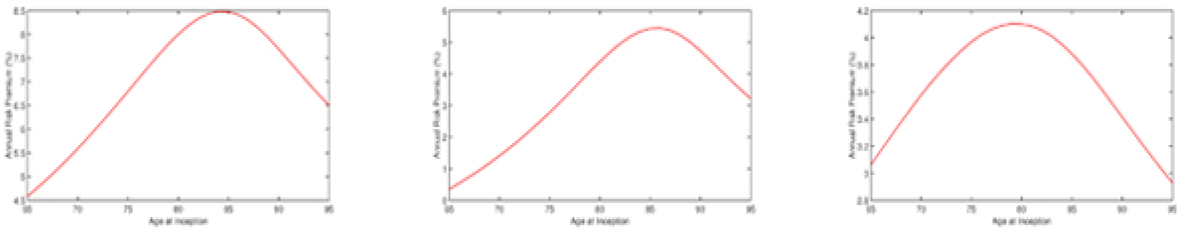


(a) Scenario 1 ($g = 1\%$)
(b) Scenario 2 ($g = 1\%$)
(c) Scenario 3 ($g = 1\%$)
(d) Scenario 1 ($g = 3.3\%$)
(e) Scenario 2 ($g = 3.3\%$)
(f) Scenario 3 ($g = 3.3\%$)

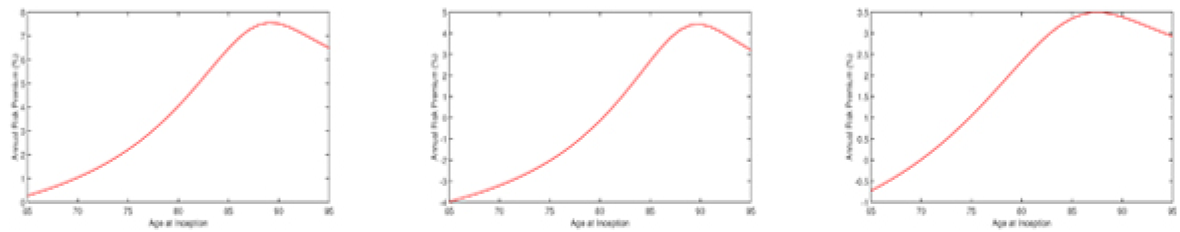
Figure 8.12 ERM risk premium as excess of return on investment over risk-free rate of interest for the three main scenarios under the GBM model.

Notes: The ERM risk premium (ERMRP) is defined as the excess over the risk-free rate of the return an investor would make from investing into the ERM market. The calculation in each scenario is based on Equation (8.11). The subplots depict the ERMRP dynamics when calculated with parameter values in Table 1.1. The initial house price is set to 1 and the LTV is 30% when calculating K_T in the ERMRP. The house price projections are generated with the GBM model, using the rental yield values displayed in the caption of the respective subplots. The risk premium calculation is done for different borrower ages at the inception of the contract. These ages are in the x-axis.

Figure 8.13 shows the ERMRP term structure for the same scenario combinations but with house prices generated under the ARMA-EGARCH model. The ERMRP term structure increases with the age of the borrower up to some age between 80 and 90 and then decreases. As opposed to the GBM case, the ERMRP is quite low for the younger borrowers, between -4% and 4%, and reaching a peak between 3.5% and 8%. These clear discrepancies when compared with the GBM model point out to serious implications of model selection risk for equity release finance calculus.



(a) Scenario 1 ($g = 1\%$) (b) Scenario 2 ($g = 1\%$) (c) Scenario 3 ($g = 1\%$)



(d) Scenario 1 ($g = 3.3\%$) (e) Scenario 2 ($g = 3.3\%$) (f) Scenario 3 ($g = 3.3\%$)

Figure 8.13 ERM risk premium as the excess of return on investment over risk-free rate of interest for the three main scenarios under the ARMA-EGARCH model.

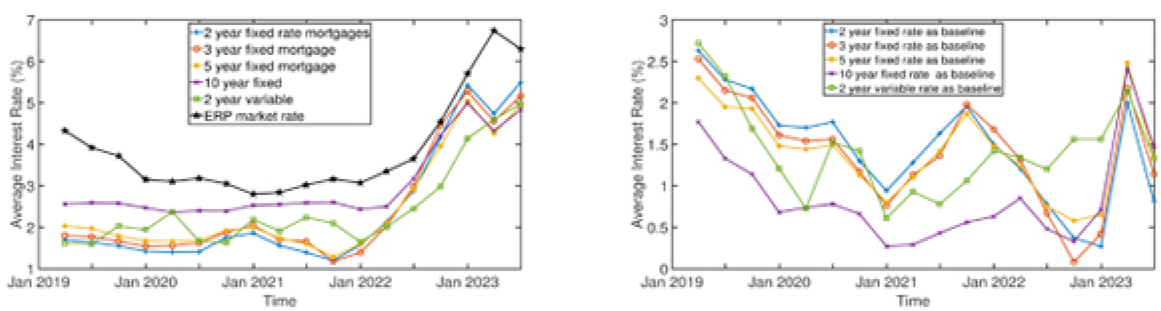
Notes: The house prices in the ERM risk premium (ERMRP) calculations are simulated from the ARMA(4,4)-EGARCH(1,1) model using the monthly Nationwide UK house price index time series. The corresponding rental yield value is displayed in the caption of the subplots and the ERMRP is defined as the excess over the risk-free rate of the return an investor would make from investing into the ERM market. This is shown in Equation (8.11). The initial house price has been set to 1 and the LTV is 30% when calculating (K_T). Each ERMRP calculation is done for different borrower ages at the inception of the contract.

8.5 NOVELTY PREMIUM

Venturing premium will account for the extra price of risk charged by investors for buying into a less familiar investment market. The same goes for first time borrowers in the ERM market space. We are also able to related venturing risk premium to the cost of financial innovation associated with its introduction as a new instrument. Innovation cost is expected to increase with limitations to efforts towards realising the full benefit of ERMs and the take-up rates amongst the elderly. It could be driven by constrained liquidity and information asymmetry associated with market prospects. The venturing premium may also be considered as an interest premium with respect to similar existing financial products. Unfortunately, the concept has received no interest in both theoretical and empirical analysis of ERMs. In a simplified view, the venturing premium (VP) may be implicit in the standard or widely recognised baseline return (BR) as an additional charge or compensation loading. We write this as:

$$Venturing\ Premium = R - Baseline\ Return \tag{8.15}$$

where the baseline return is set equal to the expected return on a familiar investment or a well-diversified portfolio, such as house prices or long maturity bonds, or mortgage rates. The VP might just be estimated with information about the year-on-year growth rate of the effective value of the ERP held on the insurer's books. Perceived unfamiliarity and VP have a positive correlation, with large VP suggesting higher perceived risk and unfamiliarity.



(a) ERP market rate & Mortgage rate (b) Venturing premium

Figure 8.14 Assessing the venturing premium in the UK: 2019Q1 to 2023Q3.

Source: The time series of quarterly average mortgage interest rate in the United Kingdom was published in February 2024 by the Bank of England. The complete survey is from March 2000 to January 2024 and is available at bankofengland.co.uk. The series is not seasonally adjusted. In this figure, we plot average mortgage rate over the period 2019Q1 to 2023Q3 to match the ERP average market rate time series which is also available over the same time period. The ERP data set is from Market Monitor H1 2023.

8.6 SUMMARY AND FURTHER READING

Housing prices were involved with many puzzles in asset pricing. There is some evidence that the average returns in housing can be similar to those observed on the equities markets, however, with a much lower volatility. This housing premium puzzle confirms the important role played by volatility. One different perspective is offered by Cheng, Lin, and Liu (2008) who are solving the puzzle by paying attention to the non i.i.d characteristic of the returns and deriving a better volatility estimator for this specific asset class.

This section points out that the risk premium from house price markets is feeding into a valuation risk premium on NNEG. The risk premium varies with the borrower's age and hence with the time to assumed maturity of the loan. It also varies with parameters estimated employing datasets with different lengths. Last but not least, it also varies with the model selected.

The regulator's decision on whether to allow real-world valuations versus “market consistent” valuations may have a dramatic impact on the evolution of the ERP markets. While the latter may be appropriate in markets like US where derivatives are offered for trading on house price indices, the former may be more relevant in other markets like the UK where no house price derivatives currently exist.

On a conceptual level, the connection between house price risk premia and NNEG risk premia may require more insight into the connection between the forward mortgage market and the ERP market. Furthermore, more research is needed to identify the main determinants of house risk premia. Furthermore, more research is also needed into studying the connectedness between risk premia in house prices and risk premia in other fundamental macro-financial variables such as interest rates and even foreign exchange, and also levels of GDP, unemployment and inflation.

The behaviour of investors participant in housing markets during extreme periods such as health pandemic, geographical hazards or geopolitical stress is also relevant for grasping a clearer picture on what can possibly go wrong in the future with house prices.

FURTHER READING

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CHAPTER **9**

Portfolio Analysis

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9.1 INTRODUCTION

THE PORTFOLIO EFFECT of house price risk when looking at cash flow analysis from a lender perspective is important. The lender of ERP, usually an insurer, must perform risk management and performance measurement calculations under the framework dictated by the regulator. This implies that those calculations are quite often performed on a loan-by-loan basis. Hence, the portfolio diversification effect is lost for these types of calculations.

However, nothing can stop the lender to perform *also* portfolio wide calculations where various effects induced by running portfolios of loans can be gauged. These second type of calculations are useful internally for comparing different business units within the same country, or even across different countries. They are also useful for designing business strategies and attracting investors.

In this chapter, we will carry out some investigations relying on hypothetical scenarios. In this type of market it is quite difficult to get full data from various market players, perhaps understandably. We will introduce various measures that we think are useful to monitor on a frequent basis. In the second part of this chapter we will review the current state of securitisation processes for ERPs defined by the three main rating agencies, namely in no particular order Standard and Poor's, Fitch Ratings and Moody's.

The lender faces the overall effect of the assumptions underlying ERM loan and NNEG valuation used in the single case. Here, the main portfolio takes the view that all borrowers live exactly to their expected lifetime. Note that the lifetime expectancy of a 60-year-old will differ from the lifetime expectancy of a 70-year-old, and so on, and it will also be different between males and females.

Given that portfolio calculations are not actually requested for ERP, the quantities that we will measure and discuss are introduced in majority by us. The measures are not standard and they can be adapted to more specific situations by risk managers and investors or academics.

9.2 RISK CONSIDERATIONS FOR A REVERSE MORTGAGES LENDER

In this section, we will consider the portfolio effects of house price risk when looking at the cash flows analysis from a lender perspective. The portfolio has 10,000 loan contracts in total, all from the UK, with 4927 female and 5073 male borrowers. There are 7500 borrowers on Flexible LTV and 2500 borrowers on Flexible Max Plus LTV. This is to preserve the dominance of the loan contracts in the market.

For the purposes of the numerical simulations in this section we assume that the value of each collateral house is equal to 1. This is so that we can gauge the various effects of the main drivers behind ERP dynamics.

9.2.1 Notations

The following notations are used here. We take $\omega_t^{(i)} = 1$ if the loan i is still active, and it is equal to 0 if it is not active. Then, $K_t^{(i)} = L_0^{(i)} e^{Rt}$ is the accumulated balance for loan i up to time t and $\widetilde{K_t^{(i)}} = K_t^{(i)} \times \omega_t^{(i)}$ is the accumulated outstanding balance for loan i up to time t, if the borrower survives to t; $\widetilde{K_t} = \sum_i \widetilde{K_t^{(i)}}$ is the portfolio outstanding balance at time t.

The evolution of accumulated loan balance $L_0^{(i)} e^{Rt} \times \omega_t^{(i)}$ in Figure 9.1 is the same for both GBM and ARMA-EGARCH models given that it ignores the NNEG risk. Up to the first 20 years in portfolio life there is almost no differentiation in the cash balance across simulated scenarios, but after that we can see a more dispersed marginal distribution. The mean of the expected portfolio loan return cash balance peaks at about 30 years. After that there is a steep decline in the expected cash-flow generated by this portfolio (ignoring NNEG risk) every year.

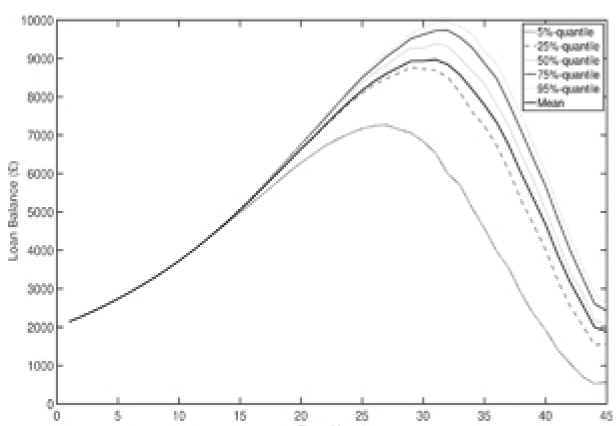


Figure 9.1 Distribution of Expected Portfolio Loan Return Cash Balance

Notes: The total accumulated loan balance of borrowers who transition from active to inactive in year t is calculated as $\sum_i (L_0^{(i)} e^{Rt} \times \omega_t^{(i)})$. The initial portfolio comprises of 10,000 borrowers with the youngest aged 55 years and oldest 95 years. There are 4927 female and 5073 male borrowers. The initial property value is set to 1 with 7500 borrowers on Flexible and 2500 borrowers on Flexible Max Plus LTV. The roll-up interest rate on each loan is 6.15%. The percentiles of the loan balance is calculated from 10,000 scenarios generated by mortality projections. The survival times or otherwise age-at-death of respective borrowers is simulated from the UK life table.

The quantity

$$C_t^{(i)} = min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)} \tag{9.1}$$

is the cash generated in year t from loan i and $\tau_t^{(i)} = 1$ if the loan i is terminated at t whilst it is zero otherwise. Then $C_t = \sum_i C_t^{(i)}$ represents the total portfolio new cash generated by loans terminating in year t. If AC_t is the total portfolio accrued cash in the money account by time t, this is calculated recursively based on the formula

$$AC_t = AC_{t-1} \times e^r + C_t. \tag{9.2}$$

For portfolio computations, one can take advantage of the fact that $C_t^{(i)}$ would be clearly zero in all years except the year when loan is simulated to be terminated. Hence, one can proceed with the following simulation procedure.

Step 1 For all times $t \in \{1, \ldots 45\}$ simulate a full path of house prices returns under the given model, GBM or ARMA-EGARCH.

Step 2 For each borrower $i = \{1, \dots, n\}$ simulate the respective time t such that $\tau_t^{(i)} = 1$; there will be n times $t_1, \dots t_n$ all belonging to $\{1, \dots, 45\}$

Step 3 For each loan i, take from the table of simulated house prices at Step 1 under the respective model the required value for the house price at the time t_i. This would be the $H_t^{(i)}$

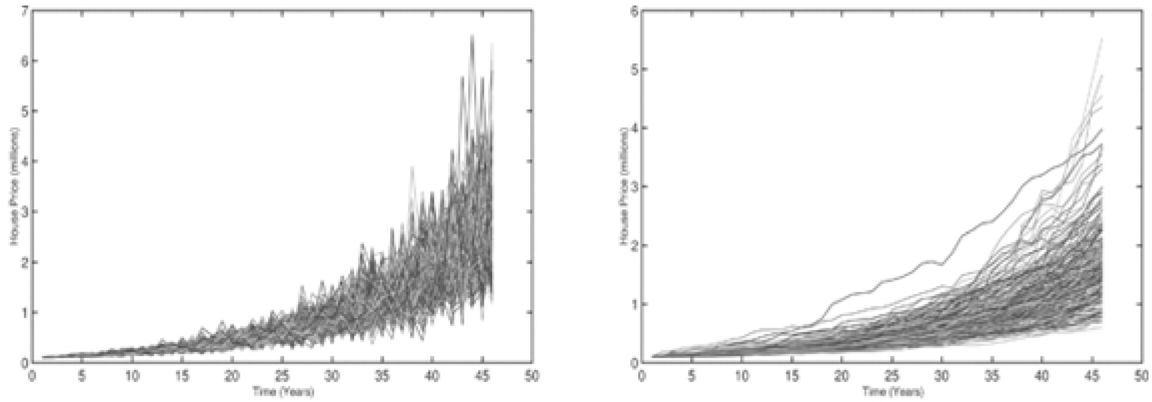
Step 4 Calculate $C_t^{(i)}$ and then C_t and AC_t.

Now, repeat the above procedure M times, with M relatively large such as 10000. By averaging the obtained C_t and AC_t over these M simulations we obtain the *expected* values $E(C_t)$ and $E(AC_t)$.

9.2.2 Loan-by-Loan Calculus and Portfolio Effects

The calculations are based on the assumption that the loans are terminated at a random time before the expected future lifetime maturities, for male and female borrowers.

The graphs in Figure 9.2 illustrate house price pathways under the GBM and the ARMA-EGARCH model, under the physical measure. The GBM price paths display more variability, indicating the possibility of house prices to be overall lower at long-term horizons. The conditional distributions at any future time horizon is more dispersed for the GBM than it is for the ARMA-EGARCH. There is clearly a much smoother evolution of simulated house prices under the ARMA-EGARCH family of models, that seems to be more reflective of real evolution of house prices.



(a) GBM scenarios (b) ARMA-EGARCH scenarios

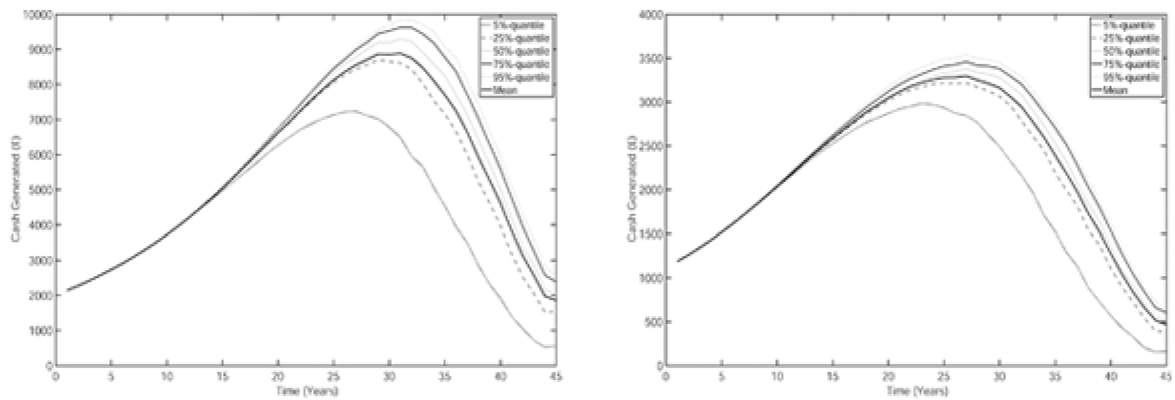
Figure 9.2 House price pathways up to 46 years under GBM and ARMA-EGARCH models.

Notes: The simulated house price paths are based on GBM and ARMA-EGARCH parameters which we estimated from the quarterly house price time series obtained from Nationwide house price database. The data series is from 1952Q4 to 2022Q4. The roll-up interest rate is 6.15% per annum and the risk-free rate of interest is 3.42% per annum. There simulation process follows as described in 9.2.1 with the number of repetitions *M* set to 10,000.

The spread between quantiles of portfolio generated cash is larger under the GBM model, as one can notice if looking carefully at the scale. This further imply riskier cash-flow projections that would automatically require larger capital reserves under Solvency 2 set of regulations. This may explain partially why the regulator likes the GBM model, since it gives more conservative estimates of risk from their perspective.

9.3 AN ANALYSIS OF A HYPOTHETICAL PORTFOLIO OF EQUITY RELEASE PRODUCTS

In Figure 9.3, the evolution of the portfolio outstanding balance under both GBM and ARMA-EGARCH models is illustrated, on the left side graphs, and the evolution of generated cash from loan terminations, again under each GBM and ARMA-EGARCH models, on the right side graphs. Under both models, there is little uncertainty in the evolution of expected portfolio generated cash with NNEG risk up to 20 years. After that, there is a clear differentiation with more uncertainty exhibited by the graph associated with the GBM, by comparison with the graph associated with the ARMA-EGARCH. There is also a difference overall in the scale, the expected cash going close to 10,000 for the GBM model between 30 and 35 years, whilst the maximum of the same quantity under the ARMA-EGARCH model does not go above 3500. For the lower bound, this is at just above 500 for GBM and at 250 for ARMA-EGARCH.



(a) GBM scenario (b) ARMA-EGARCH scenario

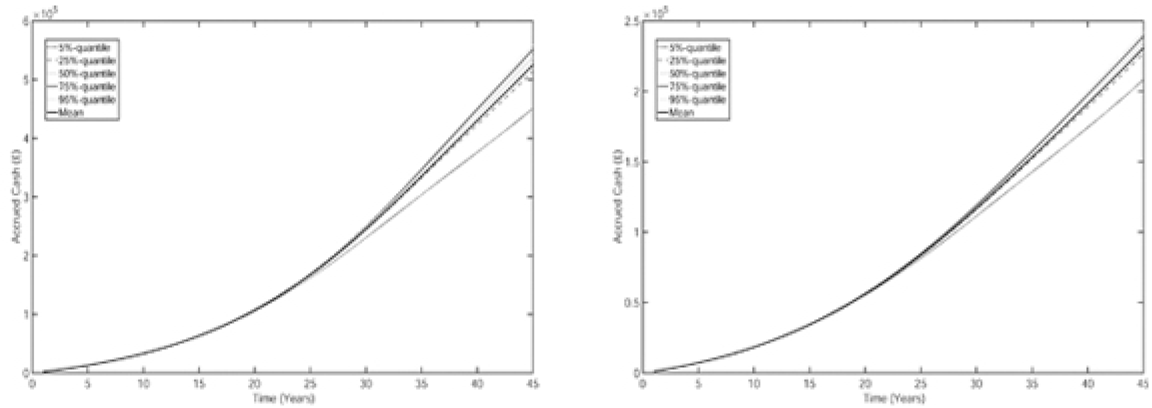
Figure 9.3 Evolution of Distributions of the Expected Portfolio Generated Cash with NNEG Risk

Notes: This figure depicts the dynamics of the total portfolio new cash generated C_t in Equation (9.1) by loans terminating in year *t* where $t = 1, \ldots, 45$. The calculation is based on separate house price simulation under the GBM and ARMA(4,3)-EGARCH(1,1) models using quarterly Nationwide house price time series. The initial portfolio comprises of 10,000 borrowers with the youngest and oldest aged 55 years and 95 years respectively. The roll-up interest rate is 6.15% per annum and the risk-free rate of interest is 3.42% per annum. There portfolio cash flow simulation process follows as described in 9.2.1 with the number of repetitions *M* set to 10,000.

The investors in an ERP backed pool would be interested in the lifetimes and magnitudes of portfolio generated cash-flows. The 5% quantile payments seem to evolve by starting at around +2,000 cash back after one year, peaking at 6,500 between 25 and 30 years and finishing at about 250 after 45 years. This would be the sort of payments that equity investors and junior tranche investors expect to get from an ERP securitisation deal based on the portfolio simulated here. Mezzanine investors, maybe following the 50% quantile or median, are looking at payments starting at around +2,000 cash back after one year, peaking at about 9,000 between 30 and 35 years and finishing at about 2,000 after 45 years. Finally, the most senior investors, following the 95% percentile, could consider payments that starting at around +2,000 cash back after one year, peaking at 10,000 between 30 and 35 years and finishing at about 2,500 after 45 years.

For the manager or the servicer of such a transaction it would be important to know how the total loan balance will evolve. One can see that the portfolio will generate most cash between 20 and 40 years.

The money that is cashed back from the loans once they terminate are not staying idle. At the minimum they can invested into a money account where they can earn the risk-free rate. This is a very important observation since the investors are waiting long periods of time before they can see any cash inflows. The portfolio accrued cash account grows as depicted in Figure 9.4 where we also illustrate the portfolio outstanding funding balance. The evolution is very similar for the two models, increasing in a convex manner over time. There is though a difference in scale, with a projected balance reaching above 55,0000 under GBM and only half of that under ARMA-EGARCH. This large discrepancy substantiates once again the danger of using the GBM model for ERP calculations.



(a) GBM Scenario (b) ARMA-EGARCH Scenario

Figure 9.4 Distribution of Expected Accrued Cash Generated by the Portfolio with added Risk-Free Interest on the Money Account

Notes: This figure depicts the percentiles of the expected accrued cash generated by the portfolio. This is denoted by AC_t in Equation (9.2). The calculation is based on the simulating procedure described in text. We present separate calculations when house price is simulated under the GBM and ARMA(4,3)-EGARCH(1,1). The house price simulations are based on quarterly Nationwide house price time series. The initial portfolio comprises of 10,000 borrowers with the youngest aged 55 years and oldest 95 years. We produce 10,000 simulations for the calculation in each year *t*. The simulations of each borrower's curtate life time is from the life table.

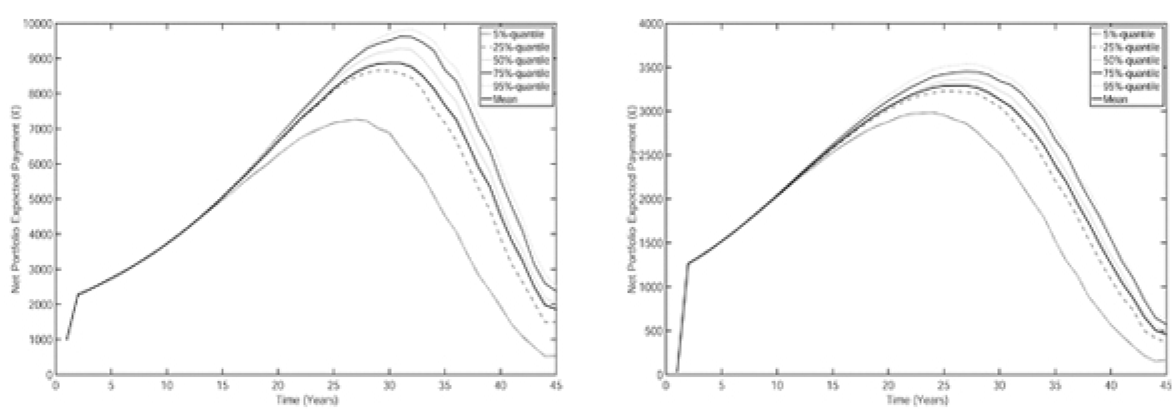
The portfolio expected payment, net of funding costs is important to look at in terms of projections of cash-flows to come. The net portfolio expected payment, denoted by *NTEP*, is calculated using the following formulae. At the end of the first year we will get

$$NTEP_1 = E(C_1) - L_0 e^{r_f} \tag{9.3}$$

where L_0 corresponds to total lump sums in the entire portfolio. This would be the difference between the money generated from the portfolio of loans during the first year minus the cash advanced that is paid on the liability side of the lender at the funding rate r_f. For the purposes of this exercise we will take $r_f = r$ the risk-free rate. Then for the following years we have the recursive formula

$$NTEP_t = E(C_t) - \min\left(NTEP_{t-1}, 0\right)e^{r_f} \tag{9.4}$$

The graphs displayed in Figure 9.5 show the NTEP under both models. In the very first few years the NTEP grows above 2,000 under GBM simulations but it reaches only about 1250 under the ARMA-EGARCH simulations. This substantial discrepancy increases over time with simulations of NTEP under GBM reaching potentially close to 10,000 just after 30 years whereas the same portfolio but using ARMA-EGARCH house price simulations may reach just about 3,500 maximum again after 30 years. One should also note that the projections under GBM are more dispersed that the projections from ARMA-EGARCH. For both calculations we ignore the interest to the advanced initial loans.



(a) GBM scenario

(b) ARMA-EGARCH scenario

Figure 9.5 Net Portfolio Expected Average Payment per Loan

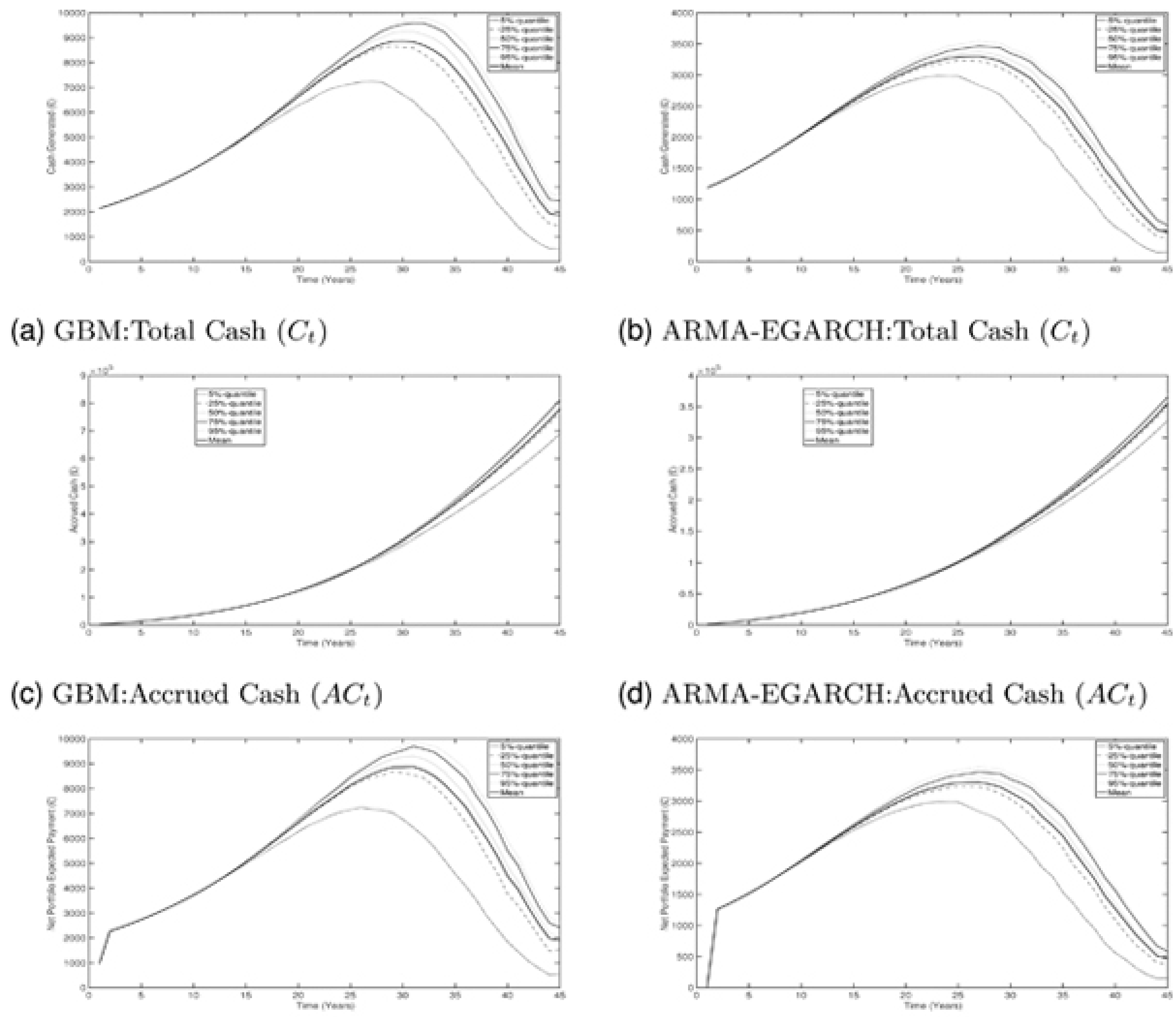
Notes: The portfolio total exposure at risk is given by $NTEP_t = E(C_t) - \min\,(NTEP_{t-1}, 0)e^{r_f}$. This figure presents the percentiles of $NTEP_t$ where $t = 1, \ldots, 45$. The portfolio simulation exercise follows as described previously. The risk-free rate of interest is 3.42% and the roll-up interest rate is 6.15%.

9.4 PORTFOLIO SENSITIVITY ANALYSIS

9.4.1 Sensitivity to changes in risk-free rate

Here we investigate the sensitivity to variations in risk-free rate on our portfolio monitoring quantities. We consider variations due to stochastic evolution of risk-free rates and other stress scenarios such as increases and decreases to 5% and a decrease to 1% in one go at various points in portfolio simulation evolution.

The results in Figure 9.6 illustrate the refreshed results of previous simulations of all types of balances when the risk-free rate moves up to 5%. The most impactful change is in the distribution of expected accrued cash generated by the portfolio with added risk-free interest on the money account. A change from $r_f = 3.42\%$ to $r_f = 5\%$ takes the balance in the money account of the portfolio managers from roughly 55,000 after 45 years under GBM and 23,000 under ARMA-EGARCH, respectively, to 80,000 under GBM and 37,500 under ARMA-EGARCH after 45 years, respectively. This is an increase of more than 50% in projected balance. The evolution of the other balances does not differ that much, showing that a relatively small change in discount rate of portfolio cash-flows does not impact the cash-flows that much in aggregate.



(a) GBM:Total Cash (C_t)

(b) ARMA-EGARCH:Total Cash (C_t)

(c) GBM:Accrued Cash (AC_t)

(d) ARMA-EGARCH:Accrued Cash (AC_t)

(e) GBM:Net Expected Payment

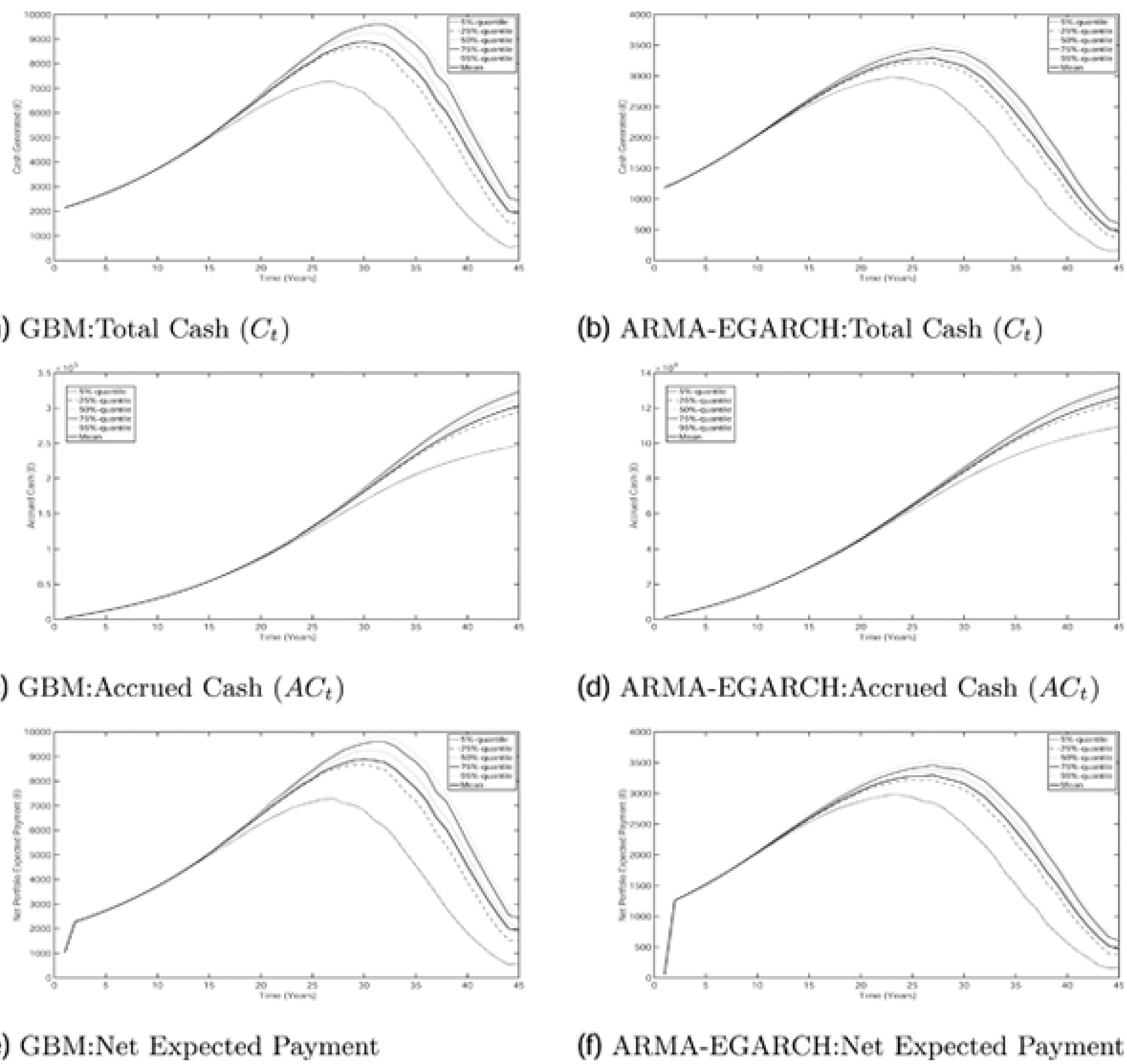
(f) ARMA-EGARCH:Net Expected Payment

Figure 9.6 Portfolio cash flow analysis using $r_f = 5\%$

Notes: This figure illustrates the distribution of portfolio total cash generated, accrued cash generated, and the net expected payment when risk-free rate of interest is increased to 5%, holding all others constant. The respective calculations are reported side-by-side for the GBM and ARMA-EGARCH house price simulation. The roll-up interest rate is 6.15% and the initial house price is set to 1 for all borrowers. There are 10,000 scenarios in each calculation.

The change to $r_f = 1\%$ is even more interesting. The graphs in Figure 9.7 depict a similar story for the balance calculations where r_f appears mainly in the discount factors of the future cash-flows. However, for the

AC_t there is a substantial reduction to 35,000 under GBM and to 13,000 under ARMA-EGARCH models, respectively. in addition, the balance evolution over time is convex first but then it changes to concave between 20 and 25 years.



(a) GBM:Total Cash (C_t)

(b) ARMA-EGARCH:Total Cash (C_t)

(c) GBM:Accrued Cash (AC_t)

(d) ARMA-EGARCH:Accrued Cash (AC_t)

(e) GBM:Net Expected Payment

(f) ARMA-EGARCH:Net Expected Payment

Figure 9.7 Portfolio cash flow analysis using $r_f = 1\%$

Notes: This figure illustrates the distribution of portfolio total cash generated, accrued cash generated, and the next expected payment when risk-free rate of interest is at 1%. The respective calculations are reported side-by-side for the GBM and ARMA-EGARCH house price simulation. The roll-up interest rate is 6.15% and the initial house price is set to 1 for all borrowers and there are 10,000 scenarios in each calculation.

Another important point to notice from both figures is that the quantile curves also indicate a very skewed distribution of all three types of balances, under both models. In other words the downside region is much larger than the upside region, relative to the mean curves.

9.5 SECURITISATION OF ERPS

Moody's rated the first US reverse mortgage transaction in August 1999, the SASCO 1999-RM1 deal arranged by Lehman Brothers, (see Structured Asset Securities Corporation, 1999). In the same year, Moody's also rated the first European SAM transaction, the Millshaw SAMS deal originated by Barclays Capital, (see Millshaw SAMS No.1 Limited, 1999). Fitch Ratings rated two equity release mortgage-supported RMBS transactions, the Equity Release Funding No. 5 plc (ERF5), that was issued in August 2005; and the Svensk Hypotekspension Fond 3 AB (Svensk) transaction that was done in February 2016.

The global financial crisis put a stop to issuance of ERP securitisations both in the US and the UK. The lack of available funding but also new regulatory regimes and an adverse market sentiment towards mortgage securitisation contributed to a rapid decrease of interest for securitisation in this niche asset class of ERPs.

The securitisation activity of reverse mortgages has started to improve in recent years. In 2021, Cerberus Capital Management has appointed Citibank to initiate the first public securitisation of equity release mortgages post global financial crisis. The ERP loans were acquired from Northern Rock and the Legal & General Group. Home equity conversion mortgages (HECM), or reverse mortgages as they are called in the US, originated between 2006 and 2021. Another securitisation deal is Ocwen's inaugural reverse mortgage RMBS covering $264.9 million. This is the first reverse mortgage securitisation for PHH Mortgage, a subsidiary of West Palm Beach, Fla.-based mortgage servicer Ocwen Financial. The collateral pool has 1,054 performing (25.14%) and nonperforming reverse mortgages. The nonperforming assets have 44.01% either in foreclosure or referred for foreclosure; 19.32% in default; 7.17% liquidated and 1.75% in bankruptcy status. Interestingly, this is an example of a securitisation pool that includes a high percentage of non-performing assets and other distressed assets. The main reason why this may work is that the U.S. Department of Housing and Urban Development (HUD), is covering for the losses in the pool.

Nationwide Securities Corporation is another example of securitisation of ERP loans. They are the manager for a pool of reverse mortgages with a notional of about $155.2 million. The pool consists of mortgage-backed securities from the Brean Asset-Backed Securities Trust 2023-RM6, the program's latest deal and it references loans issued by Nationwide Equities, South River Mortgage and SmartFi Home Loans, (see Mitchell 2023).

While the real ERP securitisation market is only beginning to flourish, academically the securitisation of ERP loans has been studied. A valuation framework under securitisation has been proposed by Wang et al. (2008). A very interesting approach to securitise the crossover risk for ERP loans has been suggested by Huang et al. (2011). They advocate the introduction of a crossover bond that is transferring the losses triggered by the crossover risk in an innovative manner. Bond buyers will receive a coupon that would land higher/lower than the Treasury bond with matched maturity when the actual loss on the ERP loan is lower/higher than expected. Lorenzo, Piscopo, Sibillo, and Tizzano (2021) expand on the above ideas by connecting the sise of the coupon to the difference between the expected and actual losses at each coupon date.

Lorenzo et al. (2021) describe a very general framework that can be easily be adopted by risk managers using particular models for each type of risk driving ERPs. We describe the framework here and then discuss various particularisations. For simplicity of exposition we work with a contract issued at time zero for a borrower aged x who received a lump sum payment equal to ΓH_0 that is matched with the value of collateral house after deducting the insurance premium for a no-negative equity guarantee. Note that $\Gamma \in (0, 1]$. The balance of the loan accrues based on a roll-up rate $R \equiv \{R_t\}_{t\geq 0}$ and it is described by the process $\{K_t\}_{t\geq 0}$ with $K_0 = \Gamma H_0$ and $K_t = K_0 e^{\int_0^t R_s ds}$. One can further decompose the roll-up rate process into a funding rate r_t and a risk premium π_t such that $R_t = r_t + \pi_t$, for any time t.

There is a subtle difference between loans issued in the UK and loans issued in the US. For the former, the NNEG must be absorbed by the lender by law and hence, this risk is priced in a higher roll-up rate R and valued separately on a loan-by-loan basis for regulatory purposes. In the US however, the no-negative equity risk is priced in the form of a no-negative insurance guarantee whose price is deducted from the initial lump sum. In essence then the borrower receives a lower lump sum amount γH_0 and the price of the insurance for no-negative guarantee is equal to $\Gamma H_0 - \gamma H_0$. It is also important to realise that the balance accrues in this case still starting from ΓH_0.

For pricing such loans, one should construct the cash-flows resulting from the potential final payments at ay future time t. The lender is entitled to

$$V_t = \min\left(K_t, H_t\right)$$

(9.5)

If τ_x represents the random stopping time corresponding to the termination event for this loan for a borrower age x then the probability of x dying within the time t is

$$F_x(t) = P(\tau_x \leq t) = {}_tq_x = 1 - {}_tp_x$$

(9.6)

Lorenzo et al. (2021) propose matching the initial lump sum of the contract with the expected value at time zero of the survival probability weighted discounted cash-flows V_t. Thus,

$$\gamma H_0 = E_{\tau_x}\left[E\left(V_t e^{-\int_0^t r_s}\right)|\tau_x\right]$$

(9.7)

Let's denote by L_t the loss made on the loan if this is settled at time t. This is equal to

$$L_t = \Gamma H_0 e^{\int_0^t R_s ds} - V_t$$

(9.8)

For a portfolio of N identical ERP loans, if D_t is the number of loans incurring termination events during the year t, we can compute the portfolio cash-flow at time t as

$$PC_t = D_t \min\left[K_t, H_t\right] \tag{9.9}$$

and the total loss or gain on the portfolio at time t is then

$$TL_t^P = N \times \gamma H_0 e^{\int_0^T R_s ds} - \sum_{u=1}^{u=t} PC_u e^{\int_u^t R_s ds} \tag{9.10}$$

The total expected loss at time zero will depend on the models specified for the house price evolution, the funding rate r_s, the mortality, morbidity and prepayment and so on.

Lorenzo et al. (2021) conceptualise the securitisation valuation as issuing a contract with a special purpose vehicle (SPV) to whom a premium is paid in exchange for getting all losses *in excess* of the expected total level covered, all the way to the maturity of the portfolio. The SPV will hedge themselves by issuing a securitised bond divisible into M contracts all with maturity T and face value F.

The SPV will pay the ERP issuer the cash-flows associated with a bond that pays each year t the coupons according to the formula

$$CB_t^l = \begin{cases} 0, & TL_t^P \leq E_0(TL_t^P); \\ TL_t^P - E_0(TL_t^P), & E_0(TL_t^P) < TL_t^P < C; \\ C, & TL_t^P \geq C. \end{cases} \tag{9.11}$$

where C is a quantity representing the cash outflows to the SPV.

The investor in the securitised notes will receive the opposite cash-flows

$$CB_t^i = \begin{cases} C, & TL_t^P \leq E_0(TL_t^P); \\ E_0(TL_t^P) - TL_t^P, & E_0(TL_t^P) < TL_t^P < C; \\ 0, & TL_t^P \geq C. \end{cases} \tag{9.12}$$

Rating agencies evidently introduced their own methodologies regarding equity release products securitisation. In the following, we review the main ideas behind each methodology for each of the three main rating agencies.

9.5.1 Standard and Poor's Reverse Mortgage Criteria

The criteria for rating U.K. equity release-backed securities for Standard & Poor's (S & P) are described in Johnstone, Naylor, Koranteng, Gilkes, and Quirk (2002). They are discussed here for convenience but one should note that those criteria were put together before the global financial crisis. We were not able to find more recent references pertinent to securitisation of equity release mortgages or reverse mortgages, perhaps to this market being stalled for a significant number of years.

The first important point made the this rating agency with regards to the risk drivers that need to be controlled for securitisation purposes is LTV. We also advocated for a long time that regulators should focus more on LTV and roll-up rates, which are both more effective to bring down negative equity risk.

The S & P recognises that “the amount initially advanced to a borrower is determined on the basis of (i) the borrower's estimated life expectancy; and

(ii) the expected future value of the property upon the borrower's death." This means that high LTVs should be applied mainly to older borrowers. By older one should recall that the minimum age for borrowers in this asset class is between 55 and 62.

S & P extracts information about mortality rates from group pension schemes, weighted by the pension amounts. These rates are then stressed such that there is an increase of the delay in overall borrower mortality at higher rating levels. Other adjustments that take into account geographical variations and so on may be applied.

Prepayments are assumed to occur on a lesser scale than with forward mortgages. To reflect this point S & P employs age-dependent prepayment step-wise functions for each rating category. Those values are illustrated in Figure 9.8.

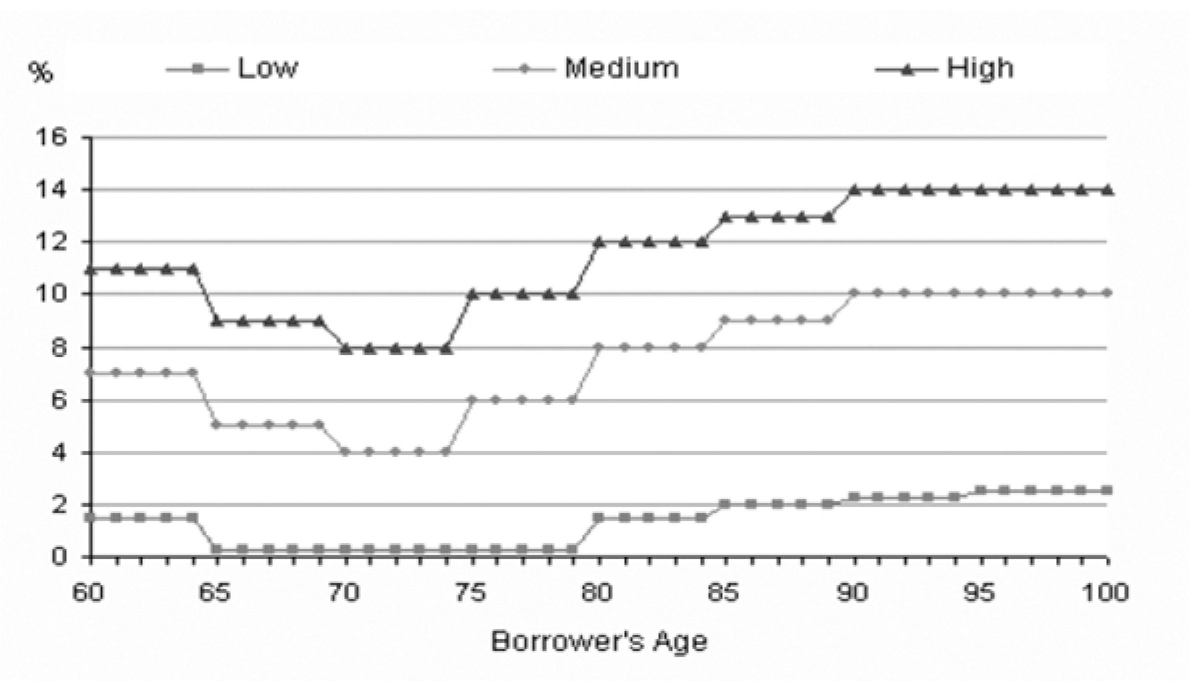


Figure 9.8 Annual Prepayment curves for changes in borrower agent used by Standard and Poor's for securitisation analysis of U.K. reverse mortgage backed securities.

Source: This graph is sourced from Johnstone et al. (2002).

The S & P have an innovative way to deal with house price risk. They employ data for the Halifax and Nationwide Building Society house price indices. The life of the transaction is divided into four distinct time periods, each period with its own house price evolution assumption. For the first year of the transaction, the current house price index is used as predictions for the next year.

For years 2-4, S & P assumes the occurrence of a recession that impacts the selling value of collateral houses for the loans in the pool in the early years of the transaction. The selling value of the house is assumed to decrease linearly for the three-year period. The declines are reflected in Table 9.1. For the third period covering years 5-15 S & P assume a recovery period all the way to year 15. To gauge the correct recovery dynamics, an annual HPI rate is calibrated from historical data to a confidence level consistent with a particular rating category. The calibration exercise employs data from the Nationwide and Halifax house price indices, using historical annual HPIs observed in non-recessionary periods for both north and south regions.

TABLE 9.1 S & P Market Value Decline Assumptions

Rating	South market value decline (% of total decrease)	South mkt. value decline % of ann. decrease over a 3-year period	North market value decline (% of total decrease)	North market value % of ann. decrease over a 3-year period

Source: The values in this table are taken from (Johnstone et al. 2002).

Rating	South market value decline (% of total decrease)	South mkt. value decline % of ann. decrease over a 3-year period	North market value decline (% of total decrease)	North market value % of ann. decrease over a 3-year period
AAA	47	19.1	25	9.1
AA	40	15.7	22	8.0
A	35	13.4	19	6.8
BBB	30	11.2	16	5.7

Source: The values in this table are taken from (Johnstone et al. 2002).

A probability distribution of possible house prices is constructed based on simulations of annualised HPI. For the last period, from year 16 to the maturity of the transaction S & P apply a stressed house price increase assumed to be 33% of the post-recession estimates obtained for the 5-15 year period.

Some other important assumptions refer to the haircut of the final sale that reflects the charges and fees due to administration, maintenance, and sale of the property The haircut rate is taken as 4% of the house value. Furthermore, the time from the loan termination event to the actual receiving of cash-flow from the sale of the property is assumed to be roughly one full year.

The securitisation analysis will also look at liquidity pressures that may appear because of large timing mismatches between the assets cash flow and the liability commitments. There is also some concern associated with the jurisdiction where the assets lie, in some countries ERPs being regulated by the Consumer Credit Act, which could delay transfer of collateral property in the hands of deal servicer.

9.5.2 FITCH RATINGS ERP Securitisation Framework

The criteria for rating ERM securitisations are outlined in (Fitch Ratings, 2017). The time of termination of each ERM is determined by a combined decrement probability covering mortality, morbidity and voluntary prepayment. For measuring and applying mortality risk Fitch uses mortality tables. The differentiation between different rating categories is gauged by applying a mortality improvement assumption to the mortality probability sourced from the mortality tables. For the Svensk deal Fitch Ratings assumed a morbidity rate equal to 15% of the corresponding mortality rate for the borrower(s) of the respective ERM. For the ERF5 deal, Fitch Ratings assumed a morbidity rate equal to 35% of corresponding mortality rate. Morbidity is modelled using assumptions derived from historical data for each country and it is also updated in time. Furthermore, historical data shows that equity release borrowers live longer than the general population. Hence, Fitch Ratings applies a mortality improvement factor that is rating category dependent as follows: 30% for AAA, 25% for AA, 20% for A, 15% for BBB, 10% for BB and 10% for B.

Prepayment rates for ERMs are naturally much lower than the prepayment rates applied to forward mortgages. For rating securitised notes of ERMs, Fitch Ratings is using the prepayment rates between a higher and a lower band, as described in Table 9.2.

TABLE 9.2 Fitch Prepayment Rates for Securitisation of Equity Release Mortgages

	High CPR(%)	Low CPR (%)
AAA	16	2.5
AA	14	2.5
A	12	2.5
BBB	10	2.5
BB	10	2.5
B	10	2.5

Source: Fitch

The most important risk for ERM is of course the house price risk. Fitch Ratings is employing a long-term house price growth rate of 2% from the loan issuance. For the ERF5 deal, Fitch Ratings maps the original property value, or subsequent if provided, onto the quarterly Halifax House Price Index and Land Registry data up to the last quarter prior to when the analysis is done.

To account for idiosyncratic risks, Fitch Ratings assumes a 100% loss for the 10 largest loans in the securitisation pool. Furthermore, Fitch Ratings assumes a house price decline scenario for each rating category. The values can be found in (Fitch Ratings, 2017) and one can see that the haircuts applied differ with geographic region as well, and also with country. Fitch Ratings assumes that risks determining the loan termination events are far less correlated with economic cycles and housing markets. This haircut should not be confused with the foreclosed sale adjustment (FSA) which is an adjustment applied to individual collateral houses associated with loans that are in negative equity. For the ERF5 deal, Fitch Ratings considers an FSA of 8.5% whilst for Svensk 3 transaction, Fitch Ratings applied an FSA of 16%.

Since a small fraction of the ERMs that are in the pool of the ERF5 transaction are index-linked such that the balance accrues based on a fixed coupon rate, plus the prevailing inflation rate, there is a swap in the transaction to hedge the inflation risk, with an upper and lower boundary to the notional.

Securitisation may also have special characteristics. The Svensk 3 deal has a four-year revolving period during which the cash receipts can be used to purchase mortgages. Fitch asks that revolving and non-revolving scenarios are run to pass the rating. In addition, Fitch Ratings requires that one year will lapse between the termination of loan event and the redemption of the loan accrued balance. Fitch is also concerned with times of illiquidity that may impair the sale of collateral houses. For the ERF5 deal Fitch Ratings applies a valuation haircut of 10% and 15% for properties in the top 5% and top 1%, respectively.

The final sale of collateral houses attract additional costs such as court fees, solicitor costs, commission payable to the estate agent for selling the property, repair costs to bring the property to a saleable condition. To account for these costs the agency assumes a level of fixed and variable costs. which are applied if the loan is in negative equity. For the ERF5 deal, the variable cost was estimated at 3% whilst the fixed cost is estimated at GBP3,000. For the Svensk deal, the variable cost was estimated at 5% while the fixed costs vary by rating category as follows: for AAAsf is SEK30k, for AAsf is SEK27k, for Asf is SEK24k, for BBBsf is SEK21k, for BBsf is SEK18k and for Bsf is SEK15k.

Servicing fees may impact the excess spread in a ERM securitised deal. The Fitch methodology employs a fee of 25bp for both the ERF5 and Svensk 3 deals.

The numerical values provided above can be used as a guide for new securitisations. However, new problems such as covid-19 pandemic and the recent geopolitical risk may require further risk assessments for future securitisations.

9.5.3 Moody's Global Approach to Rating Reverse Mortgage Securitisations

In this section we will describe the methodology behind the rating process of securitised reverse mortgage deals, following closely Moody's Investors Service (2015) and Moody's Investors Service (2022). Moody's take a quantitative approach to decide on the ratings of tranches for a securitisation of reverse mortgages. Their valuation framework is driven by stressing the risk factors contribution to the final cash-flow valuation for each of the loans in the securitisation pool. There are different levels of stress for different rating categories and Moody's employ a cash flow model of the transaction to determine whether investors would be paid in full in the respective stress scenario.

To analyse the risks, Moody's stress each of the driving factors to levels they consider are consistent with the target rating on the securities resulting from the pool. There is increased stress for the risk factors, the higher the rating. The cash flow model of the transaction aims to gauge whether investors would be paid in full under the stress scenario. The cash flow model should model how the deal allocates money from the assets, credit enhancement, and hedging vehicles among various investors in different tranches, as well as how asset losses are distributed among investors.

Here is how Moody's gauges the contributing risks for the reverse mortgage loans.

9.5.3.1 House Price Risk

Moody's base-case scenario projects a static stress overlaying a long-term house price appreciation trend with the decline in house price. The decline is assumed to take place during the first year of the deal. For Aaa (sf)-rated securities, the decline adjustments are applied directly to the house prices obtained from Moody's country-specific model for evaluating the risk of residential mortgage loans (Moody's Individual Loan Analysis, or the MILAN model). For B2 (sf)-rated securities no house price decline is applied. For Aaa (sf)-rated securities Moody's assume a zero long-term house price appreciation whilst for B2 (sf)-rated securities, they construct the stress scenario from the forecast for the change in housing prices resulting from baseline economic projections. For securities with ratings ranging from Aaa (sf) and B2 (sf), linear interpolation is applied to the stresses applied in the extreme Aaa (sf) and B2 (sf) scenarios. The analysis needs information on loan-by-loan basis on several variables such as loan balance, borrower/s age, latest available property value, valuation type, property address, property type, interest on the loan and other information specific to the loan product type The revised methodology in Moody's Investors Service (2022) assumes a decline in house prices in the first year of the deal and no decline for B2sf-rated securities, and no house price appreciation for Aaa sf-rated securities. For transactions that are off the run the decline in house prices is assumed to occur the year of the review date. The long-term assumptions for the house prices growth rate is zero for Aaa sf rating and it matches the long-run expected long-term growth rate for the B2 sf rating. For all other intermediary rating categories, Moody's uses interpolation to determine the long-term growth rate.

Moody's assumes that the collateral houses in a ERP pool will appreciate less on average than standard pool of houses because a) ERP mortgagors are relatively unlikely to repair or refurbish their houses since the upside in value will mostly benefit the lenders and b) the balance on the loan increases with time and that increases the likelihood of negative equity, reducing the incentive to maintain the home.

The stress scenarios for house price dynamics include the assumption that for this type of loans, there is a more substantial dilapidation risk. To counteract this somehow, Moody's also recognise that there is a lack of correlation in reverse mortgages between the timing of a sale and the cycle of home prices, while there is a correlation between severe home price declines and a higher rate of default among residential mortgage borrowers, see Moody's Investors Service (2015). An additional stress increase at each rating level is also required for securitisation pools that have unusual concentration, and also to account for the foreclosure costs for each liquidated loan.

9.5.3.2 Mortality, Morbidity and Prepayment Risk

Moody's assume that the mortality rates are stratified by gender and age, and adjust subjectively for possible discrepancies. They also require to differentiate between loans with a single obligor or joint obligors. The main assumption is that the loan terminates upon the death of the second individual in a couple. Hence, for joint borrowers, the joint probability of the death of both obligors is required to be modelled. Moody's allows for improvement adjustments for life expectancies resulting from improvements in living standards and in healthcare technology, but these improvements are applied more in the analysis of higher-rated securities than in our analysis of lower-rated securities, because a longer time to the termination event will put more stress on the portfolio.

Morbidity risk does not attract a penalty whilst voluntary prepayment will. If house prices increase fast that may trigger higher prepayment rates because borrowers look to withdraw equity from their homes to repay current debts. The opposite is true when house prices decline. Moody's derive the prepayment and morbidity rates, depending on the portfolio's characteristics, the rating scenario and available market data. Lack of data may lead to setting the prepayment and morbidity rates to zero.

Zhai, Stesney, and Adelson (2000) points out that in their approach Moody's takes into consideration the fact that females tend to live longer than males. Furthermore, the probability of death of both individuals in the couple is significantly lower than either individual's mortality probability, being well-known that marriage tends to increase longevity. It is important to realise the couple effect on mortality and mobility. For scenario simulations it is not beneficial to use the age of the younger borrower, or the longer average life expectancy, because that can lead to an underestimation of the couple's longevity risk. A similar rationale occurs for mobility analysis.

9.5.3.3 Interest Rate Risk

It is mainly the variable interest rate risks that may cause *future* disruptions in portfolio valuations. For loans that are exposed to variable interest rate payments Moody's assume various levels of long-term interest rates based on the applicable house price growth rate for each rating category stress. For examples, for the Aaa scenario with a severe house price decline with no recoveries, Moody's assume a low interest rate such as 1% throughout the transaction's life.

As with any other securitisation deal the structurer must gauge whether there is sufficient cash flow from the stressed assets and from other sources (such as reserve funds) to pay on time all notes. The impact of missed payments for each class of notes, as well as the likelihood and potential length of interest payment deferral is very important in determining the final rating.

The base case scenarios for US and UK, respectively, that must be passed in order to get the required ratings from Moody's are described in Table 9.3. Furthermore, additional tests are carried out under different scenarios for interest rate and house price growth rate that keep the same differential in a low rate and a high rate environment, as well as a scenario whereby the difference between the two rates is increasing over time in favour of interest rate, that is the negative equity risk is higher. These additional scenarios are illustrated in Table 9.4.

TABLE 9.3 Moody's US and UK Base Case Assumptions for Housing Price and Interest Rates

Target Rating	Aaa (sf) US	Aaa (sf) UK	Aa2 (sf) Aa2 (sf)	A2 (sf) A2 (sf)	Baa2 (sf) Baa2 (sf)	Ba2 (sf) Ba2 (sf)	B2 (sf) B2 (sf)
x% decline over 1 year	30*	35*	25	20	15	10	0
HPI returns to 100	Never	Never	20.5 yr	12.5 yr	8 yr	5 yr	N/A

Notes: *Source: Moody's Investors Service, 2015*.

*This will be based on the then prevailing country specific MILAN decline assumption

Target Rating	Aaa (sf) US	Aaa (sf) UK	Aa2 (sf) Aa2 (sf)	A2 (sf) A2 (sf)	Baa2 (sf) Baa2 (sf)	Ba2 (sf) Ba2 (sf)	B2 (sf) B2 (sf)
Growth rate after 1 year	0%	0%	1.5%	2.0%	2.5%	3.0%	3.0%
Interest Rate Level	1%	1%	2.5%	3.0%	3.5%	4.0%	4.0%

Notes: *Source: Moody's Investors Service, 2015*.

*This will be based on the then prevailing country specific MILAN decline assumption

TABLE 9.4 Moody's Stressed Scenarios Interest Rate and House Price Growth Rate

	Long-term Interest Rate	Long-term HPI Growth Rate	Interest Rate and HPI Growth Rate Lag
Aaa (sf) base case	1%	0%	1%
Aaa (sf) scenario 1	8%	7%	1%
Aaa (sf) scenario 2	8%	5%	3%

Notes: *Source: Moody's Investors Service, 2015.*

HECM loans are guaranteed by the FHA who covers the difference between the loan balance and the house value, as long as the home is sold within six months of entering real-estate-owned (REO) status. If the collateral house is not sold within six months of it entering REO status, the FHA then asks the securitisation manager to solicit an appraisal of the house from a Department of Housing and Urban Development (HUD) approved appraiser, and the FHA will only guarantee the deficiency up to the appraisal amount. In the case when the house is subsequently sold for less than the appraisal amount, the FHA will provide the negative equity differential but the securitised notes will be impaired by a loss equal to the difference between the appraisal value and the actual sales price of the house. Thus, the losses arising from HECMs loans will be considered by Moody's (see Moody's Investors Service, 2015) vis-a-vis the following factors:

- Using Moody's stress assumptions for the timing of maturity events, house price changes, and the interest rates on the loans, whether a loan is likely to have positive equity in each period after origination,
- The likelihood that a collateral house cannot be sold within six months of entering REO status
- The likely shortfall between the appraisal value (determined in the HUD-mandated appraisal conducted six months after entering REO) and the actual sale price of the house.
- One-third of the costs of foreclosure that is unreimbursed by HUD

It should also be noted that 85% of HECMs with negative equity is not going to be sold within the six-month REO window and will require FHA-mandated appraisal. Moody's assumptions regarding the likely shortfall between the appraisal value and the sale price of the house is the same 20% but it changes with the rating regarding the time horizon that is applied, as follows: Aaa and Aaa2 until maturity; A2 has 20% for 10 years and then 0 thereafter; Baa2, has 20% for 5 years and 0% thereafter, Ba2 has 20% for 3 years and 0% thereafter and B2 has 20% for 2 years and 0% thereafter.

For mortality, Moody's employ annual mortality improvement factor stresses based on rating levels and current borrower's age, see Moody's Investors Service (2015). For the Aaa (sf) case, Moody's consider a 5% annual mortality improvement in the US whilst for UK, they use a 7.5% mortality improvement for ages 60-70 and 5% for ages 70-plus. For both countries, the borrower age is capped at 120. The values used for scenario stressing are described in Table 9.5.

TABLE 9.5 Moody's US and UK Assumptions for Mortality Improvement Factor Stress

Age US	Aaa (sf)	Aa2 (sf)	Aa2 (sf)	Baa2 (sf)	Ba2 (sf)	B2 (sf)
60-120	5.0%	4.0%	3.0%	2.0%	1.5%	1.5%
Age UK	Aaa (sf)	Aa2 (sf)	Aa2 (sf)	Baa2 (sf)	Ba2 (sf)	B2 (sf)
60-70	7.5%	6.5%	5.5%	4.5%	3.5%	3.5%
70-120	5.0%	4.0%	3.0%	2.0%	1.5%	1.5%

Notes: *Source: Moody's Investors Service, 2015*

Moody's also consider an age improvement scenario, taking into account the possible sample selection bias. In other words, the borrowers for this specific type of loans may be elderly who take better care of themselves and they may live longer. To cover for this possible situation, Moody's will use an age setback of 10 years. This implies that a 80-year-old is assumed to have the mortality rates of a 70-year-old, and so on. The adjustment is the same for US and UK but it varies with the rating as follows: Aaa 10 years, Aa2 8 years, A2 6 years, Baa2 4 years, Ba2 2 years and B2 2 years.

The above components can be dealt with individually. When these are ready, in order to comply with Moody's framework for rating reverse mortgages, a structurer will then be in a position to compute the probability-weighted cash flows associated with each loan in the securitisation pool. The contractual roll-up interest rates on the loans will project the loan balance for each period in the future. The timing of settlement for each loan is also projected based on the stressed assumptions. Finally, the structurer will determine the probability that each slice of the total loans balance is paid in full in each future time period to the owners of those securitised notes.

It is useful to consider the following example, discussed in Moody's Investors Service (2015). Consider the ERP loan with the following characteristics: the loan initial balance is £ 150,000 for a collateral house worth £ 300,000 for a borrower that is male and it is aged 80. The roll-up rate is fixed at 5% and the house prices are assumed to grow by 2% per year. The time to maturity is taken as 40 years (so 120 years maximum to live) and the target rating is A2.

The cash-flow scenarios for rating by Moody's are generated using the following steps.

Step 1 Generate the paths for the house price index and the loan interest rate, based on the target rating and structural features

Step 2 Compute the adjusted annual mortality rates, based on the annual mortality rate stress for the target rating as shown in 9.5. Add the assumed morbidity and mobility rates.

Step 3 Generate the paths for the property value starting from the initial property value and the returns path for the house price index from Step 1. The following equation is used to produce the value of the collateral house in year *t*

$$H_t = H_0 \times (1 + HPIret(1)) \times (1 + HPIret(2)) \times \cdots (1 + HPIret(t$$

where $HPIret(j)$ is the returns on the house price index over the year $(j-1, j]$. Compute the cumulative loan balance using the initial loan balance and interest rate path. The following equation is used to generate the loan balance at the end of year *t*

$$OLB(t) = OLB(0) \times (1 + R(1) \times (1 + R(2)) \times \cdots (1 + R(t)))$$

where $R(j)$ is the interest rate applied to the year $(j-1, j]$.

Step 4 Determine the projected probability-weighted cash flow in each period by applying the maturity event rate for each period (from Step 2) to the minimum of the cumulative loan balance and the property value for the period (from Step 3). Make sure you apply the A2 (sf) improvement factor to the final cash flows.

In Moody's Investors Service (2022) the decrement probabilities are calculated based on the following formulae:

1. $s(t) = (1 - q(t)) * s(t-1)$ the probability that the mortgagor is alive after *t* years from the date of the analysis, with $s(0) = 1$.
2. $p(t) = q(t) * s(t-1)$ the maturity event rate or the probability that the termination of the loan arrives *t* years from the date of the analysis. Note that $\sum p(t) = 1$.
3. $Q(t) = q(t) * (1 - if)^{t-1}$ is the adjusted conditional one year probability of death $q(t)$ after accounting for the improvement factor *if*.

4. $S(t) = [1 - Q(t)] * S(t-1)$ the adjusted probability that the borrower is alive after *t* years following the date of our analysis, after accounting for the improvement factor *if*, with $S(0) = 1$.
5. $P(t) = Q(t) * S(t-1)$ the maturity event rate adjusted for the improvement factor *if*.

The effect of improvement factor for a borrower age 80 was highlighted in Moody's Investors Service (2015) and it is depicted in Figure 9.9.

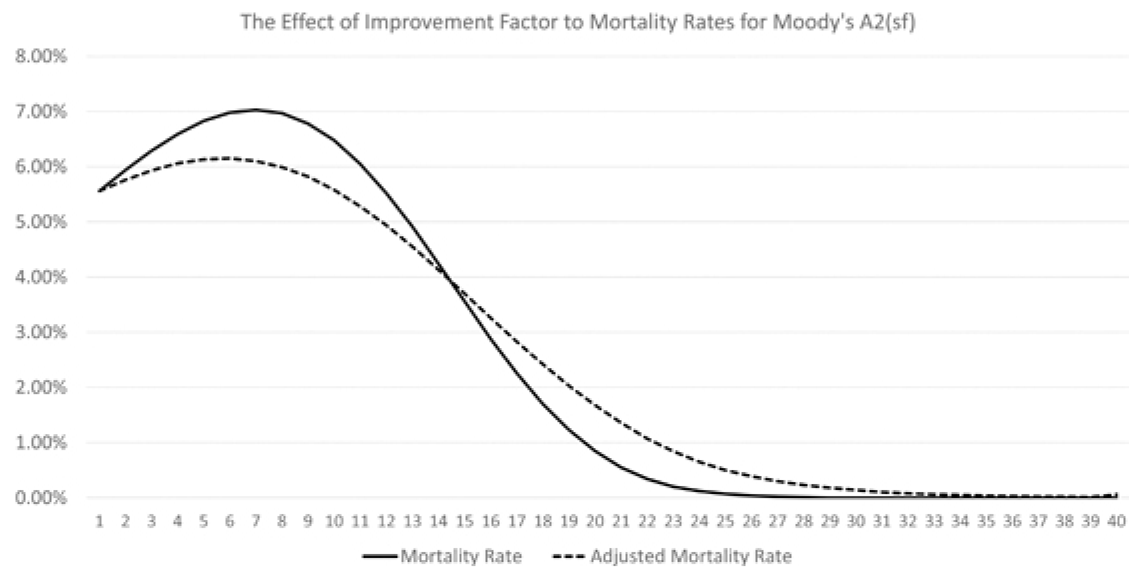


Figure 9.9 Adjusted mortality curve due to improvement factor for a 80-year-old borrower. Source: Moody's Investor Service.

Here we illustrate the calculations that would follow the latest ERM securitisation methodology as illustrated in Moody's Investors Service (2022). The calculations must be done loan-by-loan. Our example is based on a 60-year-old male borrower with a collateral house worth GBP 300,000, an LTV equal to 50%, a roll-up rate equal to 5% per annum, a fixed house price growth rate at 2% per annum and an improvement mortality factor of

2%. It is also assumed for simplicity of exposition that the loan termination event is due only to mortality. The mortality rates are sourced from the

Office of National Statistics national life tables[1], United Kingdom. The illustration in Figure 9.9 is based on the 2015-2017 life table. Table 9.6 shows the calculations for the conditional loan termination rates $p(t)$ using standard mortality rates $q(x)$ and the rates $P(t)$ adjusted for lifestyle improvement factors increasing longevity.

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[1]According to the Office of National Statistics, “National life tables, which are produced annually for the United Kingdom and its constituent countries, provide period expectation of life statistics. Period life expectancy is the average number of additional years a person can be expected to live for if he or she experiences the age-specific mortality rates of the given area and time period for the rest of his or her life.”

TABLE 9.6 Loan termination event rates using standard mortality rates anc

year	q_t	$s_t = (1 - q_t)s_{t-1}$	$p_t = q_t s_{t-1}$	$Q_t = q_t(1 - if)^{t-}$
1	0.83%	99.1687%	0.8313%	0.83%
2	0.92%	98.2541%	0.9146%	0.90%
3	1.02%	97.2540%	1.0000%	0.98%
4	1.09%	96.1894%	1.0646%	1.03%
5	1.20%	95.0327%	1.1567%	1.11%
6	1.33%	93.7659%	1.2668%	1.20%
7	1.44%	92.4122%	1.3537%	1.28%
8	1.57%	90.9577%	1.4546%	1.37%
9	1.73%	89.3851%	1.5726%	1.47%
10	1.83%	87.7505%	1.6346%	1.52%

year	q_t	$s_t = (1-q_t)s_{t-1}$	$p_t = q_t s_{t-1}$	$Q_t = q_t(1-if)^{t-1}$
1	0.83%	99.1687%	0.8313%	0.83%
11	2.03%	85.9713%	1.7792%	1.66%
12	2.23%	84.0519%	1.9194%	1.79%
13	2.55%	81.9086%	2.1433%	2.00%
14	2.81%	79.6050%	2.3036%	2.16%
15	3.14%	77.1052%	2.4998%	2.37%
16	3.51%	74.3977%	2.7075%	2.59%
17	3.88%	71.5082%	2.8895%	2.81%
18	4.35%	68.3961%	3.1121%	3.09%
19	4.81%	65.1063%	3.2898%	3.34%
20	5.40%	61.5917%	3.5146%	3.68%
21	6.01%	57.8919%	3.6999%	4.01%
22	6.65%	54.0414%	3.8505%	4.35%
23	7.54%	49.9669%	4.0745%	4.83%
24	8.48%	45.7319%	4.2349%	5.33%
25	9.47%	41.4028%	4.3292%	5.83%
26	10.69%	36.9787%	4.4240%	6.45%
27	11.86%	32.5936%	4.3851%	7.01%
28	13.34%	28.2470%	4.3467%	7.73%
29	14.99%	24.0141%	4.2328%	8.51%
30	15.93%	20.1888%	3.8253%	8.87%
31	17.86%	16.5829%	3.6059%	9.74%
32	19.62%	13.3286%	3.2543%	10.49%
33	21.51%	10.4619%	2.8667%	11.27%
34	23.82%	7.9695%	2.4924%	12.23%

year	q_t	$s_t = (1 - q_t)s_{t-1}$	$p_t = q_t s_{t-1}$	$Q_t = q_t(1 - if)^{t-}$
1	0.83%	99.1687%	0.8313%	0.83%
35	26.22%	5.8798%	2.0898%	13.19%
36	28.85%	4.1835%	1.6963%	14.22%
37	30.63%	2.9022%	1.2813%	14.80%
38	32.88%	1.9480%	0.9542%	15.57%
39	36.81%	1.2310%	0.7170%	17.08%
40	38.47%	0.7574%	0.4736%	17.50%

Figure 9.10 depicts the conditional likelihood under standard mortality rates input and under mortality rates adjusted for improvement factor. There is a cross over point after roughly 30 years suggesting that the improvement factors make a positive difference in the sense of longer borrower life, and hence higher NNEG risk, up to the age of 90; the standard borrowers surviving up to 90 are quickly becoming much riskier in the sense that their conditional mortality rates are smaller and smaller so their corresponding NNEG risk is higher and higher.

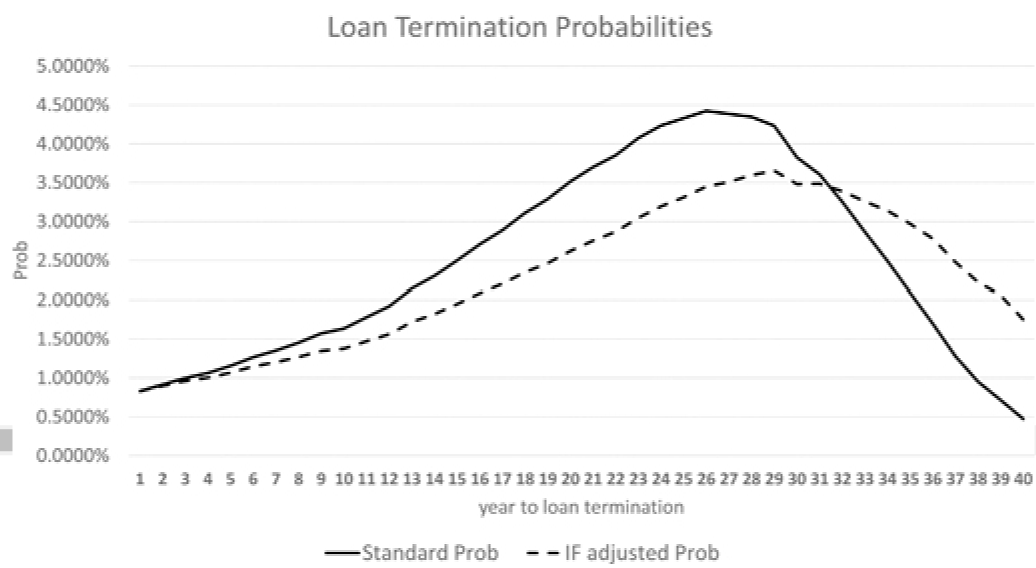

Figure 9.10 Adjusted mortality curve due to improvement factor for a 60-year-old borrower.

Table 9.7 describes the year-by-year calculations for the cash-flows incoming to the lender. Moody's is using a constant annual house price growth rate that is reminiscent of the early models applied to value ERPs. Not only is it not true that house prices grow at a constant rate but over such long periods as applied in NNEG calculus, large discrepancies can easily appear. Furthermore, house prices rally move in a more stochastic manner and therefore it would be more appropriate to use more updated technology and apply a stochastic model as a data generating process for house prices.

TABLE 9.7 Calculations of inflows cash-flows to the lender with standard and improvement factor adjusted loan termination rates.

year	$H_0(1+h)^t$	$V_0(1+R)^t$	$\min(H_t, K_t)p_t$	$\min(H_t, K_t)P_t$
1	306,000	157,500	1,309	1,309
2	312,120	165,375	1,513	1,482
3	318,362	173,644	1,736	1,668
4	324,730	182,326	1,941	1,828
5	331,224	191,442	2,214	2,045
6	337,849	201,014	2,546	2,307
7	344,606	211,065	2,857	2,540
8	351,498	221,618	3,224	2,813
9	358,528	232,699	3,659	3,136
10	365,698	244,334	3,994	3,363
11	373,012	256,551	4,565	3,778
12	380,473	269,378	5,170	4,210

year	$H_0(1+h)^t$	$V_0(1+R)^t$	$\min\,(H_t, K_t)p_t$	$\min\,(H_t, K_t)P_t$
1	306,000	157,500	1,309	1,309
13	388,082	282,847	6,062	4,860
14	395,844	296,990	6,841	5,405
15	403,761	311,839	7,795	6,075
16	411,836	327,431	8,865	6,825
17	420,072	343,803	9,934	7,566
18	428,474	360,993	11,234	8,479
19	437,043	379,043	12,470	9,345
20	445,784	397,995	13,988	10,432
21	454,700	417,894	15,462	11,506
22	463,794	438,789	16,896	12,583
23	473,070	460,729	18,772	14,039
24	482,531	483,765	20,435	15,415
25	492,182	507,953	21,307	16,294
26	502,025	533,351	22,210	17,313
27	512,066	560,018	22,455	17,967
28	522,307	588,019	22,703	18,781
29	532,753	617,420	22,551	19,465
30	543,408	648,291	20,787	18,923
31	554,277	680,706	19,987	19,328
32	565,362	714,741	18,399	19,160
33	576,669	750,478	16,531	18,788
34	588,203	788,002	14,661	18,459
35	599,967	827,402	12,538	17,825
36	611,966	868,772	10,381	17,017
37	624,206	912,211	7,998	15,490

year	$H_0(1+h)^t$	$V_0(1+R)^t$	$\min\,(H_t, K_t)p_t$	$\min\,(H_t, K_t)P_t$
1	306,000	157,500	1,309	1,309
38	636,690	957,822	6,075	14,161
39	649,423	1,005,713	4,656	13,380
40	662,412	1,055,998	3,137	11,592

If medically driven improvement factors are embedded in the risk enterprise analysis, one should expect a cross-over point between the cash flows curves with and without improvement factors applied to survival rates. This crossover point, which is clearly identifiable on the graph in Figure 9.10 may influence portfolio selection for securitisation purposes. Depending on their risk appetite, investors who are buying the ERP backed notes may require a larger or lower concentration of loans with borrowers of certain characteristics.

The other important note is that the values on the curves illustrated are representative for one scenario simulation. More insightful analysis requires many more scenario simulations and the cross-over points in different scenarios may vary across the time spectrum. Then, based on these multiple simulations, one can build a distribution of quantities of interest.

One may think that performing this type of enterprise risk management could be computationally costly with many quantities changing rapidly. This is not true by and large because many stochastic variables of interest do not change their distribution often. Mortality rates stay the same for long periods. House prices evolve with more or less the same parameters for long periods. Roll-up rates are fixed. Hence, once the analytics is finalised for an initial analysis, performing repeated analyses is facile.

In Figure 9.11 we present the cash-flows incoming to the lender of ERMs. The cross-over point after 30 years for a 60-year-old borrower means that cash flows are higher for standard borrowers for the first 30 years, and then they are better for the very old borrowers who were in the category benefitting of improvement factors such as medical improvements. For the latter, the differential in the comparative cash-flows grows rapidly with each year. It should also be noted that for the first 10 years or so, for the 60-year-old borrowers category, the improvement factors make almost no difference. Furthermore, the largest probability-weighted cash flow seems to be obtained after about 28 years, and this is around GBP 22,000. The expected cash flow can be calculated as GBP 429,859 under the standard scheme and GBP 416,955 under the improvement factor scheme.

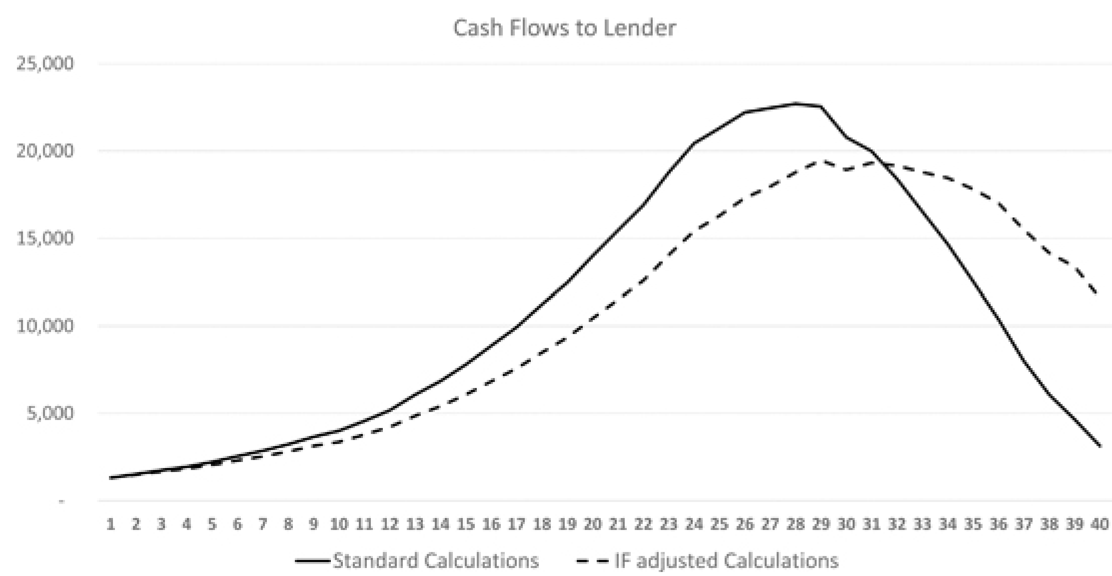

Figure 9.11 Cash-flows for the lender due to improvement factor for a 60-year-old borrower.

One major problem with all house price dynamics assumptions for all three main rating agency is the fact that they prefer to use a deterministic scenario set-up. It would be useful to understand the impact of using different data sets and different models on the cash-flows generated under a securitisation scheme. In Figure 9.12 we present the cash-flows the lender should expect for the same loan as above but using different models of evolution for house prices. The graphs on the left side are generated under the relevant ARMA-EGARCH model employing monthly and quarterly house price data for Nationwide UK index, while the graphs on the right side illustrate the cash-flows obtained under the assumption of a GBM data generating process for house prices. This is only a snapshot based on a single pathway but more simulations can be obtained by doing batches of simulations from the same model. We observe that the model could make a difference as well as the data on which it is calibrated.

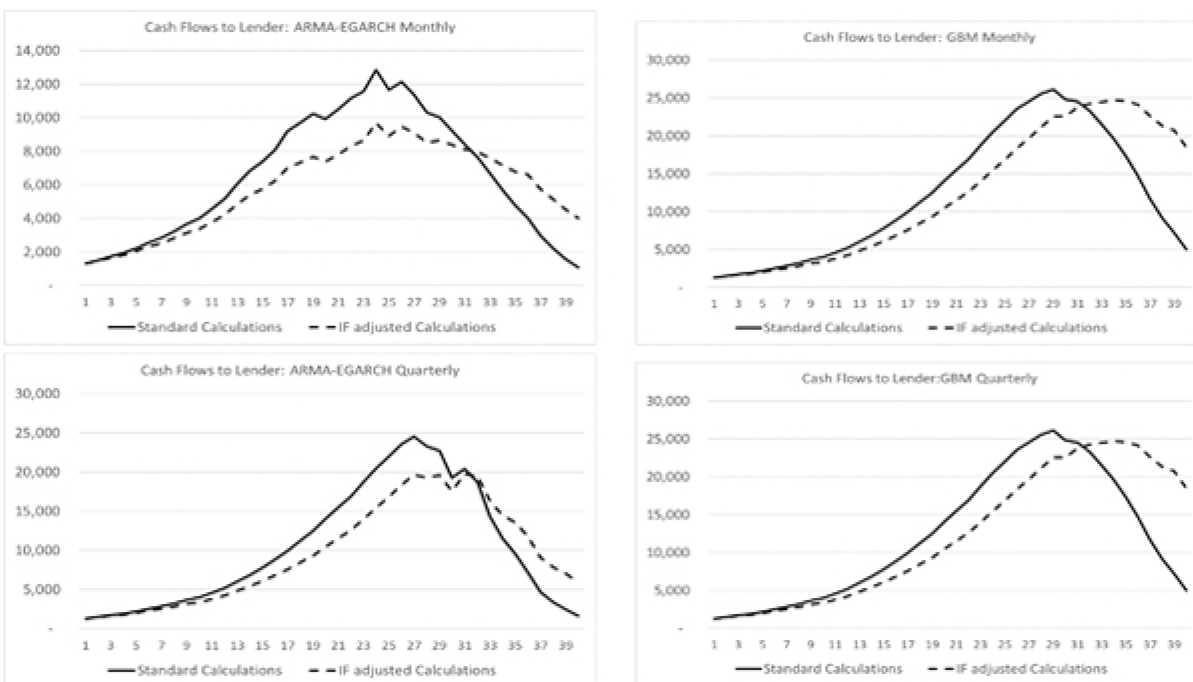

Figure 9.12 Comparative cash-flows to lender under GBM and ARMA-EGARCH models for house prices due to improvement factor for a 60-year-old borrower.

9.5.3.4 Other Considerations

For a securitisation structure, there are also some other important considerations that Moody's take into account when deciding on their rating. One of them is counterparty credit risk generated by hedge counterparties. Moody's calculates the probability of a transaction becoming unhedged and deriving additional potential losses. The ratings could be adjusted to reflect the linkage and additional loss.

Another one is operational risk, which can arise from various sources, to cause financial distress of a service provider to a reverse mortgage securitisation. It should be noted that unlike traditional mortgage transactions, ERP deals not need the servicer to process payments or make collection calls. The servicer mainly has to determine if a maturity event has occurred, updating the collateral house values and ensuring that payment of insurance is current.

A new type of risk is associated with the Environmental, Social and Governance or ESG. The ESG issues may impact the ratings of securities backed by a portfolio of ERMs. Currently this risk is gauged by Moody's following their cross-sector methodology that describes their general principles for measuring the ESG impact.

It is important to realise that the ERP as an asset class carries exposure to all dimensions of ESG. In particular the S could be very important here because these loans may help borrowers who are lower on the social scale to benefit from long-term care and advances in medicine that are not available through the national system; likewise G is also highly relevant due to the regulations around NNEG issues but also regulating participation in this market.

9.6 SUMMARY AND FURTHER READING

This chapter focused more on the investor side and looked at ERP portfolios from an investor point of view. Even if risk management is required by the regulator to be performed on a loan-by-loan basis, a portfolio aggregate analysis is important for the initial investors who may have to wait a long period of time before getting some cash-flow inflows. The same type of the analysis is relevant for lenders who want to prepare a portfolio of ERP loans for securitisations.

The results presented in this chapter emphasise that the lender/investors may be mislead in terms of projected cash-flows that they may expect to arrive in the future because of looking at this problem through the lenses of the wrong model. This could create a state of irrational exuberance and it may determine the lenders to be more risk tolerant than they should be.

The portfolio analysis based on scenario simulations is very important given that the business clock for this asset class is quite slow, the reverting cash flows needing perhaps decades to come back to the lenders. During this period of time, many assumptions and macroeconomic variables could change.

Securitisation is based on some similar assumptions across the three main rating agencies but there are also significantly different assumptions. One major criticism of methodologies for all three agencies is the assumption of constant house price increase or constant stressed house price decrease. Ignoring the stochastic evolution of house prices and working with constant HPI growth rates, whether positive or negative, may create a false sense of security. House prices historically go on long periods of positive returns, driven sometime by irrational exuberance, the Jones effect, and herding, followed by usually abrupt market corrections or even crashes.

Different rating agencies require different hurdles to be cleared in order to approve their ratings. The methodologies are quite different among themselves. One possible way to circumvent the problem of being compliant with all three rating agencies is to have an enterprise risk management system in place that is capable to produce simulated scenarios under various stresses.

An innovative approach to deal with NNEG risk using elements of securitisation is presented in Andrews and Oberoi (2015). They propose using a central intermediary that is a public-private partnership (PPP). This PPP acts more or less like a broker between lenders and homeowners. The PPP will get borrowers to apply and they will appraise the collateral houses, establish the LTVs and consider other aspects related to relevant mortality, morbidity and so on. On the other hand the lenders will advance the funds. The interest payments will include a floating base rate plus an annual spread reflecting the PPP administrative costs and also an income component driven by house returns. When a loan is terminated the lenders will get the accrued interest minus the PPP's administrative fee. The NNEG charge (similar to an insurance premium) is retained by the PPP. Note that under this structure the total return to the lenders combines fixed and floating cash-flows and it may even become negative if the house price index is less than the cumulated balance on the loan. In essence, the PPP strips off the negative equity risk and then the remaining structure is similar to standard or forward mortgages based on direct cash-flows.

FURTHER READING

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CHAPTER **10**

Summary Discussions

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THE ERP AS financial instruments will get more and more attention in the coming years. This is not going to be driven by the academic community but by real problems emerging from governments such as cost of long-term care and pension deficits.

The residential housing portfolios constitute the largest component of wealth for many developed nations. It seems feasible and preferable that a mechanism should be encouraged allowing elderly to carve a portion of the house value at some moments in time to smooth the financial evolution in their final years.

While ERP can have many benefits, regulators could be worried that elderly could get exploited and expropriated of their wealth but also that the insurance sector may collapse if pricing and risk management is not done the correct way.

This is a market that extracts first the benefits for society and only later on rewards the investors or insurers for the risk they are taken. In order for this market to thrive, a judicious balance must be obtained between regulatory driven costs and returns on investment. To this end, the interests of all investors, the regulators and the customers ought to be represented.

Given that this market depends directly on interest rates regimes and the evolution of house prices, an equilibrium achieved at some point in time may not be workable anymore in the aftermath of sudden changes in these two risk drivers. Therefore, the dialogue and collaboration between those three different groups has to be continuous.

The ERP asset class inherits most problems encountered in other more traditional asset classes. There is a clear need for more data to be made available to researchers in order to facilitate best approaches that should be more widely disseminated. Furthermore, this asset class is exposed to problems arising from actuarial science and insurance sector, as well as from financial markets and financial engineering.

In our book, we emphasised wherever it was necessary the big impact that model risk can make, parameter estimation risk as well as model selection risk. This generic issue is deeply embedded in all financial markets and perhaps less so in insurance markets.

The material developed in this book was not meant to be exhaustive. It is merely for opening the debate and motivate other researchers to contribute as well. There are still many problems that can receive better solutions.

10.1 SOME IMPORTANT POINTS

Here we would like to revise some of the main outcomes of our experience and the research done for this book. These points may serve in sharpening the policy documents, the procedures recommended to be followed in practice and to increase awareness in the community contributing to the ERP market.

The first point is that there is a different design for the same problem in the US versus in the UK, the main markets for ERPs. The former is using insurance and hence reinsurance for these kind of loans whilst the latter is asking the issuer to calculate the NNEG that effectively they need to cover in case negative equity risk manifests at the settlement of the loan. One can also think that the former is seen more as an insurance product whilst the latter is conceptualised as a hybrid product with characteristics from capital markets but also from insurance markets.

House price risk is the fundamental risk here. This is an advantage, since this type of risk is well researched and understood in both academia and real-estate industry. It is also a disadvantage because house price risk is captured as an option payoff and real-estate derivatives are less researched and understood by both academia and practice. The main reason for the latter aspect is the lack of real-world developments of markets in residential real-estate derivatives.

One may ask why we do not have models for valuing options on house price indexes or house prices in general? The problem is however not a trivial one. To start with, this type of underlying for options contracts is tricky because short-sales is not possible. Thus, a higher level of quantitative finance is required to organise option pricing on house prices on robust theoretical principles. Secondly, the time series of this underlying are low frequency and not high frequency. One immediate problem is then the inability to estimate volatility of this underlying properly. Thus, volatility specialised models may be needed and the generally speaking GARCH family does represent the state-of-the art in this area to some extent.

The other problem, which is often ignored, is the bigger parameter estimation risk or uncertainty. This points out to a real problem. One may have a very good model but may do a bad job in estimating the parameters correctly. The results may be trusted because the model is advanced and it covers all financial economics subtleties around the product but the results may be well off the mark if the parameters driving the model are mis-estimated by a large margin. This points to another problem. The results might be "right" for double the wrong reason. If the wrong model is employed and the parameters are badly estimated, it is not impossible the results to look reasonable. However, recalling that two wrongs do not make one right, this situation is actually quite dangerous. Some enthusiasts may be already committed that, in order to save the elderly, the society and who knows even the planet, a high level of volatility ought to come out of modelling house prices. It is a shame that we do not have fully functioning house price derivatives markets, the volatility would be directly recoverable then and it would reflect the views of multiple market agents. In the absence of that, one can make their pick. In order to achieve the level of valuations *a priori* decided on, the enthusiasts cannot employ complex models because they are not directly controllable. What about a model that depends mostly on volatility? That would suffice since this simple model would allow using volatility of house prices as a lever to push valuations of options on house prices to the level already decided upon. In our opinion, no matter how many books and articles are written, they will not stand ready to be convinced because they are already convinced. The only panacea for this is to introduce proper house price derivatives markets.

One other important problem we uncovered is the rental yield problem. Researchers often imitate the idea to get current stock prices from future dividends stream of cash-flows to get valuations frameworks for current house prices from rental income, net rental income to be more precise. Perhaps not surprisingly most researchers on both sides find a value of 5% for the stock dividend yield and for the house rental yield. Anyone observing closely the historical time series in the two asset classes will see that 5% is not always a great estimate but it is often a close-enough estimate. For equity stock markets, one important idea going back to Shiller (1981), see also Shiller (2003), and Shiller (2015), is that the volatility induced by dividends is in contradiction with the volatility observed directly on stock prices. To us, it looks like the same could be the case with the rental yields and the house prices, but we admit that this line of research has not yet been exploited as far as we know. Moreover, the other thing that is important is that the level of dividends paid by the constituents of a stock index (think of S & P 500) are taken exactly as they are to establish the dividend yield paid by the index. This is not and cannot be subject to negotiations. Therefore, it looks absolutely bizarre to us that for a house price index, the houses that do not pay any rental income are allocated an artificial level imposed by matching properties. One can easily spot index constituent firms that are in the same sector, roughly the same size and yet the dividend payments are quite different. Why this is not allowed for a house price index. In addition, if a constituent firm does not pay in a given year any dividend then the corresponding dividend input into the calculation of the index dividend yield is zero. This is on accounting rules, there is nothing philosophical about it. But when we talk about houses, the parallel is not kept. Parallels are there to be drawn in full not only piece-wise. Of course more research is needed on rental yields and how they contribute to the house price formation. In doing so, we need to understand what happens to rents and house prices when the percentage of houses that are rented out goes towards 100%, 50% and 0%. In some small populations countries with a lot of land, it is not possible to think of a scenario where house ownership is close to 100%. In Romania, house ownership is close to 90% and it is well known that rents are very low compared to property prices, close to 30. Same phenomenon happens though in Singapore, a country with little land given its population. The population density in Romania is 79 per square kilometer whilst in Singapore the population density is 7804 per square kilometer, about 100 time fold larger. The property price to rents ratio for Singapore is about 30 as well. The problem is that the density of population is 100 times fold higher in Singapore than it is in Romania. And yet, the property price to rents ratio is the same. This can be explained by the fact that in Romania there is high supply of rental properties whereas in Singapore there is high demand. The rental yield though in any property index should look very different in the two countries because rents are collected for most available properties in Singapore but this is not the case in Romania.

A neglected issue in relation to ERP calculus is real-estate bubbles. This phenomenon is quite perverse in the ERP context since in the first phase, when the bubble gets inflated, NNEG risk gets smaller and smaller short term. However, in the second phase, when the bubble gets burst, the NNEG risk would naturally spike up. There is plenty of research discussing real-estate bubbles, see Herring and Wachter (2003), Fabozzi, Kynigakis, Panopoulou, and Tunaru (2020). House price bubbles are not confined to the US or the UK only, they are present more internationally and they could be affecting large cities or regions or entire countries, see Hui and Yue (2006) for China, Yiu, Yu, and Jin (2013) for Hong Kong, Roehner (1999) for Paris, Akin, Montalvo, Villar, Peydro, and Raya (2014) for Spain, Ardila, Sanagdol, P., and Sornette (2017), Nneji, Brooks, and Ward (2020) and Fabozzi et al. (2020) for US, and Brooks, Katsaris, McGough, and Tsolacos (2001) and Fabozzi et al. (2020) for the UK.

More research needs to be funded in this area. Although the authors of this book made efforts to apply for research funding from the appropriate funding bodies in the UK and even if the letter from David Rule was the focal point of our request, it seems that the topic is deemed less important than other topics funded perhaps more generously than their impact would seem to suggest. We need relevant data directly related to ERMs and we need data more internationally. The data can be even synthetic, produced by regulators for model validation purposes. Then researchers could collaborate and why not compete to find out best possible ways forward. There should be an annual conference or at least a dedicated session or sessions at important annual conferences such as LIFE (IFoA) or IME.

Covid-19 highlighted yet another problem. Before the eruption of the pandemic the common view on the assumptions behind the NNEG calculations were that mortality, interest rates and house prices are independent risks. Some authors did mention that perhaps interest rates and house prices are related somehow but nobody was prepared to support the view that there could be a connection between mortality on one side and interest rates and house prices on the other. Furthermore, nobody thought that there could be a spike in mortality for elderly in a short period of time like one year. However, the covid-19 showed that yes, pandemics could trigger spikes of mortality in the elderly. Even more so, the same underlying factor, in this case covid-19, could paralyse to a great extent the real-economy which in turn will impact on house prices and ultimately on interest rates that are moved by governments to re-establish order. The necessity to consider such exogenous shocks was not there. Now we lived though it and we cannot ignore it. The problem is that this event is exogenous and it is almost impossible to bring it inside, in other words to endogenize this process. Therefore, we argue, that to cover for this scenarios, separate stress testing needs to be added to calculations leaving out this possibility.

Other possible exogenous shocks may be caused by climate change. The combination of extremely hot summers, fires over large portions of land or floodings or cyclones and clouds of dust and the inability of governments to deal swiftly with the problem may create another situation of sharp spikes in elderly mortality, a sharp drop in house prices in some areas at least and a sudden fall in GDP leading to immediate adjustments in interest rates.

Related to the last two points above, one elegant and still powerful by design solution is securitisation. There are signs of reinvigoration in this area but nowhere near the volume required. Given the continuous and growing lack of funding for the ever more expensive long-term care, securitisation may hold the key for a healthy expansion of ERP market globally.

One may think that regulators have a clear defined methodology for systemic risk, being preoccupied by the impact that extreme movements in the market/companies may have on companies/market. We should consider a similar concept of people or societal systemic risk. We need a framework of defining the concept clearly, a methodology to measure it and a policy to deal with it.

10.2 QUO VADIS

The regulator plays an important role in establishing and maintaining a viable ERP market. They can get great support from academia to run comparative studies covering different models for pricing house price derivatives, different methods of estimation of essential parameters behind those models. Furthermore, using their position on the market and making use of the information they have from the most important players, the regulators, in conjunction with academics as independent researchers and also with practitioners as the main beneficiaries and contributors, could design synthetic datasets that are necessary for a judicious investigation.

The covid-19 pandemic clearly pointed out that a separate framework for stress testing as an enterprise risk management is missing but quite urgently required. The stress should be applied to each individual risk factors, but most importantly it should be also applied to all risk drivers jointly. The future exogenous shocks cannot be guessed regarding their nature so the only thing that it can be assumed is that, whatever that exogenous shock may be, all risk drivers may simultaneously move in the wrong direction to a more extreme levels.

An important emerging point is green finance and sustainability compliant finance. The ERPs could easily become a major component into socially responsible investments (SRI) portfolios, if the loans will come with a precondition that all energy used in the collateral house would be green. This condition could be easily met if either the borrowers would use part of the released funds to replace old energy technology with green one, or even simply requesting that the energy bought from suppliers is 100% green.

For example a 65 old borrower or couple borrowers could use some of the funds released from the equity in the house, perhaps even jointly with other government backed schemes for green energy, to install solar panels and or other green energy generation devices. Then, the borrowers could possibly benefit from generating green energy and if they have surplus they could even generate further funds by giving green energy back on the grid. The proportion of green energy used would be easy to monitor and quantify and this would link directly to the ERP loans in investors portfolios. Such well selected loans would qualify as sustainable for along time

The equity release products are becoming so important for society nowadays. Academic journals should dedicate more space to articles focused on issues related to this asset class. There should be an international annual conference where best practices and new ideas should be shared and open to scrutiny.

Data is becoming a major issue for model construction and validation of risk management measures. Research funding bodies should work hand in hand with regulators and main players in this sector to put together data that can be made widely available to other practitioners and academics. Long-term care data has been an issue for a long time and morbidity tables are generally missing. Data on dilapidation is also extremely important to establish the magnitude of the haircut that should be applied to the sale value of the collateral houses in these specialised loans. Data on rental yields and even dedicated research on the linkage between the house prices and rental income is much needed; it is also expected to vary across different economic and regulatory regimes.

Modelling wise we made a very important point in this book that parameter estimation risk should not be ignored and somehow it should be measured. The regulator should be interested in the parameter estimation risk embedded in the valuations produced by different issuers using the same models. How are these discrepancies reconciled?

Product design may look into new frontiers. For a substantial proportion of borrowers the terminal risk is not given by mortality but by morbidity. In other words, the loan is terminated when the borrower moves into long-term care. But the cost of long-term care may have increased substantially a lot more than what the borrower could have extracted when the loan was issued or even as a series of regular payments. This long-term care funding gap may require further innovation of ERPs. Similarly, if the loan is taken by a couple and one of them moves first into long-term care some considerable time after taking the loan, the “surviving” borrower may face increased costs of long-term care and additional costs for themselves at a time when the equity that was released may be already consumed. Hence, the idea of competing risks becomes very important. With time, we hope that there will be mortality tables and also morbidity tables that will allow market practitioners to calibrate their models. The NNEG risk then could be carved into an NNEG risk associated with mortality and an NNEG risk associated with morbidity. More sophisticated products may treat morbidity as a preliminary stage to mortality and allow for further equity releases if a borrower has to move into long-term care.

Also on financial products, the introduction of house price derivatives would help alleviate many difficult problems regarding NNEG valuations. If futures contracts were available then one could proceed with risk-neutral valuation using a Black model. All problems embedded in the house price dynamics would be absorbed into the futures contract and the market would collectively determine via the futures contract price the implied future valuations. There could still be problems with matching maturities every year, as needed for the NNEG calculations, but a viable market on house prices futures would most likely have to consider that issue.

A less researched aspect of ERPs is seeing these products as a vehicle for boosting pension income. We have not seen any study that looks even retrospectively what would have been the optimal scheme to release equity from the house to invest it into a mix of assets, say bonds and stocks for simplicity, to generate further cash-flows later on.

All these issues point to the many benefits that will result from starting a specialised independent financial risk management and innovation lab targeting equity release products. This could be easily established as an online entity but it will require some upfront investments from various interested parties to acquire data, software, computational power and human capital. It would be a very small investment compared to the many benefits it will produce for society ultimately.

10.3 DO YOU AGREE?

After doing a lot of research work for this book, we have come to some important conclusions. We do not know if you agree and we are not asking anyone to agree with us. If you are reading this book we assume you bought the book and this is fine with us, we appreciate very much your interest. Having said that, perhaps you would like to contribute to the debate and provide further evidence. Here is our list of conclusions that may be thought provoking.

Historical volatility of house prices looks much lower than some levels imposed more or less ad-hoc. Being prudent does not mean beefing up historical evidence. One related problem may be that we should also use conditional volatility models rather than models that cover volatility as a fixed parameter. The conditional volatility models do have the advantage that they can also generate extreme scenarios but keeping their frequency calibrated with reality.

A similar problem seems to be present when it comes to rental yield. This important quantity may also impact risk management calculations differently. If rental yield is very high then a model that constructs house prices as present values of future rental income would keep house prices higher *ceteris paribus*. However, if the model imitates a continuous-time model used for equities or foreign exchange rates then the rental yield may be taken out of the drift component. This in turn will act as a break on possible upwards future paths of house prices and in this case the higher the rental yields the less likely it is to see house prices surging upwards into the future.

If someone wants to take an armageddonic view they can do so via the stress testing. Hence, they should stress the parameters of their model. The stress testing framework should also include links to the main macroeconomic variables that may either directly impact future house prices, or the cost of long-term care or interest rate regimes or changes in mortality, etc.

There is very little that is said in academic literature, practitioners technical papers and PRA documents about how the roll-up rate is constructed. The only important notable exception is Hosty et al. (2008). A loan may get into negative equity issues not necessarily because house prices collapse but because the roll-up rate R may be too large so the interest on the loan accumulates at a rate that cannot be justified on economic grounds. This is an area where PRA can do a lot more.

Given current market conditions, in the absence of financial instruments that can be used to organise any proper hedging of the risk involved with the ERPs, it looks unappealing and possibly even theoretically incorrect to insist on risk-neutral valuation or as it is called in the insurance industry the market consistent valuation. Consistent with what we may ask? Therefore, we believe that using real-world measure calculations and taking a very traditional actuarial approach may have more merits than people think.

Many pitfalls in risk management for ERPs can be solved almost instantaneously if vanilla house price derivatives are introduced on the exchanges for maturities going up to 40-50 years. We find it strange that the largest by value spot market, houses and property more generally, does not have a fully functional associated derivatives market, while at the same time there is a lot of interest in crypto currencies. The increasing interest in something that is so intangible and non-fungible is baffling. Trading real-estate does have the problem of lack of granularity. Perhaps borrowing some of the technology around blockchain and putting it to good use to widen participation in sharing real-estate risk may help society avoid future problems. Tokenisation in real-estate space could help self-organisation of a very important market. People should realise that having a house is far more important than owning crypto. For speculative activities there are plenty of opportunities all around, maybe far too many, we do not need another tulipmania type of event.

You may have noticed that we do not insist in this summary chapter on any given model. Models are there to be used as tools. There is no technical profession requiring only one tool and it is evident that the more tools you have more things one can do. The experience and the rate of success in doing things will determine which tools are more useful and which tools are also very dangerous.

We are neither interested in being politically correct and support the line promoted by PRA before seeing the results, nor are we inclined to sacrifice reason over the debate with self appointed gurus. The following text is from Jeffery and Smith (2019) and it requires in our view some clarifications:

> “This research contained one rather curious argument. In assessing the deferment rate, some parties give consideration to the rental yield that could be obtained while ownership of the property is being deferred. Tunaru estimates that as 5% but then multiplies that number by 20% because only 20% of the residential property is rented. We see no

> justification for this. It is apparent from the discussion at Staple Inn that there was little support for it.
>
> His major conclusion is that a rather complex model "ARMA-EGARCH" is better than Geometric Brownian Motion for modelling house prices. We would not argue that this might incorporate features that have been observed in the market. However we are concerned that this may transpire to be an example of Burg Khalifa modelling (The Burg Khalifa is the tallest building in the world. If you jumped off the top of its 163 stories and measured, very quickly, what had happened to you after 162 stories, you would have a lot of data saying everything was going to be alright."

The project presented at Staple Inn under IFoA umbrella had two authors, the authors of this book.[1] Regarding the first paragraph about the rental yield it is still baffling how experts jump to conclusions regarding the method how to estimate the rental yield from a portfolio of properties. First of all, rental yields cash-flows are conceptualised in the literature by analogy with dividends for stocks. Thus, this is a flow concept and not a stock concept. As with dividends for companies that are the constituents of an index, if in any given year there are no dividends paid then the rental yield for the stock index will mark a zero for the dividends of the respective companies. This is not only an accounting equation but it is also directly and strongly related to income generated by the owners of companies stock and it does have tax implications. One cannot tax proxy income as if it is real income. Please recall that one cannot as far as we know and at this moment in time offset mortgage interest rate payments against annual income tax.

Furthermore, to suggest that you can extract rental income from any property is simply not true. Those nice retirement houses in villages where there is no train station, no postal office, no GP, no schools, and where the nearest food retailer outlet is 10 km away, are clearly extremely unlikely to be “rentable” in any shape or form. And we also insists that it is the net rental yield that matters. Our experience and calculations were strictly UK based and we got the data from official sources. We recognise at the same time that we do not know how things work in Ireland, because actually the ERM market there is not really at the point to support any detailed analysis.

The second paragraph is even more eclectic. If one jumps from Burj Khalifa (sic!) we truly hope that they know they will get the same outcome looking *down* 162 floors or 163 floors. Actually we like this aphorism since anyone looking at the picture of this renowned tower can observe that depending on the direction where the temerary may decide to jump, they may end up 163 floors down but also possible several dozens floors up. It is precisely this fact that the direction of evolution of your movement may lead you to very different levels, that needs to be taken into account. One may crash much sooner than 163 floors down. If one drops only 40-50 floors we still believe that the outcome will be still a crash. To conclude on this, we should leave the poetry to poets and we should make sure that we consider all possibilities in front of us as much as possible.

Last but not least, in an era where people read less and less books, if you are reading these lines, we would like you to know that we are very grateful that you gave your time to read our thoughts organised in this book. Nowadays it is easy to find our working whereabouts, and if you want to drop us a line with your views on the topics discussed here, ideas for collaborative work, examples we may not be aware of, or links to new product development or new regulations pertinent to ERP, that would be very much appreciated.

[1]It should be said that IFoA were the sponsors of the project but they wanted to state their independence regarding the results presented. We guess this is what they do normally and we thank them for the support they gave us. We also hope that this book will allow us to have a follow up presentation in the very near future.

FURTHER READING

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APPENDIX **A**

Appendix

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In this Appendix we present further results which hopefully will help interested researchers to appreciate the diversity of results that may appear based on different sample sizes, windows of estimation of past data and other analyses carried out by the authors at different points in time.

While this may cause some concern to some readers, our aim is to invite them to reflection. After all, the type of analysis carried out in this area of research is always subject to data availability, model risk and changes in methodological views.

In A.1 we present the annual percentage changes in rental prices. Those should not be confused with rental yields but they do carry some valuable information in that they can show the evolution of rental prices over time and across regions. Structural breaks could be an issue as well. There is a need for more data on rental prices and more studies investigating the connection with house prices and their determinants.

Section A.2 presents a suite of inference on house prices that was produced with shorter data. Comparing the estimates in this appendix with the corresponding values reported in the main text and obtained for longer series points out to substantial differences in parameter estimates at different points in time. This aspect of parameter estimation risk was one of the main themes of this book.

In section A.3 we discuss some simulated paths for the main important quantities driving the calculations for ERP. The figures and tables here complement those presented inside main text.

Further simulations of future house prices, under ARMA-EGARCH and GBM models and under risk-neutral measure and physical measure are illustrated in section A.4. The differences between the achieved prices under physical and risk-neutral prices are more visible here.

In section A.6 we report a set of results for portfolio simulations cash-flows that we generated in an earlier analysis to the start of this book. We believe that these results, using parameters estimated on a slightly shorter datasets, would provide a good comparison with the exercise presented inside text in Chapter 9.

Finally, in the last section A.5 we present some additional results on NNEG and ERP risk premia when there is variation in the LTV. There is less analysis and results reported to how risk evolves around variations in LTV in general, an area that we feel deserves more research.

A.1 RENTAL PRICES

In Figure A.1 we present the changes in rental prices in England and also in some of its regions. As expected, in good times London rental prices dominate the rental prices for other regions but after any shocks to the economy such as the global financial crisis or the covid crisis, there is a dip in rental prices in London. Furthermore, the increases observed post 2021 have not been observed in the past 20 years.

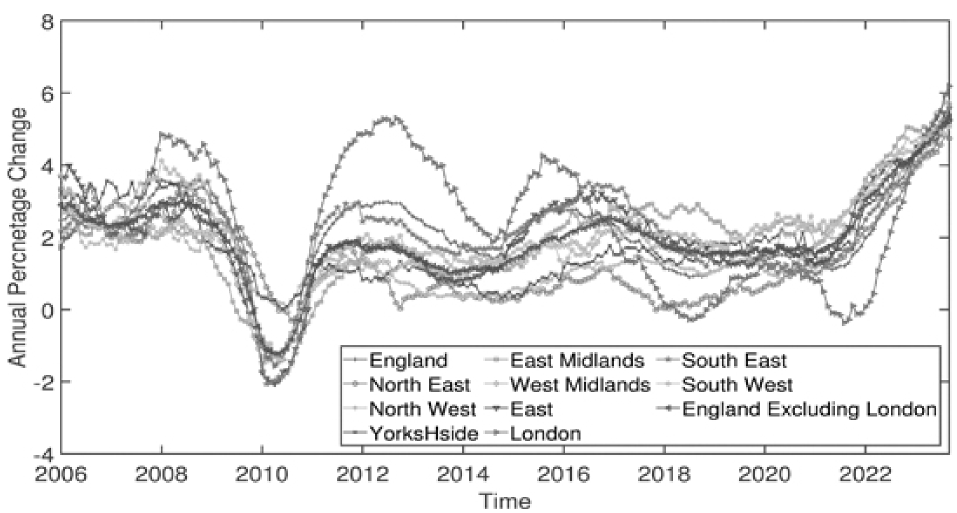


Figure A.1 Office for National Statistics (ONS) Experimental Index of Private Housing Rental prices.

Notes: The figure illustrates the time series evolution of the ONS experimental index of private housing rental prices (annual percentage) in the UK. The time series data is from January 2006 to September 2023. The ONS updates the private housing rental prices (IPHRP) date on a monthly basis with new monthly estimates. The time series data in this plot are indexed with January 2015 as base year. The plot we present here is based on time series data published by the ONS. *Source*: Source: Office for National Statistics (ONS), released 18 October 2023, ONS website, statistical bulletin, Index of Private Housing Rental Prices, UK: September 2023. https://www.ons.gov.uk/economy/inflationandpriceindices/bulletins/indexofprivatheousingrentalprices/september2023

One can also argue based on the graphs presented in this figure that there are problems with stationarity of rental yields return series, implying that they will be difficult to model.

A.2 NATIONWIDE UK, HOUSE PRICE SUMMARIES 1974-2018

Table A.1 shows the moment descriptive statistics of house price returns across different regions in the UK on a sample between 1974 and 2018. There could be significant variations from region to region. The remaining figures and tables here are showing a similar analysis with the one presented inside main text but for data ending in 2018, and hence just few years before the irruption of covid-19 pandemic. We believe that considering this period as well provides further insights to the changes that can occur at a short notice on housing markets.

TABLE A.1 Comparison of Nationwide house prices sample descriptive statistics (in %) across regions in the UK, 1974–2018. Source: Nationwide

Region	Mean (annualised %)	Skewness	Kurtosis	Volatility (annualised %)
North	6.94	2.13	1.38	9.73
Yorks The Humber	6.91	1.69	1.14	10.08
North West	7.38	1.86	1.23	9.64
East Mids	7.49	0.04	0.64	9.49

Region	Mean (annualised %)	Skewness	Kurtosis	Volatility (annualised %)
West Mids	7.31	0.51	0.89	9.10
East Anglia	7.80	0.96	0.54	10.63
Outer S East	8.10	0.31	0.19	10.13
Outer Met	8.28	-0.09	-0.01	9.59
London	8.92	0.09	0.17	10.08
South West	7.87	0.62	0.58	9.61
Wales	6.95	1.46	1.29	9.84
Scotland	6.69	-0.16	0.71	7.12
N Ireland	7.11	1.70	0.27	11.61
UK	7.51	-0.11	0.26	8.31

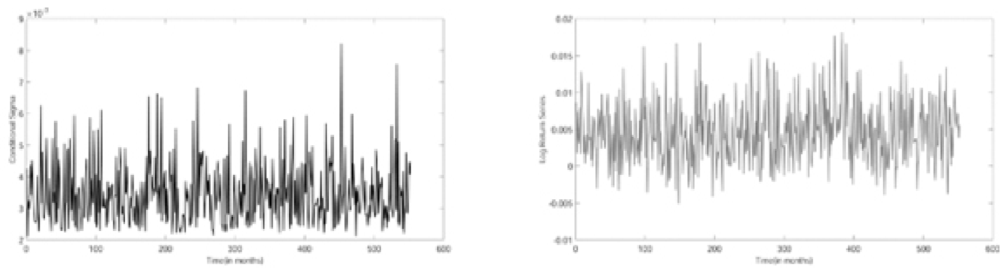
A.2.1 Estimates of the GBM Drift & Volatility Parameters with quarterly Nationwide UK data, 1974–2018

TABLE A.2 Estimation of annualised drift and volatility parameters from Nationwide quarterly time series 1974-2018 for the entire UK and also across regions, using three methods of estimation maximum likelihood estimation (MLE), method of moments (MM) and generalised method of moments (GMM).

Region	MLE		Method of Moments		GMM	
	μ	σ	μ	σ	μ	σ
Period 1974-2018						
North	6.48%	6.48%	6.69%	6.50%	4.62%	5.23%
Yorks The Humber	6.54%	6.32%	6.74%	6.34%	5.18%	5.65%
North West	6.92%	5.41%	7.06%	5.42%	5.26%	4.64%
East Mids	7.06%	5.91%	7.24%	5.93%	5.76%	4.98%
West Mids	6.92%	5.88%	7.10%	5.89%	5.50%	4.80%
East Anglia	7.27%	6.50%	7.48%	6.52%	6.25%	6.20%
Outer S East	7.47%	5.89%	7.65%	5.91%	6.87%	5.81%

Region	MLE		Method of Moments		GMM	
	μ	σ	μ	σ	μ	σ
Period 1974-2018						
Outer Met	7.69%	5.63%	7.84%	5.64%	7.49%	5.46%
London	8.29%	6.23%	8.49%	6.25%	7.96%	6.00%
South West	7.45%	5.69%	7.61%	5.70%	5.94%	5.12%
Wales	6.60%	6.44%	6.81%	6.45%	5.27%	5.67%
UK	7.07%	4.78%	7.19%	4.79%	6.03%	4.70%
Period covered in paper 1974-2006						
North	8.87%	6.83%	9.10%	6.86%	7.23%	5.82%
Yorks The Humber	8.68%	6.66%	8.91%	6.68%	7.23%	6.08%
North West	9.20%	5.48%	9.35%	5.51%	7.24%	4.60%
East Mids	8.96%	6.27%	9.16%	6.29%	7.36%	5.17%
West Mids	8.84%	6.26%	9.03%	6.28%	7.13%	5.10%
East Anglia	9.00%	6.87%	9.24%	6.89%	7.92%	6.52%
Outer S East	9.11%	6.15%	9.29%	6.17%	8.35%	6.07%
Outer Met	9.12%	5.75%	9.29%	5.78%	8.66%	5.63%
London	9.51%	6.30%	9.71%	6.32%	9.05%	6.10%
South West	9.33%	5.91%	9.51%	5.94%	7.29%	5.16%
Wales	8.82%	6.58%	9.04%	6.60%	7.03%	5.53%
UK	8.91%	4.78%	9.02%	4.80%	6.97%	4.41%
Period 2007-2018						
North	-0.22%	3.71%	-0.15%	3.75%	0.56%	3.33%
Yorks The Humber	0.51%	3.92%	0.59%	3.96%	2.15%	3.08%
North West	0.48%	3.58%	0.54%	3.62%	2.18%	2.81%
East Mids	1.75%	3.64%	1.82%	3.68%	3.89%	2.18%
West Mids	1.55%	3.41%	1.62%	3.45%	3.62%	2.29%
East Anglia	2.39%	4.54%	2.49%	4.59%	4.43%	2.95%
Outer S East	2.91%	4.35%	3.00%	4.41%	4.76%	2.90%
Outer Met	3.65%	4.69%	3.77%	4.74%	5.39%	3.59%

Region	MLE		Method of Moments		GMM	
	μ	σ	μ	σ	μ	σ
Period 1974-2018						
London	4.89%	5.70%	5.05%	5.76%	5.92%	4.76%
South West	2.15%	3.93%	2.22%	3.98%	3.88%	2.91%
Wales	0.38%	4.80%	0.50%	4.86%	1.70%	3.03%
UK	1.95%	3.72%	2.02%	3.76%	4.10%	2.58%



(a) Conditional volatility

(b) Log-returns

Figure A.2 Simulated paths for the conditional volatilities and conditional returns under the ARMA(4,3)-EGARCH(1,1) model for 45×12 months ahead.

A.2.2 Estimating ARMA(4,3)-EGARCH(1,1) with monthly Nationwide UK data, 1991–2020

TABLE A.3 Parameters estimates for the ARMA(4,3)-EGARCH(1,1) model over the monthly Nationwide house price time series between Jan 1991 and Dec 2022.

Parameter	Estimate	Std. Error	t-Stat	p-Value
c	0.0196	0.0025	7.8174	0.0000
ϕ_1	0.3543	0.0277	12.7967	0.0000
ϕ_2	-0.1517	0.0182	-8.3276	0.0000
ϕ_3	0.0802	0.0074	10.8261	0.0000
ϕ_4	0.3274	0.0280	11.7062	0.0000
θ_1	0.0453	0.0059	7.6161	0.0000

Parameter	Estimate	Std. Error	t-Stat	p-Value
θ_2	0.1838	0.0195	9.4265	0.0000
θ_3	0.3059	0.0374	8.1843	0.0000
k	-0.4270	0.0930	-4.5935	0.0000
α_1	0.2617	0.0453	5.7719	0.0000
β_1	0.9497	0.0098	97.2610	0.0000
γ_1	0.2177	0.0461	4.7217	0.0000

A.2.3 Forecast Performance GBM vs ARMA-EGARCH

In Figure A.4, we redo the same analysis for the forecasting error for the out-of-sample Nationwide monthly time series with four years out of sample. Now, the ARMA(4,3)-EGARCH(1,1) outperforms the GBM house price forecasting. Moreover, now the MLE estimates for GBM dominates the MLE and GMM method, confirming that there is substantial parameter estimation risk even for such a simple model as GBM.

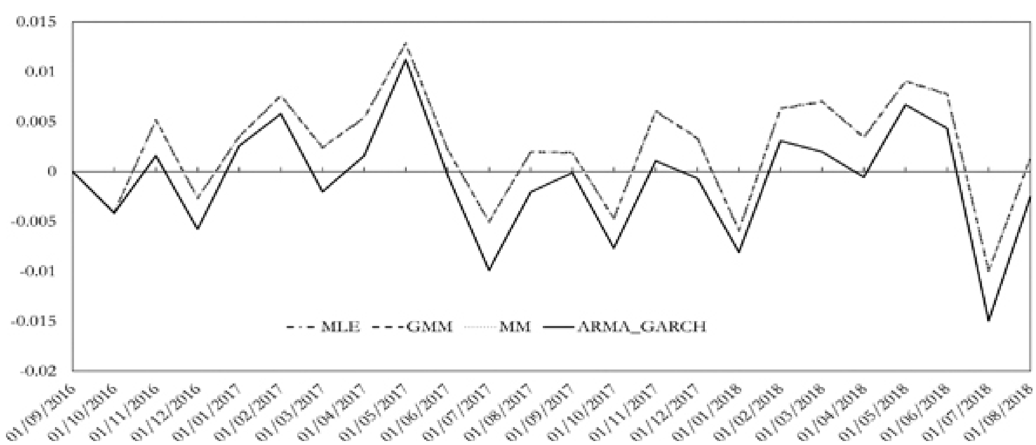


Figure A.3 Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

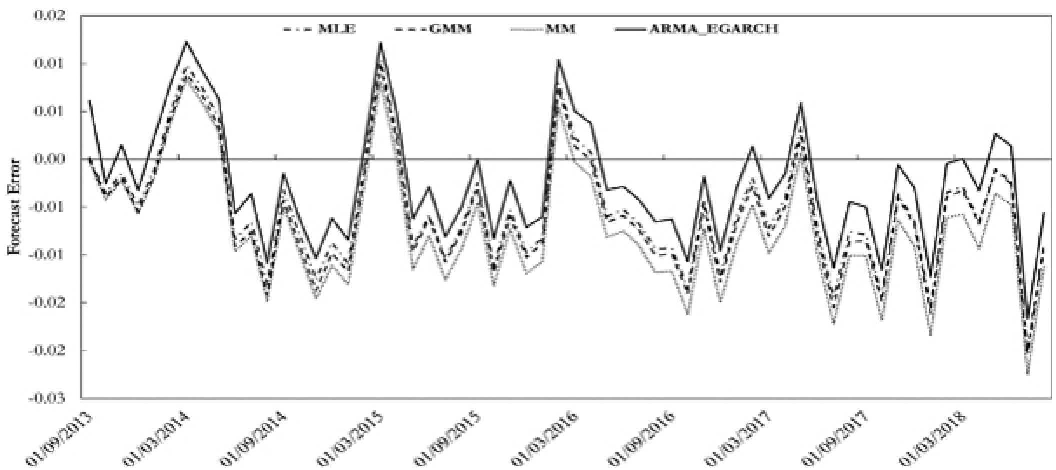


Figure A.4 Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide Average House Price Monthly (non-seasonally adjusted) for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

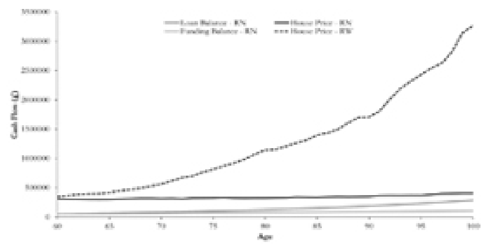
TABLE A.4 Parameter estimates for the GBM process applied to the monthly Nationwide UK house price index, between Jan 1991 and Sep 2020 and Halifax Monthly Jan 1983–Dec 2014

	Nationwide		Halifax	
Method of Estimation	Drift	Volatility	Drift	Volatility
Maximum Likelihood (MLE)	5.37%	3.64%	5.80%	3.96%
Generalised Method of Moments (GMM)	2.86%	3.26%	6.45%	2.27%
Method of Moment (MM)	5.36%	3.41%	5.88%	3.96%

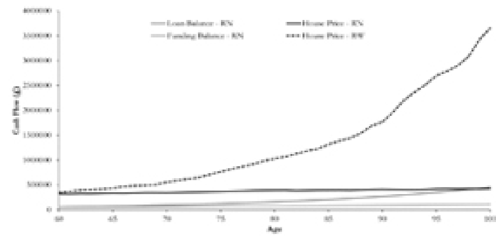
A.3 EQUITY RELEASE MORTGAGE CASH FLOWS

This section presents results on the analysis of UK equity release mortgage cash flows under different parameter values.

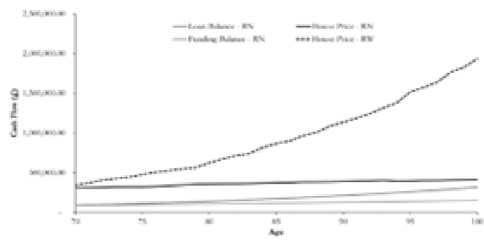
For real-world pricing practitioners use a risk-premium that is usually determined exogenously. The graphs in Figure A.6 illustrate the relative evolution of various cash-flows defining an ERM for the ARMA-EGARCH model we fitted to the Nationwide data. The house price pathways are described for the fifth Monte Carlo simulated path, under the real-world measure (where the drift and volatility parameters change monthly) and also under risk-neutral measure (where the volatility changes monthly) while the funding balance and loan balance evolution are model independent and change only with respect to the driving interest rate. In the scenarios illustrated in Figure A.6 there is very little risk for an NNEG to be in the money.



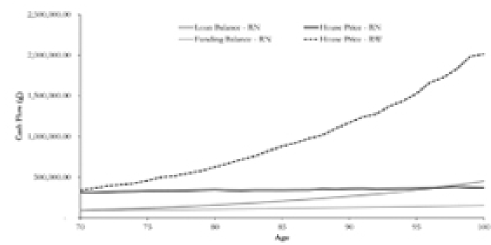
(a) Age 60 and $R = 4.15\%$



(b) Age 60 and $R = 5.25\%$

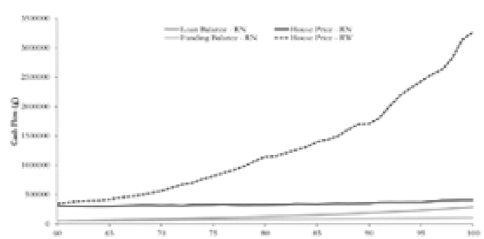


(c) Age 70 and $R = 4.15\%$

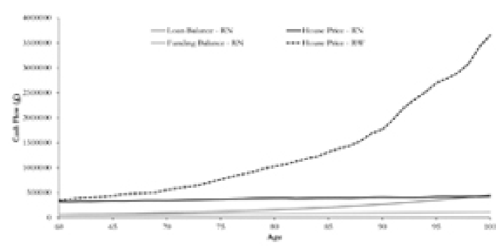


(d) Age 70 and $R = 5.25\%$

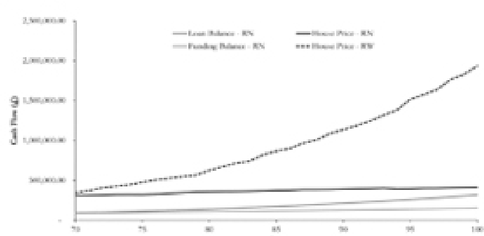
Figure A.5 Cash-flow paths for funding balance, loan balance, and the fifth Monte Carlo simulated pathway for house prices risk-neutral and real-world, under baseline scenario $r = 1.75\%, g = 1\%, \sigma = 3.26\%$.



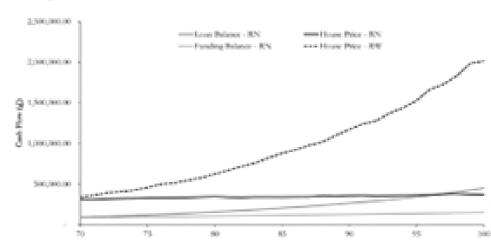
(a) Age 60 and $R = 4.15\%$



(b) Age 60 and $R = 5.25\%$



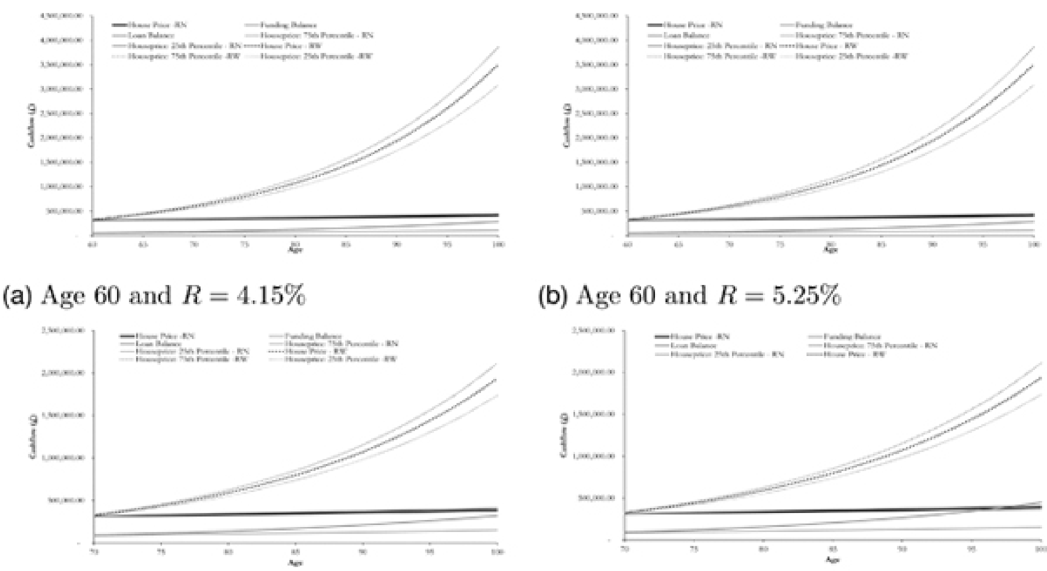
(c) Age 70 and $R = 4.15\%$



(d) Age 70 and $R = 5.25\%$

[Figure A.6] Cash-flow paths for funding balance, loan balance, and the fifth Monte Carlo simulated pathway for house prices risk-neutral and real-world, under baseline scenario $r = 1.75\%, g = 1\%, \sigma = 3.26\%$.

One may wonder whether the selected pathway analysis described above is representative for the entire sample of pathways simulated with Monte Carlo. In the scenarios illustrated in Figure A.7 there is very little risk for an in-the-money NNEG, similar to one pathway scenario. This risk occurs for a higher roll-up rate and for a 70-year-old borrower, if he/she lives close to 100 years.



(a) Age 60 and $R = 4.15\%$

(b) Age 60 and $R = 5.25\%$

(c) Age 70 and $R = 4.15\%$

(d) Age 70 and $R = 5.25\%$

Figure A.7 Cash-flow paths for funding balance, loan balance, and the average and 25% and 75% quantiles for Monte Carlo simulated pathway for house prices risk-neutral and real-world, under baseline scenario $r = 1.75\%, g = 1\%, \sigma = 3.26\%$.

TABLE A.5 Comparing GBM model under different estimation methods with the selected ARMA-EGARCH model with Diebold Mariano test over the out-sample of 24 months (Oct 2016–Sep 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0858	0.0834	
GBM-GMM	0.0428	0.0405	

MODEL	RMSE	MAE	
GBM-MM	0.0151	0.0126	
ARMA(4,3)-EGARCH(1,1)	0.028	0.021	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	**GBM-GMM**	10.4005	0.0000
GBM-MLE	**GBM-MM**	7.4334	0.0000
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	10.10	0.0000
GBM-GMM	GBM-MM	-7.9596	0.0000
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	7.0858	0.0000
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	8.2148	0.0000

TABLE A.6 Comparing forecasting (monthly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample of 60 months (Oct 2012–Sep 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0079	0.0067	
GBM-GMM	0.0081	0.0069	
GBM-MM	0.0090	0.0078	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0051	

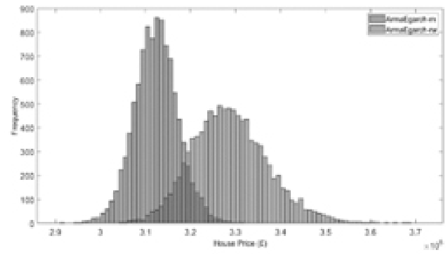
Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-3.9838	0.0002
GBM-MLE	GBM-MM	-6.7823	0.0000

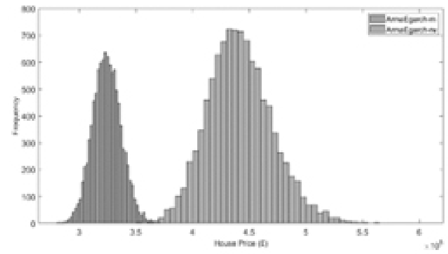
MODEL	RMSE	MAE	
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	3.7681	0.0004
GBM-GMM	GBM-MM	-6.0371	0.0000
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	3.9739	0.0002
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	4.8545	0.0000

Furthermore, there is a clear separation region between the real-world evolution and the risk-neutral evolution, suggesting that the insurers using a real-world valuation mechanism may "benefit" from higher house price values expected in the future while the insurers employing risk-neutral methods have a more conservative view on house prices, under the same market parameters. The situation may be reversed if the volatility is increased and/or the risk-neutral drift exceeds the real-world drift. If the house prices go on a downward spiralling trend, with the same volatility, the risk-neutral drift stays the same while the real-world simulate pathways will point downwards.

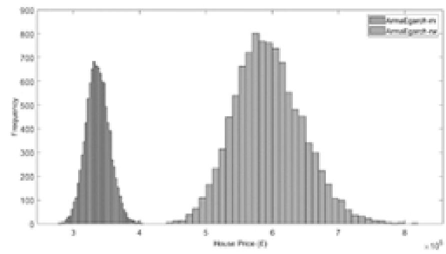
A.4 ON THE DISTRIBUTION OF PROJECTED HOUSE PRICE PATHS



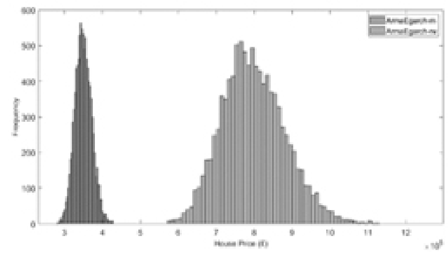
(a) Age 60



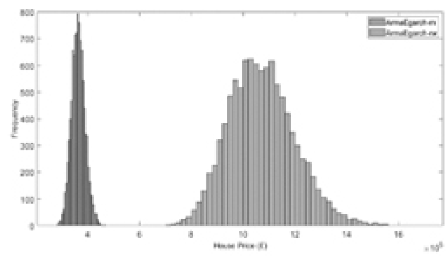
(b) Age 65



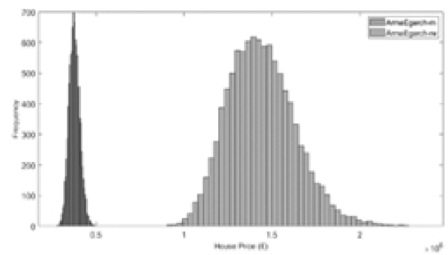
(c) Age 70



(d) Age 75



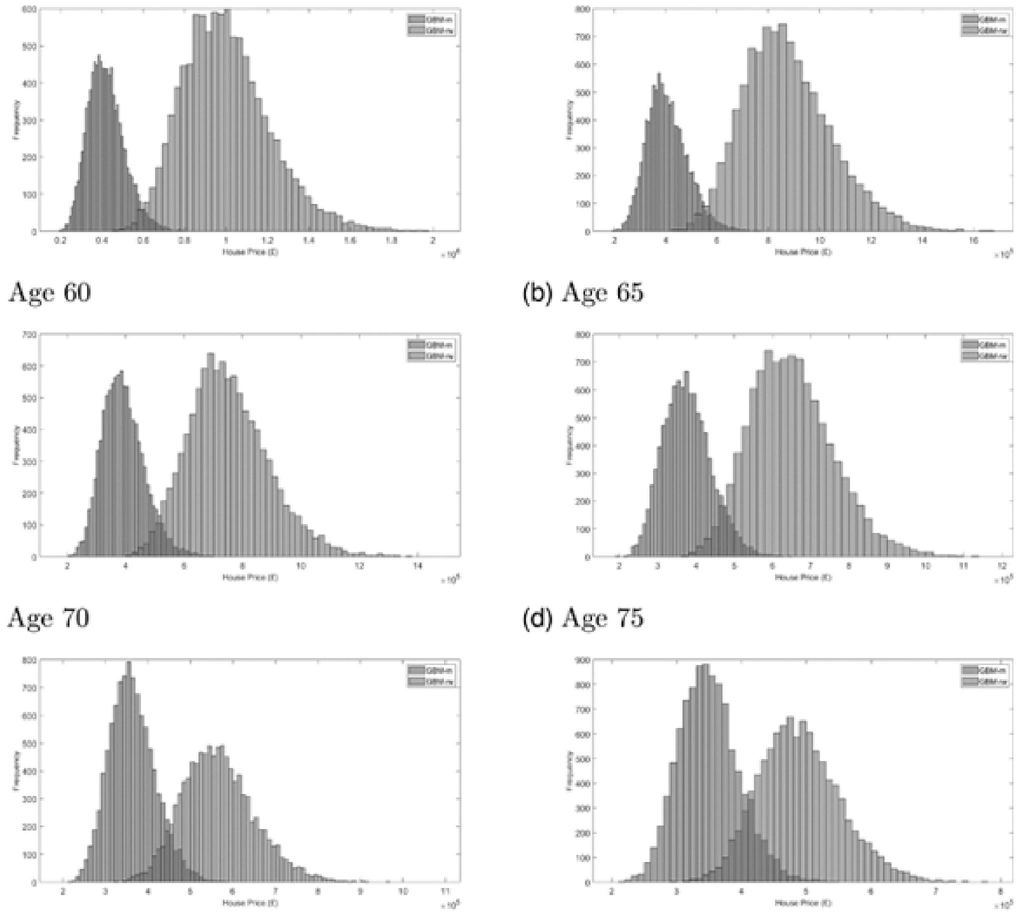
(e) Age 80



(f) Age 85

Age	60	65	70	75	80	85	90
Risk Premium (GBP)	16,500	117,468	257,675	448,959	708,740	1,063,876	1,545,548

Figure A.8 Distribution of projected house prices for UK under real-world ARMA-EGARCH and risk-neutral ARMA-EGARCH, under baseline scenario $r = 1.75\%, R = 4.15\% g = 1\%$.



(a) Age 60 (b) Age 65

(c) Age 70 (d) Age 75

(e) Age 80 (f) Age 85

Age	60	65	70	75	80	85	90
Risk Premium (GBP)	580,511	465,699	360,184	273,150	202,906	140,027	88,421

Figure A.9 Distribution of projected house prices for UK under real-world GBM and risk-neutral GBM, under baseline scenario $r = 1.75\%, R = 4.15\%, g = 1\%$.

A.5 NNEG AND ERP RISK PREMIUM

The NNEG cash intrinsic values defined as the maximum between the difference between the accumulated loan and the value of the house price at the respective time will reverse the order of distributions. A similar analysis is carried out for the GBM model, with differences calculated at future time horizons defined by borrower's age 60, 65,70,75 and 80, 85 respectively from the point of view of a 55-year-old. The two distributions being closer this time round the risk premia implied are lower than in the case of ARMA-EGARCH model. Under the GBM model the difference between the risk-neutral and real-world values simulated reflects only the parameter estimation uncertainty of inference between monthly and quarterly data. Recall that the quarterly series is historically longer than the monthly series. In addition, if the same estimates are obtained for drift and volatility for the GBM model then the same simulations would be obtained under monthly and quarterly steps. This follows from the theoretical properties of the geometric Brownian motion.

The NNEG risk premium induced by the house price valuations is calculated as the difference between the risk-neutral and physical NNEG price:

$$NNEGRP_{0,1} = \Pi^{\mathbb{Q}}_{0,1}\left(\tau_M\right) - \Pi^{\mathbb{P}}_{0,1}\left(\tau_M\right) \tag{A.1}$$

where $NNEGRP_{0,1}$ is the house price risk premium induced by the NNEG clause and $\Pi^{\mathbb{Q}}_{0,1}\left(\tau_M\right)$ is the NNEG price at inception i.e. at time (0,1) for a contract with maturity τ_M.

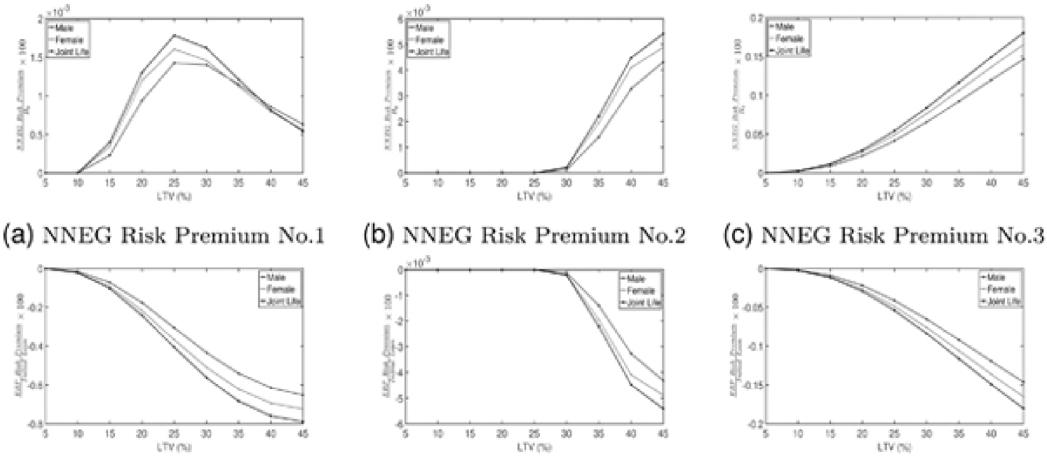
The ERP loan risk premium is defined as the difference between the risk-neutral and physical ERP price:

$$ERPRP_{0,1} = V^{\mathbb{Q}}_{0,1}\left(\tau_M\right) - V^{\mathbb{P}}_{0,1}\left(\tau_M\right) \tag{A.2}$$

where $ERPRP_{0,1}$ is the time zero hour price risk premium. $\mathbb{Q}$ is the risk-neutral measure, and $\mathbb{P}$ is the physical measure. $V^{\mathbb{Q}}_{0,1}\left(\tau_M\right)$ is the value of the ERM loan at inception under the risk-neutral measure introduced in Equation 6.53. The value of the ERM loan under the physical measure is denoted $V^{\mathbb{P}}_{0,1}\left(\tau_M\right)$.

From the way we have defined $V_{0,1}\left(\tau_M\right)$ in Equation (6.53), it is easy to see that the ERP loan risk premium can be expressed as a function of the NNEG risk premium. More specifically, this expression is the difference between $\Pi^{\mathbb{P}}_{0,1}\left(\tau_M\right)$ and $\Pi^{\mathbb{Q}}_{0,1}\left(\tau_M\right)$. The plots in Figure A.10 illustrate the characteristics of the risk-premium to initial loan ratio, and we present calculations for both the NNEG and ERP risk premiums using different initial loan values. The risk premium is calculated with Equations (A.1) and (A.2) using parameter values provided for each numbered scenario described in Table

1.1. The plots in Figure A.10 are specifically based the Black-Scholes model using a rental yield value of 1%, $H_0 = 1$, and LTVs $1\% \leq \Gamma \leq 45\%$. In fact, it is striking to observe in Figures A.10 and A.11 that the NNEG risk premium to the initial loan ratio, denoted as $(NNEGRP \div L_0)$, consistently mirrors the plot corresponding to $(ERPRP \div L_0)$. This compelling symmetry underscores the dynamics of ERMs with a fixed roll-up interest rate, highlighting a robust pattern in financial modelling. The outcome would be different when the roll-up interest rate is variable in the contract design.

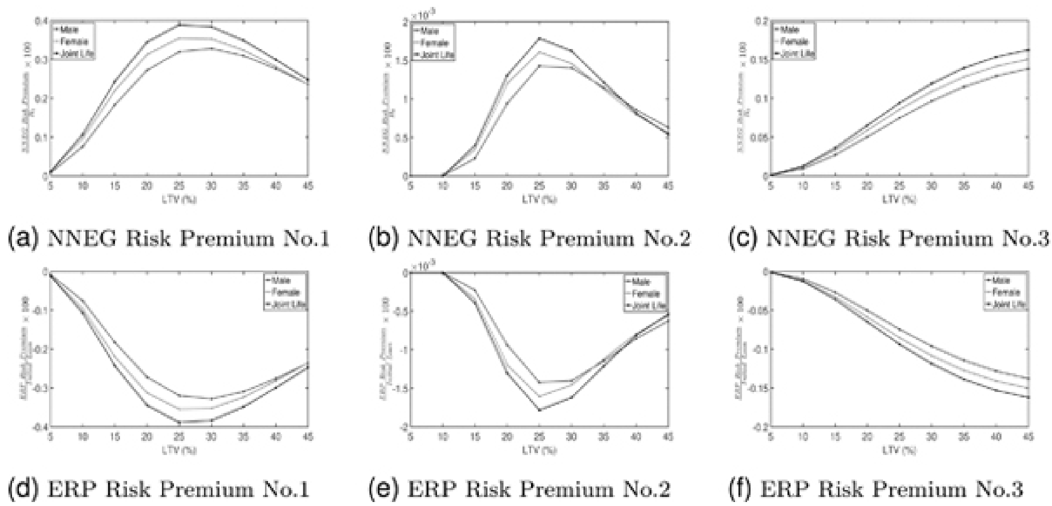


(a) NNEG Risk Premium No.1 (b) NNEG Risk Premium No.2 (c) NNEG Risk Premium No.3

(d) ERP Risk Premium No.1 (e) ERP Risk Premium No.2 (f) ERP Risk Premium No.3

Figure A.10 GBM risk premiums relative to initial loan when rental yield $g = 1\%$

Notes: This Figure illustrates the risk premium calculations under GBM pricing model. Calculations presented in subplots respectively depict the NNEG and ERP risk premium dynamics when calculated with Equations (8.9) and (8.10). The numbers in each subcaption i.e. (1), (2), and (3) respectively denote calculations using parameter values in Table 1.1. The borrower is 65 years in all instances. We calculate the risk premium at each given loan-to-value ratio $1\% \leq \Gamma \leq 45\%$. The initial house price (H_0) is set to 1 and calculations are based on the Black-Scholes model using a rental yield value (g) of 1%.



(a) NNEG Risk Premium No.1 (b) NNEG Risk Premium No.2 (c) NNEG Risk Premium No.3

(d) ERP Risk Premium No.1 (e) ERP Risk Premium No.2 (f) ERP Risk Premium No.3

Figure A.11 GBM risk premiums relative to initial loan when rental yield $g = 3.3\%$

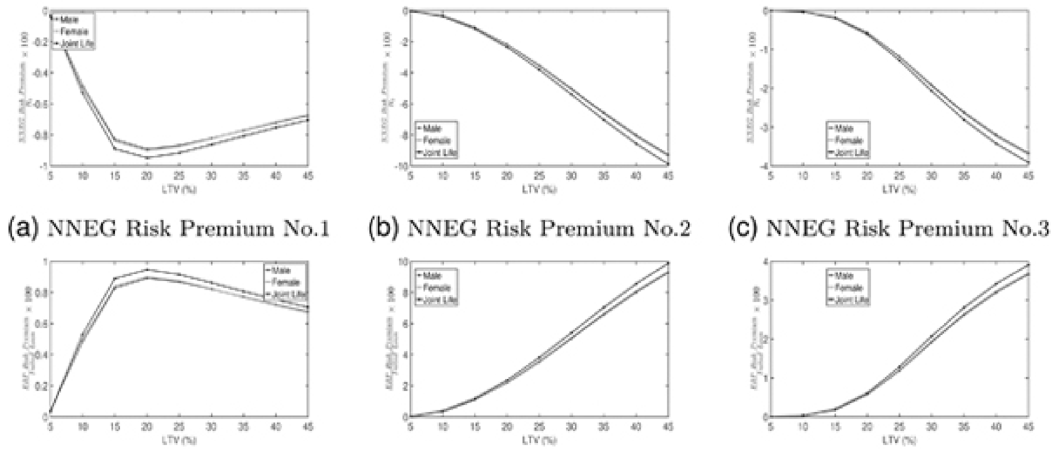
Notes: This Figure illustrates the risk premium calculations under GBM pricing model. Calculations presented in subplots respectively depict the NNEG and ERP risk premium dynamics when calculated with Equations (8.9) and (8.10). The numbers in each subcaption i.e. (1), (2), and (3) respectively denote calculations using parameter values in Table 1.1. The borrower is 65 years in all instances. We calculate the risk premium at each given loan-to-value ratio $1\% \leq \Gamma \leq 45\%$. The initial house price (H_0) is set to 1 and calculations are based on the Black-Scholes model using a rental yield value (g) of 3.3%.

There are some interesting features to note about how the risk premium changes between the three scenarios. When the initial borrower age is 65 years, the NNEG risk premium is consistently positive, but lower for males when compared to corresponding calculations for the case of females and joint life borrower. Let us delve into the intricate relationship between the risk-premium, Loan-to-Value (LTV) ratio, and the differential between the roll-up (R) and risk-free (r) rates of interest. In the first scenario, the disparity in rates, denoted by $(R - r)$, is notably pronounced, manifesting as $6\% - 0.5\% = 5.50\%$. This contrast stands in stark relief to the more subdued spreads of 1.95% and 2.50% observed in Scenarios 2 and 3, respectively. Such expansive spreads catalyse the activation of the NNEG risk premium when the LTV plunges below 10% for borrowers who are over 65 years of age at the inception of the loan agreement. This observation infers that loan contracts involving borrowers aged between 55 to 65 years will also experience positive risk premia, particularly when the LTV descends further below the 10% threshold. The presence of a positive NNEG risk premium vividly underscores the expectation that investors will demand substantial compensation for shouldering the NNEG risk. The range of values of the NNEG risk premium relative to the initial loan clearly increases with the disparity between the roll-up and risk-free interest rates. This relationship is positive; i.e., a broader spread generates a more extensive range of risk premiums, whereas narrower spreads constrict it.

The rental yield value also impacts the risk-premium dynamics in different LTV ratios. In Figure A.11, we redo the NNEG and ERP risk premium calculations using a rental yield value (g) set to 3.3%. The NNEG risk premium kicks in and peaks earlier this time around across all ranges of LTVs. This observation is consistent across all three scenarios. There is a convergence in the risk premium values for male, female, and joint life contracts at higher LTV ratios. A particularly interesting observation is the way the risk premiums converge to the same value in Scenarios 1 and 2, which both respectively explore extremely high and low spreads between the risk-free rate of interest and the roll-up interest rate. As noted earlier, these notable features will kick in earlier for younger borrowers. Essentially, the results seem to suggest that the risk premium of the contract is independent of the age and gender of the borrower, when we encounter design-specific features like high rental yield coupled with economic conditions that result in either extremely low or extremely high disparity between roll-up and risk-free interest rates.

Overall, the positive risk premiums implied by the GBM pricing model suggest that the NNEG clause in the UK equity release scheme carries some level of risk that investors will demand additional returns to compensate for. There is therefore a risk-return trade-off to observe when valuation of the NNEG is discussed and valued in a Black-Scholes framework. It is very important to note that this risk premium calculation exercise is done on a loan-by-loan basis. It would be interesting to explore the same in a portfolio of loans. This exercise will help determine diversification benefits when dealing with the NNEG risk in a large portfolio.

We repeat similar analysis in the ARMA-EGARCH case, with results illustrated in Figure A.12.



(a) NNEG Risk Premium No.1 (b) NNEG Risk Premium No.2 (c) NNEG Risk Premium No.3

(d) ERP Risk Premium No.1 (e) ERP Risk Premium No.2 (f) ERP Risk Premium No.3

Figure A.12 ARMA-EGARCH risk premiums relative to initial loan when rental yield $g = 1\%$

Notes: This Figure illustrates the risk premium calculations under ARMA-EGARCH pricing model. Calculations presented in subplots respectively depict the NNEG and ERP risk premium dynamics when calculated with Equations (8.9) and (8.10). The numbers in each subcaption i.e. (1), (2), and (3) respectively denote calculations using parameter values in Table 1.1. The borrower is 65 years in all instances. We calculate the risk premium at each given loan-to-value ratio $1\% \leq \Gamma \leq 45\%$. The initial house price (H_0) is set to 1 and calculations are based on the ARMA(4,4)-EGARCH(1,1) model using a rental yield value (g) of 1%.

A.6 PORTFOLIO CASH FLOWS

Table A.7 reports the sensitivity analysis for NNEG calculations for a joint couple loan in the UK at the end of 2019 first quarter. Panel A shows the results under GBM model preferred by the regulator while Panel B shows the corresponding results, for the same scenarios, under the ARMA-EGARCH model identified as a suitable model for forecasting well house prices in the UK. Similar results are reported in the Appendix, for the single female borrower and single male borrower.

TABLE A.7 Non-negative equity guarantee sensitivity analysis for joint couple borrower from a baseline flexible LTV in the UK.

Age	60	65	70	75	80	85	90
LTV	17.0%	22.5%	28.5%	32.4%	36.5%	41.5%	41.5%
Initial Loan	65,100	82,150	102,300	114,700	130,200	145,700	145,700

Notes: We report the NNEG cost as a percentage of initial cash advanced to borrower.

Age	60	65	70	75	80	85	90
Panel A: Joint Life - GBM							
Baseline							
g=1%,σ=4.88%	12.02	12.21	9.96	4.19	1.38	0.37	0.01
R=5.25%, r=1.75%							
Rental yield							
g = 0.5% ($\downarrow 0.5\%$)	4.22	4.98	4.50	1.86	0.60	0.16	0.00
g = 2.5% ($\uparrow 1.5\%$)	86.52	69.04	48.54	22.53	8.45	2.56	0.08
g = 4.0% ($\uparrow 3\%$)	220.23	164.48	112.64	58.13	24.97	8.86	0.58
House price volatility							
$\sigma = 2\%(\downarrow 2.88\%)$	1.80	2.80	2.69	0.67	0.08	0.00	0.00
$\sigma = 8\%(\uparrow 3.12\%)$	35.42	31.34	24.21	12.29	5.35	2.10	0.20
$\sigma = 13\%(\uparrow 8.12\%)$	92.06	75.36	56.73	33.03	17.29	8.42	1.88
Risk-free rate							
r = 0.75% ($\downarrow 1.00\%$)	73.15	58.78	40.74	17.53	6.03	1.66	0.04
r = 1.25% ($\downarrow 0.5\%$)	32.42	28.83	21.42	9.08	3.04	0.82	0.01
r = 2.50% ($\uparrow 0.75\%$)	1.81	2.41	2.39	1.02	0.33	0.09	0.00
Roll-up rate							
R = 3.50% ($\downarrow 1.75\%$)	0.07	0.14	0.21	0.10	0.03	0.01	0.00
R = 6.15% ($\uparrow 0.90\%$)	59.43	48.97	34.53	14.78	5.04	1.38	0.03
Panel B: Joint Life - ARMA-EGARCH							
Baseline							
g=1%,σ=4.88%	0.18	0.93	1.20	0.06	0.00	0.00	0.00
R=5.25%, r=1.75%							

Notes: We report the NNEG cost as a percentage of initial cash advanced to borrower.

Age	60	65	70	75	80	85	90
Rental yield							
g = 0.5% ($\downarrow 0.5\%$)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
g = 2.5% ($\uparrow 1.5\%$)	62.80	51.71	35.93	14.10	3.76	0.36	0.00
g = 4.0% ($\uparrow 3\%$)	207.78	154.51	104.21	51.07	19.95	5.76	0.00
House price volatility							
$\sigma = 2\%(\downarrow 2.88\%)$	0.18	0.92	1.20	0.06	0.00	0.00	0.00
$\sigma = 8\%(\uparrow 3.12\%)$	0.18	0.94	1.20	0.06	0.00	0.00	0.00
$\sigma = 13\%(\uparrow 8.12\%)$	0.18	0.96	1.21	0.06	0.00	0.00	0.00
Risk-free rate							
r = 0.75% ($\downarrow 1.00\%$)	39.50	34.92	24.50	7.92	1.42	0.00	0.00
r = 1.25% ($\downarrow 0.5\%$)	8.53	10.10	8.12	1.99	0.09	0.00	0.00
r = 2.50% ($\uparrow 0.75\%$)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Roll-up rate							
R = 3.50% ($\downarrow 1.75\%$)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R = 6.15% ($\uparrow 0.90\%$)	28.00	26.25	18.75	5.90	0.86	0.00	0.00

Notes: We report the NNEG cost as a percentage of initial cash advanced to borrower.

The baseline scenario comparison across the two models indicates an over-conservative NNEG valuation for the GBM model. The main reason for that is that the variance of house price returns increases linearly with time. The NNEG has an almost insignificant weight for near-term maturities given that mortality probabilities are increasing to more significant levels towards long-term maturities. Furthermore, in the first few years, the relative low LTV ratio protects the accumulated value of the loan and so there is very little risk that the loan will not be repaid if it comes due. The NNEG risk manifests after the accumulated balance has had sufficient time to increase and, at the same time, the house prices had time to experience a crash or market correction. On the other hand, for the ARMA-EGARCH model the variance of house price returns does not increase linearly with time.

A decrease in the rental yield parameter g will decrease substantially the NNEG values while an increase in g will determine very large values for NNEG. A similar effect occurs for the changes in the volatility, smaller volatility leads to smaller NNEG and larger volatility means larger NNEG values. The risk-free rate r should be assessed vis-a-vis g, since the market NNEG valuation depends on the drift $r - g$ combination. Hence, the effect of changing r is the opposite of g. When r decreases the NNEG increases, and when r increases the NNEG value decreases. The roll-up rate R has the expected effect, decreasing the NNEG value when R decreases and increasing the NNEG value when R is increasing. Overall, g and σ are the parameters inducing the highest sensitivity to the NNEG value. Their estimation is therefore a very important exercise.

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Index